PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXXXVI.

PART I.

LONDON:

PRINTED BY RICHARD TAYLOR, RED LION COURT, FLEET STREET.

MDCCCXXXVI.

ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume: the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body,

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upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

A List of Public Institutions and Individuals, entitled to receive a copy of the Philosophical Transactions of each year, on making application for the same directly or through their respective agents, within five years of the date of publication.

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The Royal Academy of Sciences at Stockholm.
The Royal Society of Sciences at Upsal.
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Dr. William Olbers, of Bremen.
The Dépôt de la Marine, Paris.
The Bowden College, United States.
The Waterville College, United States.

ROYAL MEDALS.

HIS MAJESTY KING WILLIAM THE FOURTH, in restoring the Foundation of the Royal Medals, graciously Commanded a Letter, of which the following is an extract, to be addressed to the Royal Society, through His Royal Highness the DUKE OF SUSSEX, K.G., President:

"Windsor Castle, March 25, 1833.

"It is HIS MAJESTY'S wish,—

"First, That the Two Gold Medals, value of Fifty Guineas each, shall "henceforth be awarded on the day of the Anniversary Meeting of the "Royal Society, on each ensuing year, for the most important discoveries "in any one principal subject or branch of knowledge.

"Secondly, That the subject matter of inquiry shall be previously settled "and propounded by the Council of the Royal Society, three years pre- "ceding the day of such award.

"Thirdly, That Literary Men of all nations shall be invited to afford the "aid of their talents and research: and,

"Fourthly, That for the ensuing three successive years, the said Two "Medals shall be awarded to such important discoveries, or series of in- "vestigations, as shall be sufficiently established, or completed to the "satisfaction of the Council, within the last five years of the days of award, "for the years 1834 and 1835, including the present year, and for which "the Author shall not have previously received an honorary reward.

(Signed) "H. TAYLOR."

The Royal Medals for the year 1833 were awarded to

SIR JOHN FREDERICK WILLIAM HERSCHEL, K.H. F.R.S.,

for his Paper on the Investigation of the Orbits of Revolving Double Stars; and to

PROFESSOR AUGUSTE PYRAME DE CANDOLLE, of Geneva, Foreign Member of the Royal Society,

for his Discoveries and Investigations in Vegetable Physiology.

Those for 1834 were awarded to

JOHN WILLIAM LUBBOCK, ESQ., V.P. & TREAS. R.S.,

for his Papers on the Tides published in the Philosophical Transactions; and to

CHARLES LYELL, ESQ.,

for his Work entitled "Principles of Geology."

Those for the present year have been awarded to

MICHAEL FARADAY, D.C.L.,

for his Investigations and Discoveries contained in the Series of Experimental Researches in Electricity, published in the Philosophical Transactions, and more particularly for the Seventh Series, relating to the definite nature of electro-chemical action; and to

SIR WILLIAM ROWAN HAMILTON, Andrews' Professor of Astronomy in the University of Dublin, and Royal Astronomer of Ireland,

for the Papers published by him in the 16th and 17th volumes of the Transactions of the Royal Irish Academy, entitled "Supplement to an Essay on the Theory of "Systems of Rays," and more particularly for those Investigations at the conclusion of the third and last Supplement, which relate to the discovery of Conical Refraction.

The Council propose to give one of the Royal Medals in the year 1836, to the most important unpublished paper in Astronomy, communicated to the Royal Society for insertion in their Transactions, after the present date (May 13th, 1833,) and prior to the month of June in the year 1836.

The Council also propose to give one of the Royal Medals in the year 1836 to the

most important unpublished paper in Animal Physiology, communicated to the Royal Society for insertion in their Transactions, after the present date (May 13th, 1833,) and prior to the month of June in the year 1836.

The Council propose to give one of the Royal Medals in the year 1837 to the most important unpublished paper in Physics, communicated to the Royal Society for insertion in their Transactions, after the present date (November 27th, 1834,) and prior to the month of June in that year.

The Council also propose to give one of the Royal Medals in the year 1837 to the author of the best paper, to be entitled "Contributions towards a System of Geo-"logical Chronology founded on an examination of fossil remains, and their attendant "phenomena," such paper to be communicated to the Royal Society after the present date (December 1st, 1834,) and prior to the month of June 1837:—but in case no paper is presented to the Society fulfilling the conditions implied by the above Resolution, or possessing sufficient merit, the Council propose to give one of the Royal Medals in the year 1837 to the author of the best paper in Geology and Mineralogy, communicated to the Royal Society for insertion in their Transactions after the present date and prior to the month of June in that year.

The Council propose to give one of the Royal Medals in the year 1838 to the most important unpublished paper on Chemistry, communicated to the Royal Society for insertion in their Transactions, after the present date (November 12th, 1835,) and prior to the month of June 1838.

The Council also propose to give one of the Royal Medals in the year 1838 to the most important unpublished paper in Physics, communicated to the Royal Society for insertion in the Philosophical Transactions, after the present date (November 19, 1835,) and prior to the month of June 1838.

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APPENDIX.

Meteorological Journal kept at the Apartments of the Royal Society, by order of the President and Council.

PHILOSOPHICAL TRANSACTIONS.

I. *Researches on the Tides.—Fourth Series.* On the Empirical Laws of the Tides in the Port of Liverpool.* By the Rev. W. WHEWELL, *M.A. F.R.S.*

Received November 10,—Read November 19, 1835.

1. IN the Philosophical Transactions for 1831 Mr. LUBBOCK published the results of a discussion of nineteen years of the London tide observations; and from the materials there given, I endeavoured to obtain the mathematical laws of the inequalities of the phenomena, in a memoir which was published in the Transactions for 1834. Mr. LUBBOCK having now, in Part II. of the Transactions for the present year, published the results of a similar discussion of nineteen years of the Liverpool tide observations, I intend in the present paper to use these results in testing and improving the formulæ to which I was led by the London observations.

Perhaps the precise object of such investigations as this may be best understood by comparing them with corresponding steps in the history of other parts of astronomy; as, for instance, in the progress of our knowledge respecting the Moon's motions. After HIPPARCHUS had singled out and reduced to rule the great inequality of the Moon's motion, the *Equation of the Centre*, it was the employment of succeeding astronomers, as, for instance, PTOLEMY and TYCHO, to discover, by examination of long-continued observations, other smaller inequalities, and the laws which they follow; as the *Variation*, *Evection*, and others. In the same manner, the great inequality of the tides, the *Semimenstrual Inequality* of the time, is now well understood; and the agreement which Mr. LUBBOCK showed to exist between the London observations and the formulæ leaves nothing to desire. But formulæ for the observed effects of *lunar* and *solar parallax* and *declination* (although some such formulæ may have been

* For convenience of reference I shall take the liberty of thus numbering the papers in the Philosophical Transactions in which I have attempted to make out the Laws of the Tides. The preceding papers are,

First Series. Essay towards a First Approximation to a Map of Cotidal Lines.—1833, Part I.

Second Series. On the Empirical Laws of the Tides in the Port of London.—1834, Part I.

Third Series. On the Results of Tide Observations made in June 1834 at the Coast Guard Stations in Great Britain and Ireland.—1835, Part I.

employed by the calculators of Tide Tables) have never been published, as far as I am aware, except in the memoir already mentioned. It was therefore a matter of great interest to examine whether the formulæ obtained from the London tides are confirmed by those of Liverpool, and whether any further light is thrown upon the subject by this addition to our materials.

2. The results of this examination have been very satisfactory. The Liverpool observations have both confirmed, in general, my formulæ, and have given me the means of very much improving them. The corrections for lunar parallax and declination, which, as far as they depended on the former investigation, might be considered as in some measure doubtful, and probably only locally applicable, have been so fully verified as to their general form, that I do not conceive any doubt now remains on that subject; and the nature of the local differences in the constants of the formula has also in part come into view. This investigation shows, that notwithstanding the great irregularities to which the tides are subject, the results of the *means* of large masses of good observations agree with the formulæ with a precision not far below that of other astronomical phenomena; as, for example, a fraction of a minute in the times, and a fraction of an inch in the heights.

This precision is the more worthy notice, because the formulæ which we obtain point directly to a very simple general law of the tides; namely, that the tide at any place occurs in the same way as if the ocean imitated the form of equilibrium corresponding to a certain antecedent time. This Equilibrium-Theory (the constant quantities which it introduces being suitably modified,) expresses, with very remarkable exactness, most of the circumstances in my results: I will therefore, before stating them, explain it a little further.

3. The theoretical formula for the position of the pole of the equilibrium-spheroid is

$$\tan 2\,\theta' = -\frac{h \sin 2\varphi}{h' + h \cos 2\varphi},$$

where h and h' are the elevation of the spheroid due to the sun and the moon respectively, φ the angular distance of the moon from the sun, θ' the angular distance of the pole of the spheroid from the moon's place.

In the case of the tides, we may suppose the actual ocean-spheroid to follow the equilibrium-spheroid at an angular distance λ', the spheroid being that which corresponds to a distance of the sun and moon $\varphi - \alpha$, instead of φ. Thus we have

$$\tan 2\,(\theta' - \lambda') = -\frac{h \sin 2\,(\varphi - \alpha)}{h' + h \cos 2\,(\varphi - \alpha)}.$$

In the same manner the theoretical height of the pole of the equilibrium-spheroid above the mean surface is $\surd\{h'^2 + h^2 + 2\,h\,h' \cos 2\,\varphi\}$; and on the equilibrium-theory the height of the tide above the mean surface is $\surd\{h'^2 + h^2 + 2\,h\,h' \cos 2\,(\varphi - \alpha)\}$.

By assuming properly the values of h and h', α and λ', these expressions may be made to agree very closely with the mean results of observation. This was shown with

respect to the expression for the time by Mr. Lubbock from the London observations. The Liverpool observations give a still closer agreement, assuming $\lambda' = 11^h\,6^m$, $\alpha = 1^h\,15^m$, $\frac{h}{h'} = \sin 89^m = \sin 22^\circ\,15'$.

The expression for the heights also agrees very nearly with observation, as I shall show, but for this purpose we must suppose $\alpha = 1^h$, $\frac{h}{h'} = \sin 23^\circ\,30'$.

The agreement in these cases is the more remarkable, on account of the want of symmetry in the functions which thus occur. The curve, the ordinate of which represents the time of high water (reckoned from the moon's transit), is not symmetrical with regard to its maximum ordinates. The curve, the ordinate of which represents the height of high water, is not symmetrical with regard to its mean line of abscissas. Yet in both these cases the theoretical and observed curve agree within a minute and an inch during their whole course. It is impossible to doubt, under these circumstances, that the theoretical formula truly represents the observed facts.

4. But this agreement belongs to the mean of all the observations; and we have further to seek for the alteration in the formula, which is requisite in order to represent the effect of changes in the parallax and declination of the sun and moon. In these respects also we find a near agreement of the theory and observation. By the equilibrium-theory, the height h' of the lunar tide ought to be proportional to the cube of the moon's parallax; it is exactly or nearly so: by the same theory the height h' ought to diminish when the moon's declination increases, and by a quantity proportional to the square of the sine of the declination. It is found to do so with great precision.

5. But the equilibrium-theory, since it does not point out the existence of the quantities λ' and α, does not indicate what changes these quantities may be expected to undergo, when the moon's force is altered by the effects of parallax and declination. We find that in that case, these quantities also are altered, and the resulting change in the phenomena may be conceived in the following manner.

If we suppose the moon to revolve about the earth by the diurnal motion, perpetually drawing the waters towards the position of equilibrium, we may conceive that the ocean would form a spheroid, the pole of which would revolve round the earth, following the moon at a certain distance of terrestrial longitude. For the sake of distinctness, let this distance be called the *Retroposition* of the theoretical tide *in longitude*. Its mean value is what I have termed in other communications the "*corrected establishment*" of a place in the open ocean.

If, from an original equilibrium-tide, a derivative tide were sent off, along any channel in which it is no further influenced by the forces of the moon and sun, it would take a certain time in reaching any place in that channel; and the circumstances of the tide at that place would not depend upon the positions and distances of the moon and sun at the time when the tide happens, but upon the positions and

distances of those luminaries at a certain time, anterior to the time of the tide by the interval occupied in the transmission of the tide along the channel. Let this interval of time be called the *Retroposition* of the theoretical tide *in time*. It is what, on former occasions, I have called the "*age of the tide*."

6. This phraseology being adopted, the phenomena of the Liverpool tides may be expressed as follows.

(1.) *The effects which the changes of the Moon's force produce upon the Tides, are the same as the effects which those changes would produce upon a Retroposited Equilibrium-tide.*

(2.) *The Retroposition of the Tide in longitude is affected by small changes, which changes are proportional to the variations in the moon's tidal force.*

(3.) *The Retroposition of the Tide in time is also affected by small changes, which changes depend on the variations of the moon's force.*

7. The former of these propositions is proved by the verification of the formulæ already mentioned, since these agree with the formulæ for equilibrium-tides, except in the circumstance of having $\varphi - \alpha$ for φ. Now this difference is equivalent to a retroposition of the tide in time, of such magnitude that, during this time, the distance of the sun and moon is changed from $\varphi - \alpha$ to φ. If α be $1^h\ 15^m$, as collected from the law of the times, the retroposition in time is the time requisite for the moon to increase its right ascension from the sun by $1^h\ 15^m$; that is, it is $\frac{75}{48}$ days nearly, or 1 day $13\frac{1}{2}$ hours. The tide at Liverpool agrees nearly with an equilibrium-tide produced in the southern ocean, $37\frac{1}{2}$ hours previously to the moon's transit at that port, and transmitted thither unchanged.

8. The second of the above propositions is proved by tracing the effect of changes of lunar parallax and declination upon the results compared with the above formula for the times and heights, namely,

$$\tan 2(\theta' - \lambda') = -\frac{h \sin 2(\varphi - \alpha)}{h' + h \cos 2(\varphi - \alpha)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a.)$$

$$y = \surd \{h'^2 + h^2 + 2\, h\, h' \cos 2(\varphi - \alpha)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b.)$$

By the equilibrium-theory, the change which would be produced by any alteration of the moon's force would correspond to the effect of an alteration in the value of h', the amount of the lunar tide. It appears from the examination of the observations, that this change takes place in fact, but that we must also suppose a change in λ' in order that the formula (*a.*) may represent the observed intervals of time. This change in λ', the retroposition of the tide in longitude, is

$2^m{\cdot}5\ (p - 57)$ for parallax p minutes. (See Art. 15.)
$84^m \sin^2 \delta$ for declination δ. (Art. 21.)

Now, by the theory, the effect of a change in the moon's parallax on the equilibrium-tide is as the change of parallax; and the effect of the moon's declination is a change

proportional to the square of the declination. Therefore the second of the above three propositions is established.

According as the moon's parallax is less, and according as her declination is greater, the moon's tidal force is less, and h' in the above formulæ is less. Yet it is remarkable that these two circumstances affect the magnitude of the retroposition of the tide in opposite ways. In one case λ' is augmented, in the other case it is diminished. When the moon's force decreases, by her receding further from the earth, the tide follows the moon at a greater interval; the mean interval increasing from $10^h\ 55^m$ to $11^h\ 12^m$, while the parallax diminishes from $61'$ to $54'$. But when the moon moves away from the equator, which also diminishes her tidal force, the tide follows her more closely, the interval decreasing from $11^h\ 12^m$ to $10^h\ 55^m$, while the declination increases from $0°$ to $27°$.

The Liverpool tide happens about 11 hours after the next preceding transit; and as the retroposited tide happens about $37\frac{1}{2}$ hours before this transit, we must suppose the Liverpool tide to be produced at an interval of $48\frac{1}{2}$ hours preceding the time at which it is observed, in order to make it agree nearly with the equilibrium-theory; and we may suppose this time to be employed in the transmission of the tide along its channel. If we suppose the original tide to lag behind the position of equilibrium, we may suppose the amount by which it lags to vary with the changes of the moon's force, to the amount above stated as the variation of λ'. On this supposition we may suppose the time of transmission of the tide along its channel to be constant. Or we may suppose that the changes of the moon's force not only affect the lagging of the original tide behind the equilibrium position, but also affect the velocity of transmission to Liverpool. In either of these ways the circumstances of the tide may be hypothetically represented; but it will, of course, be understood that we use such hypotheses at present only for the sake of connecting and representing the facts.

9. The effect which changes in the moon's force produce upon the retroposition of the tide in time, that is, on the value of α in the formulæ (*a.*) and (*b.*), is more difficult to determine with any precision. It is, however, manifest from the general course of the quantities in the Tables, that α is greater as the moon's parallax is greater, and as her declination is greater. This is proved by each Table independently. Thus I have collected as the amount of this change,

$2^m.5\ (p - 57')$ from the effect of parallax on the times (Art. 18.),
$4^m\ (p - 57')$ from the effect of parallax on the heights (Art. 20.),
$75^m \sin^2 \delta$ from the effect of declination on the times (Art. 23.);

the effect of declination on the heights offers no clear evidence of a change in α.

Since, in the change of parallax from $54'$ to $61'$, the value of α, as given by the times, changes from about $1^h\ 8^m$ to $1^h\ 24^m$, the retroposition of the tide in time varies from about 34 to 42 hours.

10. The circumstances of the Liverpool tides may be represented hypothetically

in the following way. Let it be supposed that the ocean-spheroid assumes a form agreeing with the equilibrium-spheroid at the moment, and that the pole of this spheroid follows the position of the pole of the equilibrium-spheroid at a certain mean interval, say 90°. Let it be supposed that at a certain time a tide is sent off from this ocean-spheroid along a channel in which it is no longer affected by the moon or sun, and thus reaches Liverpool, producing the tide there. The following assumptions will then represent the facts.

When the horizontal parallax is 54′, the tide is sent off along the channel in longitude 48½° east of Liverpool, at 45^h 6^m before the time of Liverpool high water, and the pole of the ocean-spheroid follows the position of equilibrium at a distance 90° 24′.

When the horizontal parallax is 57′, the tide is sent off along the Liverpool channel in longitude 94½° east, at 48^h 36^m before the Liverpool high water, and the actual spheroid is 90° behind the position of equilibrium.

When the horizontal parallax is 61′, the tide is sent off along the channel in longitude 159½° east, at 53^h 0^m before the occurrence of high water, and the actual spheroid is 89° 16′ behind the position of equilibrium.

This hypothesis thus modified represents the circumstances of the Liverpool tide as affected by lunar parallax. The effect of lunar declination might be represented in a similar manner.

It is not to be imagined that this hypothetical representation is near to the true state of the case. The changes in the lagging, in the length of the channel of transmission, and in the velocity of transmission, are not such as the forces can be supposed likely to produce. Nor is it likely that the original tide is exactly what it would be if the condition of equilibrium were fully attained. The tide-spheroid not only lags behind the position of equilibrium, but deviates from the form of equilibrium; and other differences, besides the retroposition in longitude and in time, are introduced by the waters being in motion instead of at rest. This is seen in our results; for the tidal force of the moon, which, in the equilibrium-spheroid, varies as the cube of the parallax, appears in the observations to vary more nearly as the square of the parallax: and though this difference may be referred to the inaccuracy of the observations, it may, I think with more probability, be considered as resulting from the condition of the waters being a condition of motion, not of equilibrium. The temporary variations of the force do not affect the form of the waters in the same proportion as the mean force, which is constantly dragging the waters after it, round the earth.

11. In what has been hitherto stated with regard to the hypothetical representation of the tides, we have had a reference solely to Liverpool. It cannot, however, be doubted that the general laws of the tides at other places would resemble those of that port, and therefore might be represented in a similar manner. It has already been shown in my former memoir, though less satisfactorily and precisely than in this, that the tides of London follow the same rules as those now described.

The numbers, however, which enter into these formulæ will not necessarily be the

same at two places; and since the empirical formulæ have not been determined for any places except those of London and Liverpool, we have not the means of discovering the relation of the constants at various places. The following comparison of the data of observation at London and at Liverpool is instructive as far as it goes.

The greatest difference arising from the mean semimenstrual inequality is the same at the two ports, being 88^m at both. This coincidence is striking; yet I am disposed to believe it accidental, although, according to theory, this quantity ought to be the same at all places, since it depends only upon the mean ratio of the solar and lunar tidal forces; for the semimenstrual inequalities at different places differ so much by other observations, (varying from 79^m at Brest to 96^m at Plymouth,) that I do not conceive the difference can arise from the incompleteness of the observations.

The effects of the parallax and declination at London were given by approximate formulæ, less exact than those which we have now obtained for Liverpool; but, comparing the London formulæ with corresponding approximations at Liverpool, we have the following results.

If Λ' represent the value of λ' for the mean parallax $57'$ and the declination $0°$, we have for the parallax p, and declination δ,

at London $\quad \lambda' = \Lambda' - 3^m\,(p - 57) - 132^m \sin^2\delta$,

at Liverpool $\quad \lambda' = \Lambda' - 2\tfrac{1}{2}{}^m\,(p - 57) - 84^m \sin^2\delta$, by Art. 15 and 21.

Also the maximum semimenstrual inequality,

at London $\quad = 40^m + 3^m\,(p - 57) + 84^m \sin^2\delta$,

at Liverpool $\quad = 41^m + 2^m\,(p - 57) + 30^m \sin^2\delta$, by Art. 17 and 23.

Also if H' be the value of h' for the mean parallax and the declination $0°$, we have,

at London $\quad h' = H' + 0^{ft}{\cdot}17\,(p - 57) - 3^{ft} \sin^2\delta$,

at Liverpool $\quad h' = H' + 1^{ft}{\cdot}47\,(p - 57) - 6^{ft} \sin^2\delta$, by Art. 19 and 24.

And at London $\quad h = 1^{ft}{\cdot}7$.

at Liverpool $\quad h = 2^{ft}{\cdot}8$.

12. The resemblances of the formulæ at the two places are remarkable, but the differences are still more so. The differences in the heights of the tide at different places are indeed what we know to prevail universally, and to depend upon local circumstances in an intelligible manner: but the differences in time are more difficult to explain, since both the tides come from the same origin. The difference in the effect of parallax may indeed be due to the inaccuracy of the data, but it is scarcely possible that this can be true of the difference in the effect of declination, which appears to be in the ratio of 132 to 84 for the non-periodical, and 84 to 30 for the periodical, part. Similar discussions of observations at other places will best throw light on this difficulty.

I now proceed to state the method by which the above empirical formulæ have been obtained.

The Semimenstrual Inequality.

13. In order to obtain the semimenstrual inequality of the *times* of high water, I take Mr. Lubbock's Table VII., and from each column of intervals (of tide and moon's transit) I subtract the mean of that column; and I thus obtain Table VII. (a), which exhibits the semimenstrual inequalities for each minute of parallax. I then take the means of the horizontal lines in this, interpolating in H. P. 60′ and omitting H. P. 61′. The resulting intervals are those of the mean tide.

Table VII. (a.)

Mean of each column subtracted from the column "*Interval*" of times.

H. P.....	54′.	55′.	56′.	57′.	58′.	59′.	60′.	61′.	Mean.
Mean Interval	h m 11 12·7	h m 11 11·5	h m 11 8·3	h m 11 6·5	h m 11 3·7	h m 11 0·3	h m 10 58·5	h m 10 54·5	h m 11 6
☽ 's Transit. h m	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.
0 30	+13·7	+11·6	+ 8·0	+11·6	+12·2	+12·9	+13·3	+11·8	+12·2
1 30	− 5·2	− 6·8	− 6·5	− 3·9	− 5·3	− 2·8	− 2·5	− 2·8	− 4·6
2 30	−23·2	−22·0	−22·0	−20·8	−18·6	−17·5	−16·6	−17·1	−20·0
3 30	−41·4	−36·7	−34·7	−33·0	−32·1	−29·3	−30·4	..	−33·9
4 30	−49·0	−47·8	−44·0	−41·8	−40·4	−38·4	−38·7	..	−42·8
5 30	−47·7	−45·7	−43·6	−43·2	−39·4	−38·5	−37·5	..	−43·2
6 30	−27·2	−26·4	−24·5	−25·7	−25·6	−24·8	−21·5	..	−25·0
7 30	+14·3	+13·4	+11·9	+ 9·2	+ 6·5	+ 1·6	+11	..	+ 9·6
8 30	+44·8	+41·8	+40·1	+37·4	+34·1	+31·1	+20·2	..	+36·6
9 30	+50·9	+49·3	+47·9	+45·0	+44·1	+41·6	+39·8	+39	+45·6
10 30	+42·5	+41·8	+41·3	+40·1	+38·6	+38·0	+36·1	+35·8	+39·8
11 30	+28·5	+28·1	+26·6	+25·7	+23·6	+25·7	+24·4	+25·4	+26·1
Max. Diff.	99·9	95·1	91·9	88·2	84·5	80·1	78·5		88·8

On comparing the mean numbers in the last column with the theoretical formula

$$\tan 2\,(\theta' - \lambda') = -\frac{c \sin 2\,(\varphi - \alpha)}{1 + c \cos 2\,(\varphi - \alpha)},$$

it appears that they may be very accurately represented by making $\lambda' = 11^h\ 6^m$, $\alpha = 1^h\ 15^m$, $c = \sin 1^h\ 29^m$. The agreement of this formula with observation is as follows:

Moon's Transit.	Formula.	Obs.	Diff.
h m	m s	m s	m s
0 30	+12 16	+12 12	− 0 4
1 30	− 4 7	− 4 36	− 0 29
2 30	−20 6	−20 0	+ 0 6
3 30	−34 0	−33 54	+ 0 6
4 30	−43 6	−42 48	+ 0 18
5 30	−42 40	−43 12	− 0 32
6 30	−25 8	−25 0	+ 0 8
7 30	+ 9 2	+ 9 6	+ 0 4
8 30	+36 28	+36 36	+ 0 8
9 30	+44 30	45 36	+ 1 6
10 30	+39 40	39 48	+ 0 8
11 30	+27 36	26 6	− 1 30

This accordance is complete, the difference amounting in only two cases to 1^m.

14. The semimenstrual inequality of the *heights* for each minute of horizontal parallax, and the mean semimenstrual inequality of the heights, are in like manner obtained by subtracting from each column of heights in Mr. LUBBOCK's Table VII. the mean of that column, and taking the means of the horizontal lines as is done in Table VII. (a.)

TABLE VII. (b.)

Mean of each column subtracted from column "*Height* of Tide."

H. P.....	54′.	55′.	56′.	57′.	58′.	59′.	60′.	61′.	Mean.
Mean height	14·20	14·41	14·84	15·22	15·63	16·02	16·43	16·66	
☽'s Transit. h m	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.
30	+2·38	+2·47	+2·45	+2·31	+2·33	+2·35	+2·19	+2·51	+2·35
1 30	+2·26	+2·23	+2·25	+2·41	+2·49	+2·43	+2·66	+2·73	+2·39
2 30	+1·62	+1·61	+1·83	+1·82	+1·91	+2·02	+2·32	+2·21	+1·88
3 30	+0·49	+0·70	+0·74	+0·95	+0·95	+1·17	+1·27		+·90
4 30	−0·61	−0·55	−0·64	−0·33	−0·27	−0·15	−0·10		−·38
5 30	−1·92	−1·92	−1·93	−1·83	−1·62	−1·53	−1·57		−1·76
6 30	−3·13	−3·05	−3·14	−2·69	−2·79	−2·73	−2·85		−2·91
7 30	−2·81	−2·96	−2·74	−2·89	−2·97	−3·06	−3·14		−2·94
8 30	−1·52	−1·62	−1·69	−1·86	−1·86	−2·09	−2·33		−1·85
9 30	−0·06	−0·08	−0·18	−0·43	−0·54	−0·60	−0·75	−0·60	−·38
10 30	+1·18	+1·16	+1·13	+0·82	+0·80	+0·60	+1·61	−0·65	+1·04
11 30	+2·10	+1·89	+1·95	+1·78	+1·63	+1·64	+1·65	+1·78	+1·81
Max. diff.	5·51	5·52	5·59	5·30	5·46	5·49	5·80	.....	5·33
Mean ..	5·44								
Half ...	2·74								

The theoretical height of the high water above the mean surface of the ocean is $\surd\{h'^2 + h^2 + 2\,h\,h'\cos 2(\phi - \alpha)\}$; and therefore if k be the mean of all the high-water heights, we shall have for the semimenstrual inequality of height the expression

$$\surd\{h'^2 + h^2 + 2\,h\,h'\cos 2(\phi - \alpha)\} - k.$$

This will agree very nearly with the result of observation if we make

$$h = 2{\cdot}74,\ h' = 6{\cdot}872,\ k = 7{\cdot}19,\ \alpha = 1^{h}.$$

The accordance is as follows:

Moon's Transit.	Formula.	Obs.
h m	f.	
0 30	2·35	2·35
1 30	2·35	2·39
2 30	1·83	1·88
3 30	0·84	0·90
4 30	−0·48	−0·38
5 30	−1·89	−1·76
6 30	−2·90	−2·91
7 30	−2·90	−2·94
8 30	−1·89	−1·85
9 30	−0·48	−0·38
10 30	0·84	1·04
11 30	1·83	1·81

The greatest deviation is about an inch, the mean a small fraction of an inch.

Effect of the Moon's Parallax.

15. In Mr. LUBBOCK's Table VII., which contains the effect of lunar parallax, and has a column for each minute of parallax, we have, in Art. 13, taken the mean of each column, and subtracted it from every number in the column. In this way, it is evident that the *mean* contains the *non-periodical* part of the effect, and the *remainder* contains the part which goes through its *period* in a semi-lunation.

The *non-periodical* part of the *interval* stands in the uppermost line of Table VII. (a) Art. 13.; and its variations are manifestly nearly or exactly proportional to the variations of the parallax. If we take $57'$ as the mean parallax, we may express these means very nearly by the formula

$$11^h\ 6^m{\cdot}5 - 2{\cdot}5\ (p - 57'),$$

p being the H. P. The agreement of the formula with observation is as follows, and is a near approximation.

H. P.	54′.	55′.	56′.	57′.	58′.	59′.	60′.	61′.
	h m	h m	h m	h m	h m	h m	h m	h m
Obs.	11 12·7	11 11·5	11 8·3	11 6·5	11 3·7	11 0·3	10 5·85	10 54·5
Formula....	11 13	11 11·5	11 9	11 6·5	11 4	11 1·5	10 5·9	10 56·5

The column for H. P. $60'$ is completed by interpolation, and the column for H. P. $61'$ is omitted. The latter is defective in half the hours of moon's transit, which arises from the effect of the Moon's Variation on the parallax. The parallax has a term depending on the sine of twice the distance of the moon from the sun, and cannot be so great as $61'$ except near syzygy. The "observed" mean for $61'$ is that which makes the numbers in that column follow nearly the same law as the rest.

The *periodical* part of the effect of lunar parallax is shown in the lower part of Table VII. (a). It appears there that the *intervals* for all the values of H. P. follow nearly the same law as the mean interval already considered, but with a difference in the maximum value of the inequality. If we add together the greatest positive and negative numbers in each column, we obtain the double of the maximum inequality nearly, but not exactly, since the maximum does not correspond exactly to times of moon's transit contained in the Table. Making a slight addition on this account, we have,

H. P.	54′.	55′.	56′.	57′.	58′.	59′.	60′.	61′.
	m	m	m	m	m	m	m	
Sum	99·9	95·1	91·9	88·2	84·5	80·1	78·5	
Double Max.	101	96·1	92·8	89	85·2	80·7	79	
Formula....	101	97	93	89	85	81	77	

Now in the expression

$$\tan 2\,(\theta' - \lambda') = -\frac{c \sin 2\,(\varphi - \alpha)}{1 + c \cos 2\,(\varphi - \alpha)}$$

the maximum value occurs when $\cos 2\,(\varphi - \alpha) = -c$, and is equal to $\frac{c}{\sqrt{1 - c^2}}$. If

we make $c = \sin\gamma$, we have for the maximum $\tan 2(\theta' - \lambda') = \tan\gamma$; and therefore maximum $2(\theta' - \lambda') = \gamma$. Hence we have γ for each H. P. by reducing the above double maxima to arcs, and thence we have c, by finding the sines of these arcs.

16. By the equilibrium-theory, c should be inversely as the cube of the H. P.; therefore, if C be the value of c for H. P. 57′, we have $\frac{c}{C} = \frac{57^3}{p^3}$; $\log c + 3\log p = \log C + 3\log 57$; and therefore this quantity, $\log c + 3\log p$, should be constant. It is found that we get a quantity much more nearly constant by taking $\log c + 2{\cdot}2 \log p$. The following is the result:

H. P.	γ.	$\log c$ ($c = \sin\gamma$).	$\log p$.	$\frac{22}{10}\log p$.	$\log c\, p^{2\cdot2}$.
54′.	° ′ 25 15	9·62999	1·73239	346478	3·44125
				3·46478	
55.	24 15	9·61354	1·74036	348072	3·44233
				3·48072	
56.	23 12	9·59543	1·74819	349638	3·44145
				3·49638	
57.	22 15	9·57824	1·75587	351174	3·44115
				3·51174	
58.	21 18	9·56021	1·76343	352686	3·43976
				3·52686	
59.	20 10	9·53751	1·77085	354170	3·43338
				3·54170	
60.	19 45	9·52881	1·77815	355630	3·44074
				3·55630	
61.					

Hence, $c = C\left(\frac{57}{p}\right)^{2\cdot2}$ nearly.

17. We may, however, express the result more conveniently for some purposes by expanding this expression; for the variation of the maximum will be very nearly as the variation of the parallax; and the double maximum may be nearly expressed by the following formula:

$$89^m - 4^m(p - 57).$$

The accordance is shown in the lowest line of the second Table in Art. 15.

18. By comparing, in Table VII. (a), the inequalities for moon's transit $0^h\ 30^m$, $1^h\ 30^m$, and for $6^h\ 30^m$ and $7^h\ 30^m$, it is clear that they are equal to 0 at a later hour for the larger than for the smaller parallaxes, which also appears by the maxima. Hence α is larger for large parallaxes than for small ones. The exactness of the observations hardly allows us to determine its variation exactly. It appears, however, that it may be sufficiently well represented by $\alpha = 1^h\ 15^m + \cdot 2^m{\cdot}5\ (p - 57)$.

19. The effect of the lunar parallax on the *heights* will be found from Table VII. (b.) in the same way as the effect on the times, by taking the *mean* of each column as the *non-periodical*, and the *remainder* as the *periodical*, part of the inequality. The origin of the measurements is arbitrary, the low water not being given. The *non-periodical* part is represented with great accuracy (except for the extreme parallaxes) by the formula $15{\cdot}22 + \cdot 4\ (p - 57)$. The accordance is as follows:

H. P. ...	54′.	55′.	56′.	57′.	58′.	59′.	60′.	61′.
Obs. ..	14·20	14·41	14·84	15·22	15·63	16·02	16·43	16·66
Formula	14·02	14·42	14·82	15·22	15·62	16·02	16·42	16·82

We cannot compare this effect of parallax on the heights with the whole height of the tide, or with theory, from not having any observations of low water for this series of tides.

The periodical part of the heights, as appears by the remainder of Table VII. (b.), whatever be the parallax, follows nearly the law of the mean, which has already been explained; and the magnitude of the maximum differences does not appear to be steadily different for different H. P. In fact, theory would lead us to expect it to be the same in all these cases, because the amount of this inequality is $2\,h$, double the mean solar tide.

20. But it appears from the Table VII. (b.) that the time of moon's transit, when the periodical inequality vanishes, is later for the larger parallaxes, and the maxima indicate the same change: the amount of the change is about $4^{m}\,(p - 57)$, at the mean between the greatest and least values of the height.

When $\varphi = \frac{\pi}{4} + \alpha$, the formula for the inequality becomes $h' - h$, which is the mean between the greatest and least values. In this case $\alpha = 1^{h}$. Hence the value of α is $60^{m} + 4^{m}\,(p - 57)$.

Effect of the Moon's Declination.

21. The effect of the changes of lunar declination upon the tide will be found in nearly the same way as the effect of changes in the parallax. Mr. Lubbock's Table XII. gives the *intervals* for each 3° of declination. By finding the mean of each column, and subtracting it from the column, we obtain the *non-periodical* and the *periodical* part respectively of the inequality as is done in Table XII. (a.)

Table XII. (a.)

[*Intervals* of times.] Mean of each column subtracted from the column.

Decl.	0°.	3°.	6°.	9°.	12°.	15°.	18°.	21°.	24°.	27°.
Mean Interval	h m 11 12·1	h m 11 11·0	h m 11 11·3	h m 11 10·4	h m 11 8·4	h m 11 6·5	h m 11 3·6	h m 11 1·2	h m 10 59·0	h m 10 55·6
☽'s Transit. h m	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.
0 30	+10·0	+ 8·4	+ 8·8	+ 9·1	+ 9·5	+10·8	+12·1	+12·3	+13·2	+13·6
1 30	− 7·1	− 6·3	− 5·6	− 6·1	− 5·5	− 5·3	− 4·6	− 3·2	− 3·8	− 3·3
2 30	−20·7	−20·4	−20·5	−21·1	−20·4	−20·5	−20·0	−20·8	−20·8	−20·6
3 30	−33·0	−31·9	−32·7	−31·5	−33·0	−33·7	−30·4	−33·0	−34·4	−33·7
4 30	−40·5	−41·2	−40·9	−40·3	−41·1	−42·2	−42·0	−43·3	−44·6	−45·7
5 30	−37·6	−36·0	−37·2	−38·3	−40·2	−40·4	−43·4	−43·5	−45·7	−46·6
6 30	−17·4	−17·2	−19·6	−20·7	−22·2	−23·1	−25·3	−26·9	−28·4	−31·7
7 30	+11·3	+14·0	+11·5	+12·0	+10·0	+11·4	+ 8·6	+ 3·4	+ 9·3	+ 4·3
8 30	+35·0	+33·0	+37·1	+34·3	+37·2	+36·4	+36·5	+39·7	+37·3	+40·9
9 30	+41·7	+41·2	+44·7	+43·1	+42·2	+43·6	+44·1	+46·0	+46·6	+49·9
10 30	+36·0	+31·8	+34·7	+36·0	+38·5	+37·9	+38·5	+40·9	+42·1	+43·6
11 30	+22·3	+24·6	+21·9	+23·9	+25·6	+25·6	+26·6	+28·2	+28·9	+30·0
Greatest Diff.	82·2	82·4	83·6	83·4	83·3	85·8	87·5	89·5	92·3	96·5
Excess above 82	0·2	0·4	1·6	1·4	1·3	3·8	5·5	7·5	10·3	14·5

The first line of that Table contains the *non-periodical* part. In order to find its law, subtract each mean from $11^h\ 12^m$ corresponding to decl. 0. We obtain a series of numbers which increase faster than the declination; and it is found that they may be nearly represented by the expression $84 \sin^2 \delta$, δ being the declination. The agreement is as follows:

Declination.....	0°.	3°.	6°.	9°.	12°.	15°.	18°.	21°.	24°.	27°.
Obs. Diff.....	0	1·0	0·7	1·7	3·6	5·5	8·0	10·8	13·0	16·4
Formula	0	0·2	0·8	2·0	3·6	5·6	8·0	10·8	13·9	17·3

Hence the *non-periodical* part is $11^h\ 12^m - 84^m \sin^2 \delta$.

The *remainder* of the Table XII. (a.) exhibits the periodical part of the inequality; and it will be seen that each column follows nearly the law of the mean semimenstrual inequality as already obtained. In order to obtain the law of the coefficients, I take, as before, the sum of the two maximum values. This sum converted into arc gives γ, and $c = \sin \gamma$.

Decl.	γ.	cosec $\gamma = \frac{1}{c}$.	Excess of Decl. 0°.	Log. Excess.	Log. $\sin^2 \delta$.	Difference.
°	° ′					
0	20 33	2·8488028				
3	20 36	2·8421877				
6	20 54	2·8031777				
9	20 51	2·8091995	·0396033	$\bar{2}$·59769	18·38866	·20903
12	20 50	2·8117471	·0370557	$\bar{2}$·56878	18·63576	$\bar{1}$·93302
15	21 27	2·7345630	·1142398	$\bar{1}$·05778	18·82600	·23178
18	21 52	2·6849391	·1638637	$\bar{1}$·21447	18·97996	·23451
21	22 23	2·6260406	·2227622	$\bar{1}$·34783	19·00866	·33917
24	23 5	2·5505680	·2982348	$\bar{1}$·47455	19·21862	·25593
27	24 8	2·4458163	·4029865	$\bar{1}$·60528	19·31410	·29118

For the smaller declinations, the differences are too small to be depended on. The numbers corresponding to the resulting logarithms from 15° to 27° are from 1·7 to 2·1. If we take the mean 1·85 as the number, we have for $\frac{1}{c}$ the value $2{\cdot}85 - 1{\cdot}85 \sin^2 \delta$, which is sufficiently near.

22. By the theory of equilibrium $\frac{1}{c} = \frac{h'}{h}$. And by the same theory, if H′ be the height of the lunar tide at the equator when the declination is 0, we shall have in latitude l, when the declination is δ, two tides, of which the heights are $H' \cos^2 (l + \delta)$ and $H' \cos^2 (l - \delta)$. Now as we have not distinguished these two tides, our result will be the mean of them. Therefore,

$$\begin{aligned} h' &= \tfrac{1}{2} H' \{\cos^2 (l + \delta) + \cos^2 (l - \delta)\} \\ &= H' \{\cos^2 l \cos^2 \delta + \sin^2 l \sin^2 \delta\} \\ &= H' \{\cos^2 l - (\cos^2 l - \sin^2 l) \sin^2 \delta\} \\ &= H' \cos^2 l \{1 - (1 - \tan^2 l) \sin^2 \delta\}. \end{aligned}$$

But we have found that h' varies as

$$2{\cdot}85 - 1{\cdot}85 \sin^2 \delta.$$

Hence $1 - \tan^2 l = \frac{1{\cdot}85}{2{\cdot}85} = \frac{37}{57}$; $\tan^2 l = \frac{20}{57}$

$$l = 30^\circ\ 38'.$$

Hence it appears that the amount of the effect of declination on the time H. W. at Liverpool corresponds to the effect on an equilibrium-spheroid at a place in latitude 30°.

23. It will be convenient to express the double maximum of the semimenstrual inequality by a formula. The formula $60^m \sin^2 \delta$ gives the following accordance with the excess above 82^m:

Decl.	0°.	3°.	6°.	9°.	12°.	15°.	18°.	21°.	24°.	27°.
Obs. ...	0·2	0·4	1·6	1·4	1·3	3·8	5·5	7·5	10·3	14·5
Formula	0	0·2	0·3	1·4	2·6	4·0	6·0	7·7	9·9	12·4

Hence $82^m + 60^m \sin^2 \delta$ nearly expresses the double maximum.

It appears also that the times when the inequality is 0 and max. are later as the declination is greater, the difference being about 15^m from decl. 0° to decl. 27°, and apparently increasing as the square of the decl. or of sin decl. Now $\sin^2 27 = \frac{1}{5}$ nearly: hence the correction of α is nearly $75^m \sin^2 \delta$, and we have $\alpha = 65^m + 75^m \sin^2 \delta$.

24. The effect of lunar declination on the *heights* will be found in the same manner from Mr. Lubbock's Table XVI. The mean of each column is obtained in the first line of the following Table.

Table XVI. (b.)

[Heights.] Mean of each column subtracted from the column.

Decl.	0°.	3°.	6°.	9°.	12°.	15°.	18°.	21°.	24°.	27°.
Mean Height ...	15·74	15·78	15·77	15·72	15·54	15·41	15·23	14·92	14·74	14·39
☽'s Transit. h m	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.	Remainder.
0 30	+2·59	+2·64	+2·30	+2·52	+2·65	+2·58	+2·27	+2·53	+2·18	+2·38
1 30	+2·53	+2·52	+2·73	+2·57	+2·61	+2·51	+2·46	+2·32	+2·34	+2·40
2 30	+1·75	+1·97	+1·95	+1·82	+1·99	+1·81	+2·03	+1·86	+1·99	+2·10
3 30	+1·01	+0·69	−0·77	+0·88	+0·80	+0·84	+1·10	+1·03	+1·21	+1·21
4 30	−0·70	−0·48	−0·53	−0·74	−0·54	−0·52	−0·70	−0·27	−0·22	−0·22
5 30	−1·62	−2·10	−1·85	−1·96	−1·83	−1·81	−2·01	−1·86	−1·75	−1·61
6 30	−2·66	−2·74	−3·00	−2·86	−3·08	−2·96	−2·88	−3·12	−3·05	−3·24
7 30	−2·83	−2·66	−2·70	−3·10	−2·95	−3·02	−2·91	−3·01	−3·10	−3·20
8 30	−1·61	−2·28	−2·11	−1·72	−1·89	−1·99	−1·90	−1·95	−1·94	−1·88
9 30	−0·60	−0·37	−0·42	−0·32	−0·53	−0·41	−0·30	−0·40	−0·21	−0·39
10 30	+1·12	+0·92	+1·03	+0·84	+0·95	+1·12	+0·99	+1·01	+0·80	+0·85
11 30	+1·09	+1·93	+1·84	+2·14	+1·87	+1·81	+1·90	+1·87	+1·78	+1·60
Greatest Diff.	5·42	5·38	5·73	5·67	5·73	5·60	5·37	5·65	5·44	5·64
Mean	5·63									

If we subtract these mean heights from 15·8, the remainders are very nearly as $\sin^2 \delta$. The formula 6 $\sin^2 \delta$ gives the following accordance:

Decl.	0°.	3°.	6°.	9°.	12°.	15°.	18°.	21°.	24°.	27°.
Obs.	·06	·02	·03	·08	·26	·39	·57	·88	1·06	1·41
Formula ..	·00	·02	·08	·14	·26	·40	·60	·77	·99	1·24

Hence 15·8 — 6 $\sin^2 \delta$ nearly is the non-periodical part of the Table.

The *periodical* part, as appears by the remainder of Table XVI. (b.), follows nearly the law of the mean height already explained. The sum of the maximum inequalities is not definitely different for the different declinations, which agrees with the theory, according to which it is constant and equal to 2 h.

Also by comparing the columns for decl. 0° and 27°, it appears that the interval between the times when the inequality is 0, is less for the greater decl., which also agrees with the theory, for in that case the fraction $\frac{h}{h'}$ is greater, and the defect of symmetry in the curve increases with this fraction.

There is no clear evidence of a variation of α in this Table.

Trinity College, Cambridge,
November 12, 1835.

II. *Researches towards establishing a Theory of the Dispersion of Light. No. II. By the Rev.* BADEN POWELL, *M.A. F.R.S. Savilian Professor of Geometry in the University of Oxford.*

Received Nov. 5,—Read December 17, 1835.

IN my paper inserted in the last part of the Philosophical Transactions, I have commenced a comparison between the results of M. CAUCHY's system of undulations, expressing the theoretical refractive index for each of the standard rays of the spectrum, and the corresponding index found from observation in different media. This comparison is there carried on for all the results obtained by M. FRAUNHOFER. But these include only a limited range of transparent bodies; and close as is the accordance in these instances, the theory cannot be considered as fully verified, until we shall have extended a similar examination to a greater number of media, and especially to those of higher dispersive power. In this research I am now engaged: but as it will necessarily occupy a considerable period to carry it on, from time to time, as data are furnished, I venture for the present to submit to the Royal Society the following portion of my calculations in continuation of the preceding.

In my former communication I had referred to M. FRAUNHOFER's results as affording the *only* precise data which observation had as yet furnished. But through the kindness of Prof. MILLER, of Cambridge, I have since become acquainted with the series of results obtained by M. RUDBERG. They are given in POGGENDORFF's Annalen, band xiv. and xvii., and comprise the indices observed by him for the standard rays, or the ratios of the velocities in air to the velocities within the crystal, in a direction perpendicular to the axis of the rhombohedron, in a prism of calcareous spar, having its edge parallel to that axis; and in a prism of quartz similarly cut; in either case, both for the ordinary and extraordinary ray: also the ratios of the velocities in the direction of the three axes of elasticity respectively, in aragonite and topaz.

This valuable series of data I have now examined: and the comparison of them with theory constitutes the present communication. The calculations are made by precisely the same method as those described in my former paper; and the results are here stated in exactly the same tabular form, which will consequently need no explanation. The coincidences of observation and theory will be found at least as close as those already obtained from M. FRAUNHOFER's results, and I think will be allowed to afford a satisfactory extension of the theory to the cases here discussed.

Thus the hypothesis of undulations assigns the law and cause of dispersion in ten new cases, in addition to the ten considered in my former paper.

Oxford, November 1, 1835.

Postscript.

It may be right here to mention, that since my former paper was printed, I have learned from M. Cauchy that he has also investigated the relation between the length of a wave and the refractive index. And in a memoir on his new method of interpolation he has applied that method to this case, and has also given an example of the comparison of numerical values. This, however, is only made for one single case, viz. the Flint Glass, No. 23. of Fraunhofer.

Also, while this paper has been passing through the press, some other important observations closely connected with the subject have been made, for which the reader must refer to the London and Edinburgh Philosophical Magazine and Journal of Science, Nos. 44 and 45.

Comparison of Refractive Indices from Cauchy's *Theory and from observation.*

Calcareous Spar. Rudberg.

The edge of the prism parallel to the axis of the rhombohedron.

Ordinary Ray.

Ray.	Observed value of μ.	$\left(\frac{\theta}{\lambda}\right)$	Ratio $\left(\frac{\text{arc}}{\text{sine}}\right)$.	Calculated value of μ $= \text{const} \times \left(\frac{\text{arc}}{\text{sine}}\right)$.
B	1·6531	13° 16′ 0″	1·009	1·6531
C	1·6545	13 55 2	1·010	1·6547
D	1·6585	15 29 59	1·0123	1·6584
E	1·6636	17 19 45	1·0156	1·6638
F	1·6680	18 47 30	1·0181	1·6680
G	1·6762	21 14 30	1·0233	1·6765
H	1·6833	23 1 30	1·0277	1·6834
			const = 1·6384	

Extraordinary Ray.

B	1·4839	9° 30′ 0″	1·0045	1·4838
C	1·4845	9 57 59	1·0051	1·4847
D	1·4863	11 5 58	1·0063	1·4864
E	1·4887	12 24 38	1·0080	1·4889
F	1·4907	13 17 20	1·0092	1·4908
G	1·4945	15 12 30	1·0119	1·4948
H	1·4978	16 29 15	1·0140	1·4978
			const = 1·4772	

Quartz. Rudberg.

The edge of the prism parallel to the axis of the Rhombohedron. Extraordinary Ray.

Ray.	Observed value of μ.	$\left(\frac{\delta}{\lambda}\right)$.	Ratio $\left(\frac{\text{arc}}{\text{sine}}\right)$.	Calculated value of μ $= \text{const.} \times \left(\frac{\text{arc}}{\text{sine}}\right)$.
B	1·5499	° ′ ″ 10 33 0	1·0056	1·5497
C	1·5508	11 4 0	1·00635	1·5508
D	1·5533	12 19 50	1·008	1·5533
E	1·5563	13 46 50	1·0097	1·5560
F	1·5589	14 56 50	1·0114	1·5585
G	1·5636	16 53 15	1·0147	1·5636
H	1·5677	18 18 30	1·0173	1·5677
			const. = 1·541	
		Ordinary Ray.		
B	1·5409	10 20 0	1·0054	1·5409
C	1·5418	10 50 30	1·006	1·5418
D	1·5442	12 4 20	1·0075	1·5442
E	1·5471	13 30 0	1·0093	1·5469
F	1·5496	14 38 15	1·0109	1·5493
G	1·5542	16 32 45	1·0141	1·5541
H	1·5582	17 56 0	1·0166	1·5582
			const. = 1·5326	

Aragonite. Rudberg.

Ray in the direction of the axes of elasticity. First Axis.

B	1·5275	9 40 0	1·0047	1.5275
C	1·5282	10 8 27	1·0051	1·5282
D	1·5301	11 17 34	1·0065	1·5303
E	1·5326	12 37 40	1·0081	1·5328
F	1·5348	13 41 25	1·0095	1·5348
G	1·5388	15 28 33	1·0123	1·5390
H	1·5423	16 46 36	1·0144	1·5424
			const. = 1·5204	
		Second Axis.		
B	1·6763	12 50 0	1·0084	1·6763
C	1·6778	13 27 53	1·0092	1·6776
D	1·6816	14 59 35	1·0115	1·6815
E	1·6863	16 45 56	1·0144	1·6863
F	1·6905	18 10 42	1·0168	1·6903
G	1·6984	20 32 55	1·0217	1·6984
H	1·7051	22 16 27	1·0257	1·7050
			const. = 1·6623	
		Third Axis.		
B	1·6806	° ′ ″ 13 0 0	1·0086	1·6805
C	1·6820	13 38 20	1·0095	1·6820
D	1·6859	15 11 17	1·0118	1·6858
E	1·6908	16 59 0	1·0148	1·6908
F	1·6951	18 24 45	1·0175	1·6952
G	1·7032	20 48 52	1·0223	1·7033
H	1·7101	22 33 50	1·0263	1·7101
			const. = 1·6662	

Topaz. Rudberg.

First Axis of elasticity.

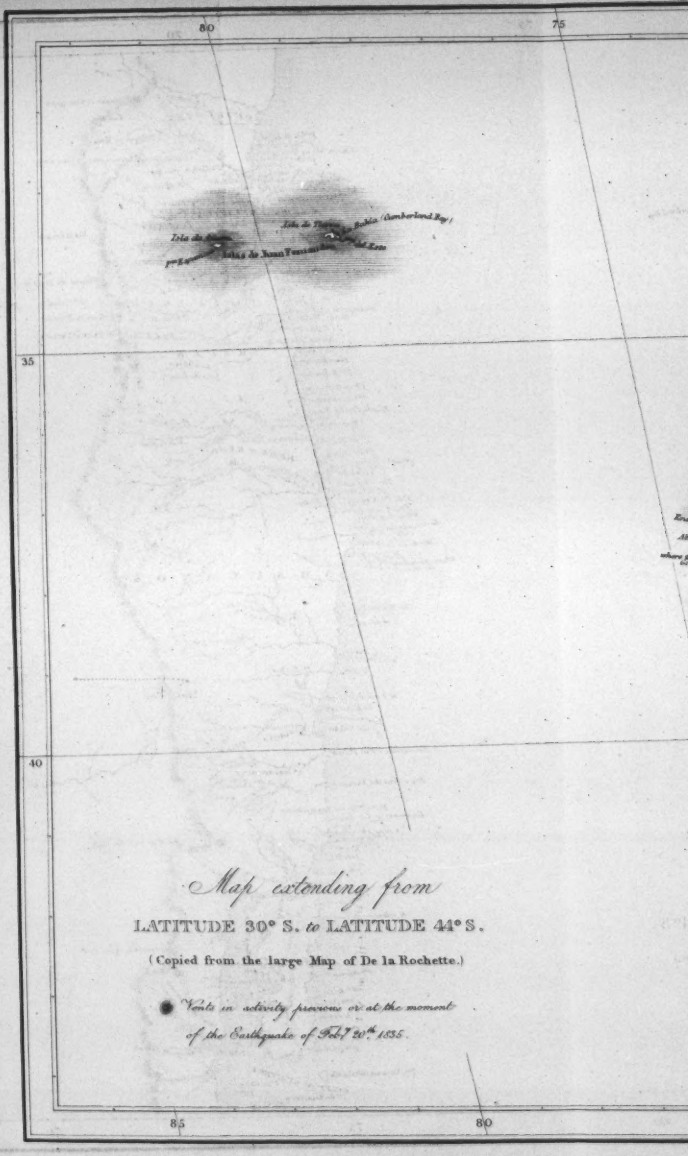
Ray.	Observed value of μ.	$\left(\frac{\delta}{\lambda}\right)$.	Ratio $\left(\frac{\text{arc}}{\text{sine}}\right)$.	Calculated value of μ = const. $\times \left(\frac{\text{arc}}{\text{sine}}\right)$
B	1·6084	° ′ ″ 10 5 0	1·0051	1·6085
C	1·6093	10 34 42	1·0056	1·6092
D	1·6116	11 46 48	1·0070	1·6114
E	1·6145	13 10 22	1·0089	1·6145
F	1·6170	14 16 53	1·0104	1·6172
G	1·6215	16 8 40	1·0133	1·6216
H	1·6254	17 30 2	1·0157	1·6254
			const. = 1·6003	

Second Axis.

B	1·6105	° ′ ″ 10 7 0	1·00515	1·6105
C	1·6114	10 36 47	1·0058	1·6115
D	1·6137	11 49 10	1·0071	1·6136
E	1·6167	13 12 58	1·0090	1·6165
F	1·6191	14 19 45	1·0104	1·6189
G	1·6236	16 11 53	1·0133	1·6236
H	1·6274	17 33 29	1·0158	1·6275
			const. = 1·6022	

Third Axis.

B	1·6180	° ′ ″ 10 7 0	1·00515	1·6180
C	1·6188	10 36 47	1·0058	1·6189
D	1·6211	11 49 10	1·0071	1·6209
E	1·6241	13 12 58	1·0090	1·6240
F	1·6265	14 19 45	1·0104	1·6264
G	1·6312	16 11 53	1·0133	1·6310
H	1·6351	17 33 29	1·0158	1·6351
			const. = 1·60955	



Map extending from

LATITUDE 30° S. *to* LATITUDE 44° S.

(Copied from the large Map of De la Rochette.)

● *Vents in activity previous or at the moment of the Earthquake of Feb.ʸ 20.th 1835.*

70
Aconcagua
QUILLOTA
Quillota
Valparaiso
MELIPILA
SANTIAGO
Mendoza
Volcan de Maypu
Chiquillanes
35
MAULE
Isla de Mocha
CHILLAN
Chillan
ARAUCANO
Maquegua
Valdivia
Charancabis
Vol. de Quechu cabi
Pampas de Buenos Ayres
40
75
70

III. *An Account of the great Earthquake experienced in Chile on the 20th of February, 1835; with a Map.* By Alexander Caldcleugh, *Esq. F.R.S. F.G.S., &c.*

Received October 15,—Read November 26, 1835.

THE phenomena attending this great disturbance of the surface of the earth have been so varied, and the extent of its effects so considerable, that I should almost deviate from my duty if I did not endeavour to draw up and transmit to the Royal Society some account of a convulsion which has laid in ruins three provinces, and caused incalculable damage to the southern part of this country. I am the more inclined to take this step from a happy concurrence of circumstances having drawn several scientific observers to Concepcion shortly after the catastrophe, who have obligingly confided their notes to me. I trust therefore the Royal Society will not consider that I am about to trespass upon its time.

An idea, in some degree fanciful, prevailed for some time after the conquest of these countries by the Spaniards, that these convulsions of the earth's crust occurred at intervals of a century; afterwards it was supposed that about fifty years was the term which usually elapsed between great shocks; but, since the commencement of this century, the repeated catastrophes which have occurred, especially in the years 1812 in Caraccas, 1818 in Copiapo, 1822 in the province of Santiago, 1827 in Bogota, 1828 in Lima, 1829 in Santiago, and 1832 in Huasco, have prepared the minds of the inhabitants to expect at all times these frightful oscillations of the earth, which, although they cause little sensation at first, after some time affect the nerves in a manner not easy to account for by ordinary causes. That they happen at all times and in all states of the atmosphere seems clearly decided. The finest weather, and the most variable, equally prevail at the moment; but many are the fancied signs by which the coming earthquakes are predicted, and in the faith of which the inhabitants confide, as they think their experience bears them out. While some place great confidence in rats running violently over the ceilings of the room, others prepare for a shock when they observe the stars twinkling more than usual, and all fears are removed when much lightning coruscates in the Cordillera. As far as my own observations go, little reliance can be placed on the two former prognostics; something more certain seems to be due to the latter. A few hours previous to the earthquake which I am about to describe, immense flocks of sea birds proceeded from the coast towards the Cordillera, a circumstance which occurred prior to the great shock of 1822; and it is affirmed by too many respectable persons not to be

entitled to some degree of credit, that on the morning of the convulsion all the dogs disappeared from Talcahuano.

The summer in Chile had been rather colder than in preceding years. The mean of the thermometer in Santiago (two thousand feet above the level of the sea) for the months of January and February was 72° of Fahr. The mean of the barometer for the same period was 28·25, which is about one tenth of an inch below its usual height.

From the 1st of February the barometer was unusually low in Santiago; and on the 14th, six days prior to the earthquake, the barometer at half past 6 A.M. stood at 28·1, the thermometer at the same time being 73°. A slight oscillation, which lasted twenty seconds, was felt on this day; on the 20th the barometer marked 28·17, and the thermometer rose to 76°: the weather fine. In Concepcion, in the night of the 17th to 18th the barometer fell four tenths of an inch, but gradually recovered itself, and indicated nothing extraordinary on the morning of the 20th. In Valdivia, according to the observations most obligingly communicated to me by Capt. FitzRoy, of the Beagle Surveying Ship, the barometer stood on the 16th of February at 29·92, and continued to rise gradually until the end of the month, with an increased temperature. From my own observations, deduced from many oscillations, I have remarked that the barometer usually falls shortly before any considerable shock, and then returns to its ordinary mean. On the 26th of September 1829, a very severe earthquake was experienced in this city, which did much damage to most buildings; the front of the house I then inhabited fell down; and it is worthy of remark, that the instant after every shock a burst of rain fell, which soon moderated, until a fresh tremor caused it to recommence.

The igneous vents of the whole range of the Cordillera may be said to have been in remarkable activity both preceding and at the moment of the late convulsion. From the flat-topped volcano of Yanteles, in front of Chiloe, to the lofty range of the Andes in Central America, all the information which has been obtained gives details of violent eruptions. On the 20th of January the volcano of Osorno, north-east of Chiloe, burst forth with inconceivable fury; and the lava was seen at night rushing out of the crater and rolling down the side of the mountain, elevated 3900 feet above the level of the sea. The reflection of the flame reached double that height, and is described to me by Mr. P. G. King, of the Beagle, as presenting the most magnificent object he had ever beheld. From the plains of Talca, eighty leagues to the south of the capital, two volcanos were observed in activity a few days after the 20th of February. They are both situate near the lake of Mondaca, twenty-five leagues eastward in the Cordillera: and another new rent was observed on the estate called Cerro colorado, on the right bank of the river Maule, and near its source. The volcano of Petoroa, and another near it whence a stream of asphaltum flows, and those of Maypu and Aconchagua have also been for some months in a state of activity.

In the month of January the volcano of Coseguina in Central America became

exceedingly active, and ejected a body of lava which covered a circumference of eight leagues three yards and a half deep, burying all the farm-houses, sugar-works, and cattle: the ashes continued falling for five days, and reached upwards of three hundred leagues from the centre of desolation and ruin.

It was at half past eleven o'clock on the morning of the 20th that the earthquake commenced, with an atmosphere as serene and beautiful as the elements beneath were convulsed and threatening. The first oscillation, gentle, and attended with little noise, was but the precursor of the two succeeding undulations, which were extremely violent; the duration from the first to the last vibration was about two minutes and a half, and the direction appeared to be from south-west to north-east. The sensation occasioned by the undulatory movements, seemed to me to be similar to that which would be produced by standing on a plank the ends of which rose and fell two feet from the ground. The small streams of water which run down the streets were checked and thrown over the edges of their channels. In Talca, eighty leagues to the south, the effects were still more violent: the oscillation commenced without being accompanied by that rumbling noise which usually is the forerunner of these awful phenomena. In Concepcion, where the great violence of the earthquake was felt, it was the second undulation which caused the havoc in the buildings; and previous to that and the many succeeding shocks, a violent report was heard, proceeding from the southward, as from a volcano in that direction. All the houses in the port of Talcahuano, which were situate on the low lands beneath the hills, were laid prostrate; and about half an hour after the vibration, when the inhabitants were returning to their houses from the heights and open spaces, it was remarked that the sea had retired so much beyond its usual limits that all the rocks and shoals in the bay were visible. It flowed again, and again retired, leaving the ships dry which were at anchor in the harbour. Then an enormous wave was seen slowly approaching the devoted town, from the direction of the Boca Chica. For ten minutes it rolled majestically on, giving time to the inhabitants to run to the heights, whence they saw the whole place swallowed up by this immense breaker.

In this moment of terror, men saw the roller with little accordance as to size; some compared it to the height of the loftiest ship, others to the height of the island of Quiriquina. It carried all before it, and rose by accurate measurement twenty-eight feet above high-water mark. A small schooner of eighty tons, nearly ready for launching, was lifted over the remains of the walls, and found lying among the ruins three hundred yards from her stocks. The reflux of this roller carried everything to the ocean. Another and a larger wave succeeded; but taking a more easterly direction, the ruins of Talcahuano escaped, but the Isla del Rey was ravaged by it. A fourth and last roller, of small dimensions, advanced, but nothing was left for further devastation. While these great waves were rushing on, two eruptions of dense smoke were observed to issue from the sea. One, in shape like a lofty tower, occurred in the offing; the other took place in the small bay of San Vicente,

and after it had disappeared, a whirlpool succeeded, hollow, in shape like an inverted cone, as if the sea were pouring into a cavity of the earth. In every direction in this bay, as well as in Talcahuano, vast bubbles broke, as if an immense evolution of gas were taking place, turning the colour of the water black, and exhaling a fetid sulphureous odour.

At San Tomé, on the other side of the bay, the roller did immense damage; and on the island of Quiriquina the cattle dashed off the cliffs from panic. In this island the waves injured houses forty feet above the present level of high water, and during the three following days the sea ebbed and flowed irregularly.

In the bay of Concepcion, the strata of clay slate have been visibly elevated, from about three to four feet. This alteration of the relative position of sea and land is clearly distinguishable, by a rock off the landing-place, which previous to the shock was nearly level with high water, being subsequently found to be raised three feet higher; and the buoy on the Belen Bank has now four feet less water than formerly. A vessel lying at anchor had one fathom less water alongside her than before the shock; but it is very likely that she changed her position. At the port of San Vicente, a little to the south of Talcahuano, the land has also risen about a foot and a half; and along the shore of the latter bay, even at high water, beds of dead muscles were left as proofs of the upheaval of the strata.

To the southward of the entrance of the bay of Concepcion there is a small island called Santa Maria, about seven miles long and two wide. Capt. FitzRoy examined with great care the line of beach in the southern cove, as well as the northern part of the island; and from the visible evidence of beds of dead shell-fish, from soundings, and from unbiassed oral testimony, it appears placed beyond the shadow of doubt, that on the latter side the elevation of the land has not been less than ten feet, in the centre of the island about nine, and in the southern cove about eight feet. This upheaving has almost destroyed the southern port of the island, for it now affords but little shelter to vessels, and the landing is bad. Everywhere around the island the soundings have been diminished a fathom and a half, and the cliffs, of the height of 150 or 200 feet, are split and rent in all directions, and huge masses precipitated below. Both Capt. FitzRoy and Capt. Simpson, of the Chilian Navy, are of opinion that the uprising of the strata, both in this island and in Concepcion, at the time of the earthquake, was considerably greater, and that the many subsequent minor oscillations may have caused a subsidence to the level before recorded. At Subul, a little to the south-east of Santa Maria, the elevation of the strata appears to have been about six feet.

At Nuevo Bilbao, the port of the river Maule, seventy leagues north of Concepcion, about an hour and a half after the shock, the sea flowed above the usual water mark, and continued for half an hour in that state before a reflux took place. Fifty minutes afterwards the sea, greatly agitated, rolled on the coast and up the river with extraordinary violence, and reached a height of twelve feet above the water mark.

By this last inroad, two schooners, anchored in the port, carried away their cables, and were found among the bushes one hundred and fifty yards from the beach.

A third rush of the sea occurred half an hour afterwards, which ascended to a height of nine feet; and for the space of forty-eight hours repeated rollers came forward, but with diminished violence. No elevation of the coast has been discovered at this port, but on the bar at the mouth of the river, which has always rendered the entrance to the port both difficult and dangerous, two feet more water has been remarked; and in consequence of the fall of an immense point of a mountain into the sea, it is hoped that, owing to the new direction given to the current, no further accumulation of sand will take place.

In Valparaiso the sea was observed to advance and recede rapidly, but gently and without violence.

It would be of little avail to distress the Society with the details of the ruin caused in all the southern provinces of Chile by this convulsion. To the southward of Talca scarcely a wall has been left standing, and even to the north of this line the damage caused to every description of building has been most serious. Throughout the provinces of Canqueues and Concepcion, the entire crust of the earth has been rent and shattered in every direction. In some places fissures of several feet in depth and width have been discovered intersecting the country for considerable distances. On one estate near Chillan, thirty leagues from the coast, extensive fissures have been the vents of muddy eruptions of salt water, which have made large deposits of a kind of grey pulverulent tufa; and on the same estate a great many circular pools were discovered of salt water, and many new thermal springs have burst forth. In many places the ground swelled like a large bubble, and then bursting, poured forth black and extraordinarily fetid water.

The limits to which the oscillations extended were, to the north as far as Coquimbo, and to Mendoza on the eastern ridge of the great chain of the Andes. Vessels navigating the Pacific within a hundred miles of the coast experienced the shock with considerable force. The bark Glenmalia, bound to Valparaiso, when ninety-five miles from the coast and in front of the Maule, had her course through the water suddenly checked, and her rate of sailing altered from seven knots to one, and the master conceived the vessel was dragging over a sand-bank. The sea was strongly agitated, and appeared to lift the vessel twenty feet. Such was the alarm that the boats were nearly lowered: no soundings were met with.

The Island of Juan Fernandez, a mass of basalt three hundred and sixty miles from the coast, experienced the earthquake, but with less violence; the sea rose to the height of the Mole in a similar manner to that of Concepcion, and then receded, leaving the bottom of Cumberland Bay dry to some distance from the shore, and in the second rush rose fifteen feet above the usual level, carrying all before it. At the same time the Governor, Major Sutcliffe, observed a dense column of smoke issuing from the sea about a mile off the Point Bacalao, which lasted until 2 o'clock

in the morning, when an immense explosion took place, which threw the water in every direction; during the rest of the night great bursts of flame rising from the same spot illumined the whole island. Captain Simpson, about a month after, sounded near the spot in every direction and found no bottom in less than sixty-nine fathoms. It is worthy of remark, that when on the 24th of May 1751 the city of Concepcion was destroyed by an earthquake, and by the inroad of the sea, the rising colony of Juan Fernandez was swallowed up in a similar manner by immense rollers. The Governor, his family, and thirty-five persons perished by the catastrophe.

After the earthquake the usual atmospheric changes occurred. In many places the most awful hurricanes completed the dismay of the inhabitants and added to the catastrophe. To these succeeded deluges of rain, a circumstance most unusual at that period of the year. At the Hot Springs of Canqueues, where the water issues at the temperature of 118° of Fahr., the heat was lowered after the earthquake to 92°, a circumstance which occurred after the shock of 1822. The diminished temperature lasted but a short time.

At the risk of being tedious, I have given a detail to the Society of the changes effected in the earth's surface by this violent convulsion. After examining the extensive area of its vibration, after observing the uprising of an island and the adjacent coast, together with the eruption of a submarine volcano, it is difficult to deny that the same causes are still in operation which ages since raised tertiary formations to their present lofty site in the great range of the Cordillera. Surrounded with these continued changes on the surface of the earth, it is impossible not to respect the opinions of those philosophers who conceive that the Continent of America has risen into existence at a more modern period than that which therefore may, with more propriety, be termed the Old World.

Owing to the early hour on the 20th that the oscillation commenced, comparatively few lives were lost, but the frequent occurrence of these catastrophes, by causing organic defects, may very probably explain the causes of the short duration of human existence in these countries.

Santiago de Chile,
12th June, 1835.

IV. *Some Account of the Volcanic Eruption of Cosegüina in the Bay of Fonseca, commonly called the Bay of Conchagua, on the Western Coast of Central America.* By Alexander Caldcleugh, *Esq., F.R.S. F.G.S., &c.*

Received December 29, 1835,—Read January 7, 1836.

THERE is perhaps no country on the face of the globe which shows more signs of vast geological disturbances than that part of the western hemisphere which, situate between its great northern and southern divisions, has obtained in more modern times the name of Central America. Its shores, extending to both oceans, are in spots precipitous, while other and extended lines of coast are low, and abound in mangrove creeks, intersected by mountains and volcanic vents, and excavated by a series of lakes, which in the province of Nicaragua interrupt and appear to replace the great chain of the Andes. The finely comminuted scoria affords a soil which produces the richest vegetation, and a vast and new field is offered to the man of science who will boldly face the miasma of the forest, or penetrate the rich mines with which one part of the country abounds.

At the termination of a narrow promontory, which runs in a northerly direction from the southern and eastern side of the Bay of Fonseca, stands the volcanic mountain of Cosegüina, washed on three sides by the ocean, of insignificant height, and flat-topped; two eruptions are on record, viz. those of the years 1709, and 1809. Since this last date it has remained in a state of quiescence, until the period of that stupendous eruption on the 20th of January last, the details of which I now beg permission to lay before the Royal Society. These details I have drawn up partly from official documents transmitted from the various towns to the government of Centro-America, and partly from the information of intelligent friends, eye-witnesses of all that occurred in those days of terror. The reports to the Government, which are voluminous, fully agree on the main points; in others, probably owing to the changes of locality and consequent variation in the direction of the wind, some slight differences are observable. It is, however, impossible to read these official reports, written too by persons little versed in classical learning, without being struck with the similarity of their description, even to the very terms he used, with that of the younger Pliny in relating to Tacitus the commencement of that eruption of Vesuvius which, nineteen centuries since, buried two cities under its ashes.

On the 19th of January, after twenty-six years of repose, a slight noise attended with smoke proceeded from the mountain Cosegüina. On the following morning

about half-past six o'clock, a cloud of very unusual size and shape was observed by the inhabitants of the neighbouring places to rise in the direction of this volcano. When viewed from San Antonio, about sixteen leagues to the southward, it took the appearance of an immense plume of the whitest feathers, rising with considerable velocity, and expanding in every direction. Its colour, at first of the most brilliant white, became tinged with grey, then passed into yellow, and finally became of a crimson hue. Columns of fire shot up directly through what was still imagined to be but a nebulous exhalation of extraordinary appearance; a severe shock of an earthquake was then felt. During the whole of the 20th the cloud preserved its appearance, although unattended by that magnificence which at first predominated.

At three P.M. on the 21st, two severe shocks were felt at San Antonio and Realejo, and at midnight five others were experienced; the two first undulations were not severe, but the third and the last were terrific.

On the morning of the 22nd the sun shone brightly at San Antonio, but a line of intense darkness was observed in the direction of the cloud which had excited so much attention two days before. At the same time a fine white ash was observed to fall around, the black line rose rapidly, the light began to fail, and darkness commenced with such quickness, that in half an hour it was more utterly dark than during the most clouded night. So intense was this darkness that men could touch without seeing each other, the cattle came in from the country showing all the signs of alarm and uneasiness, and the fowls went to roost as on the approach of night. This state of complete darkness prevailed until the following day, when at twelve o'clock it became a little brighter, and objects became visible at ten or twelve yards distance. This atmospheric darkness prevailed two days longer, during all of which time a fine white impalpable dust continued to fall. This deposit covered the ground at San Antonio about two inches and a half in three layers of different colours, the lower stratum being of a darker hue, the next of a greyish, and the upper of a whitish appearance. For ten or twelve days a murky light continued to prevail.

At Nacaome, a city in a northerly direction, eight leagues distant from the volcano, the same cloud was observed to rise at half-past six o'clock in a pyramidal shape. At half-past eleven on that day the darkness commenced, and at twelve nothing whatever could be distinguished; shortly before this a kind of ash had begun to fall, and at five o'clock had covered the earth to the depth of three inches. At six o'clock it fell in diminished quantity, and respiration became relieved. During the following night various undulations of different degrees of intensity were experienced, preceded either by heavy rumbling noises, or loud explosions. On the 21st at Nacaome the morning broke clear, but at eight o'clock the atmosphere became again thick and hazy, and during the twenty-four hours following, the volcanic matter continued to fall, attended with repeated noises and undulations of the earth. The darkness continued to prevail during the 22nd, and the depth of the ashes was from four to five inches; a fetid sulphurous smell proceeded from this deposit, which the

slightest breath of air drove into every interstice. At midnight violent explosions were heard, and, a quarter of an hour after, a severe shock was experienced, the forerunner of new eruptions. At 5 o'clock on the morning of the 23rd it was sufficiently light to observe that a fresh eruption had taken place from Cosegüina, and at 8 o'clock the darkness had returned as on the 20th. At 9 o'clock the obscurity was complete, and new and new awful echoes of vast discharges of volcanic matter, attended with flashes of lightning in all directions, convinced the panic-stricken inhabitants that the day of judgement had arrived.

No pen, says the Governor, is capable of describing the scene of dismay which prevailed. On the 24th the atmosphere became clearer. The houses were covered to the depth of eight inches with the ashes which had fallen, and many small birds and animals were found suffocated in them. Deer and other wild animals sought the town for protection, and the banks of the neighbouring streams were strewed with dead fish.

It would be but useless to tire the Society by giving extracts of all the reports made from different places within the sphere of the eruption. I shall confine myself to stating that at Macuelizo, in Segovia, the colour of the sand which fell was black; and on the Hacienda of Cosegüina, belonging to Don Bernardo Benevio, eight leagues to the southward of the crater, the ashes covered the ground to the depth of three yards and a half, destroying the fine woods and dwellings.

It also avails little to mention the great mortality which prevailed among the cattle. Thousands perished, and those which after the eruption reached the abodes of men, presented sad spectacles; their bodies in many instances being one mass of scorched flesh.

Within the bay of Fonseca, and two miles from the volcano, it is stated that two islands have been thrown up, of from 200 to 300 yards in length, their surface, but a few yards above the sea, presenting, it is said, a mass of scoria and ashes: their elevation has probably been caused by the heavy fall of scoriaceous matter on shoals previously existing in those places. However probable, the evidence is not conclusive, although the fact of the beach on the eastern or inner side of the promontory being extended by the ashes about 800 feet further out, gives additional reason to credit the statement.

On the 3rd of March, nearly two months after this great eruption, the volcano remained in a state of activity, but not ejecting ashes. By some geologists it has been considered that heavy eruptions of fine scoriaceous matter tend, by their falling again into the crater, to restore the volcano to a quiescent state, and that therefore this phenomenon more usually attends the conclusion of an explosion. In this particular instance it appears that the first effect of the explosion was to blow out of the crater, and finely triturate, the scoria and ashes left there twenty-six years before.

In the districts of Segovia, Comagagua, Choluteca, Nacaome, and Tegusigalpa, immense deluges of rain followed these clouds of ashes, and again gave rise to a fetid,

disagreeable odour. At this season such an occurrence was extraordinary, and almost unprecedented in Central America.

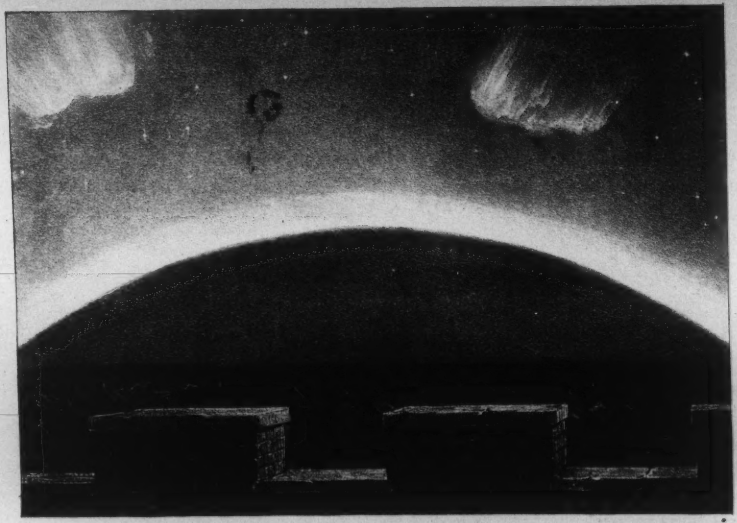
I shall conclude by stating that the ashes reached as far as Chiapa to the north, upwards of 400 leagues to windward of the volcano: thus proving the existence of a counter current of wind in the higher regions of the atmosphere. At St. Anne's, Jamaica, on the 24th and 25th of January, the sun was obscured, and not only there but over the whole island, showers of fine ashes were observed to fall. The distance in a direct line north-easterly is about 700 miles; consequently the ashes must have travelled at the rate of about 170 miles per diem.

Captain Eden, of His Majesty's ship Conway, informs me, that in lat. 7° 26′ north, and long. 104° 45′ west, when 900 miles from the nearest land and 1100 from the volcano, he ran forty miles through floating pumice, some of which was in pieces of considerable size.

The latitude of Cosegüina is 13° north, and longitude 87° 3′ west. Its height above the sea is computed at 500 feet.

No volcanic eruption in modern times has been recorded that reached the frightful extent of the one of which I have now had the honour of laying an account before the Royal Society. The explosion of Tomboro in Sumbaya in 1815, described in the Memoirs of the late lamented Sir Thomas Stamford Raffles, more nearly approaches it than any other with which I am acquainted.

Santiago de Chile,
18*th August*, 1835.



N° 1. *The Aurora Borealis at 9h 5m*

N° 2. *The Aurora Borealis at 9h 15m*

C. C. Christie del.

J. Basire lithog.

V. *Memoranda made during the appearance of the Aurora Borealis on the 18th of November, 1835. By* CHARLES C. CHRISTIE, *Esq. M.A. Communicated by* S. HUNTER CHRISTIE, *Esq. M.A. F.R.S. &c.*

Received and Read December 10, 1835.

AS the following Memoranda, noted down during the remarkable Aurora of the 18th of November, have been considered of sufficient interest to be read before the Royal Society, I think it right to state, that I did not anticipate that this would be the case, and that my only object in making them was, to record carefully whatever remarkable appearance might present itself, for the information of one who, I knew, felt a deep interest in every phenomenon of the kind, and whose simultaneous observations I hoped to have the pleasure of comparing with my own. Being, however, unwilling to make any alteration in a statement of facts, I have preferred leaving them exactly as they were communicated, merely offering a few additional explanations and remarks, suggested by striking exhibitions of the phenomenon.

Memoranda.

Wednesday evening, 9 o'clock.—Remarked, on looking accidentally from the drawing-room window of Deal Castle, a bright light over Ramsgate, exactly as if the moon were about to rise in that quarter. Saw, on proceeding to the roof, a perfect and very bright arch to the north; the lower edge being sharply defined on the dark cloud beneath, the upper shaded off into the sky. The sky, except beneath the luminous arch, perfectly cloudless; the stars shining brightly down to its upper edge. The wind rather high and gusty, not particularly cold, and north-north-west (by the ventometer). Altitude of the arch about that of γ Ursæ Majoris. Western extremity terminating exactly below α Aquilæ.

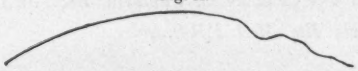
$9^h\ 5^m$.—The arch itself motionless, but large bodies of faint vapoury light continually ascending from it, and whirled in every direction, across the zenith, &c., as if by the wind, and with such rapidity as scarcely to be followed by the eye. These frequently rose perpendicularly, and were then sharply whisked off towards the south-east. (See Sketch 1. Plate II.)

$9^h\ 15^m$.—A fine outbreak of pencils of light from the centre and eastern extremity of the arch; none of them stationary, or in straight lines, but waving more or less and flickering, as if with the wind; masses of vapoury light whirled up occasionally; the whole presenting the appearance of an immense and not distant, conflagration,

while the paleness of the light and the absence of noise gave it a spectral and unearthly character, which was very striking. The gusts of wind increased the illusion. (See Sketch 2. Plate II.)

$9^h\ 20^m$.—The arch becoming very irregular; a large indentation on the eastern side, thus:

Fig. A.



The pseudo-flames have almost entirely subsided; they still exhibit the same appearance of burning, but in a steadier manner.

The centre of the arch is about equidistant from γ Ursæ Majoris and α Lyræ.

$9^h\ 25^m$.—The western extremity suddenly blazed up; one very broad pencil of rose-coloured light forming the western boundary to the rest; through it α Aquilæ shone with great brilliancy. All these westerly pencils perfectly straight, of greater altitude, and of a more defined and steady light than the easterly. Extremely narrow brilliant jets of light issuing from the central part, and having their base in the midst of the dark cloud. (See Sketch 3. Plate III.)

$9^h\ 27^m$.—Arch dilapidated. (See Sketch 4. Plate III.)

$9^h\ 30^m$.—Arch entirely broken up.

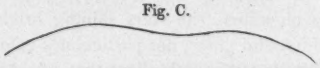
$9^h\ 35^m$.—The arch restored, but of an irregular undulating form, thus:

Fig. B.

The light fainter, as also the pencils which continue to rise from every part, but more distinctly from the two extremities.

$9^h\ 55^m$.—The arch much depressed, and in form thus:

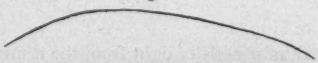
Fig. C.



The cloud below much darker, the pencillings very faint.

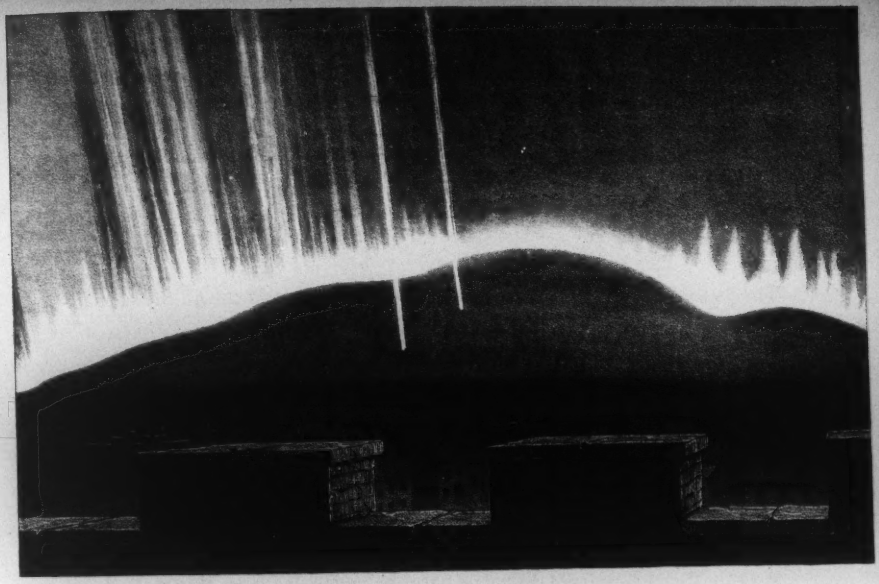
$10^h\ 20^m$. The arch strongly defined and steady; but occasional gleams, as of the vapoury light rising in a body simultaneously from the whole extent of it. The cloud very dark, and of this form:

Fig. D.



I regret that I was unable to wait for the total disappearance of the phenomenon, though for nearly an hour there had been no promise of any fresh display.

Note.—By the term *arch* is indicated the under surface of the body of light.



N° 3. *The Aurora Borealis, at 9h. 25m.*

N° 4. *The Aurora Borealis at 9h. 27m.*

C. C. Christie del.

J. Basire lithog

Remarks.

Having at the time no instruments for determining the bearing or altitude of the arch, I was obliged to depend upon the positions of some conspicuous stars, which were conveniently situated for that purpose. According to these rough data, the altitude was 18°; the angle subtended by the span of the arch about 130°; the bearing of the centre of the arch north-north-west, true, or very nearly magnetic north; and the arch was consequently at right angles to the magnetic meridian.

The body of light was nearly colourless; its brightness was similar to that seen on the edge of a cloud when the moon is about to rise behind it, with, however, this striking difference, that the stars were distinctly seen through the diffused light of its upper surface, and those in the tail of the Bear shone clearly in the very body of the light on the right hand.

With regard to the sketches with which I have attempted to illustrate the preceding notes, it is necessary to observe, first, that the extent of horizon renders it impossible to give in one view any idea of the magnificent scale on which the original was depicted, or even to preserve very correctly the relative proportions of height and breadth; and next, that in sketches I. and II. the rapid motion of the bodies of vapoury light, and of the flame-like pencils, must be held in mind: the former bore an exact resemblance to the faint reflected light darting across the sides of a room from a mirror turned sharply in the hand, and the latter to the lambent flames which diluted spirit of wine, poured on a flat surface and ignited, will exhibit when half extinguished.

The pencils which appeared in front of the dark cloud, of which there were not more than three, were very distinct in their character from the others; they were of a yellower tinge, and extremely narrow throughout their whole height. I have stated, that they *issued from* the dark cloud; perhaps it would be more correct to say, that they *pierced through* it; for although I did not observe the instant of their appearance, being at the moment engrossed by the display on the left, yet, in each, the brightness of the base, which was, as it were, the nucleus of its light, seemed to warrant this idea. The mere circumstance, however, of their appearance in front of the cloud, tends to elucidate a point on which there exists much difference of opinion, the height of the aurora in the atmosphere. The dark cloud itself can scarcely be supposed to have occupied a very elevated region; and it is manifest, that if these brilliant pencils had their origin *in*, or *in advance of*, the cloud, their bases must have been of inferior altitude to its upper portion, and equally so, if they were identical with any continuation of the luminous matter of the arch, concealed by the cloud.

The first appearance of the aurora at nine o'clock, was that of a dark convex cloud, cutting off the luminous arch, and concealing a body of light behind, the eye naturally referring the light to a more distant region, while the sharp line of division threw the cloud forward. Subsequent appearances, however, did not seem to confirm this

notion, but, on the contrary, induced me to consider, whether the dark cloud might not be a *substratum* of matter differing in nature and density from the superincumbent arch of light. The following are the facts which appear to favour this supposition. First; every great outbreak of coruscations from the luminous arch produced a corresponding disturbance in the part of the cloud immediately below. Thus, during the display at $9^h\ 15^m$, the arch was gradually losing its regular form on the right, and at $9^h\ 20^m$ I noted it "very irregular, with a large indentation on the eastern side," while on the west, where the body of light was undisturbed, the arch remained perfect. Thus also, immediately after the western half had been in vivid coruscation, the whole of the arch was "dilapidated," and finally, "entirely broken up." Secondly; neither the straight nor waving pencils appeared to proceed from behind the cloud, but always from the upper surface of the light. Thirdly; when the arch was "dilapidated" (see Sketch IV.), it was not merely its upper surface which was of irregular form, but masses of it were lying in confusion, separated from each other by a boundary of light, not appearing in the least as if light behind were shining through, but rather as if the substances of the arch and cloud had unwillingly interpenetrated each other, and refused to mix together more intimately, while at the same time the light above became more diffused and of diminished brightness. Lastly; when at $9^h\ 35^m$ the continuity of the arch is restored, it remains "of irregular undulating form" (fig. B.), while fainter pencils continue to rise from every part; at $9^h\ 55^m$ the cloud is "much darker," "the pencillings very faint," while its form is evidently becoming more regular (from fig. B. to fig. C.); and at $10^h\ 20^m$ it is nearly perfect in form (fig. D.), "strongly defined and steady," the pencillings having entirely ceased. All these circumstances struck me as so closely resembling the disturbance of two fluids, the one superposed on the other, mutually repulsive, but compelled to mingle by forces, of whose action the vividness of the pencillings seemed to indicate the intensity, and requiring intervals of repose to re-collect their scattered energies, that I cannot but conclude the luminous matter of an aurora to be a superincumbent stratum, and, consequently, that its altitude is dependent on that of the dark mass immediately beneath.

Bath, January 4th, 1836.

Nº II.

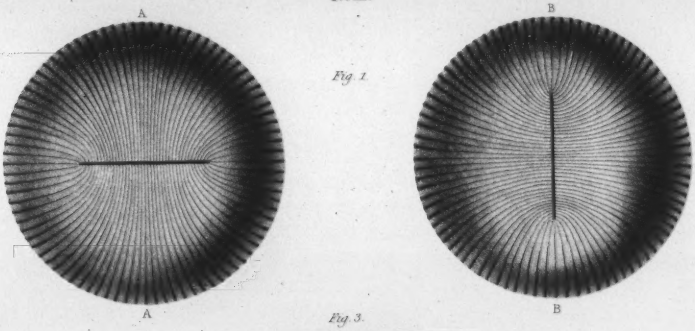
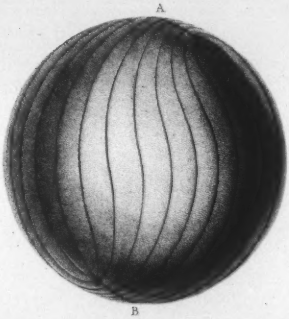


Fig. 1.

Fig. 3.



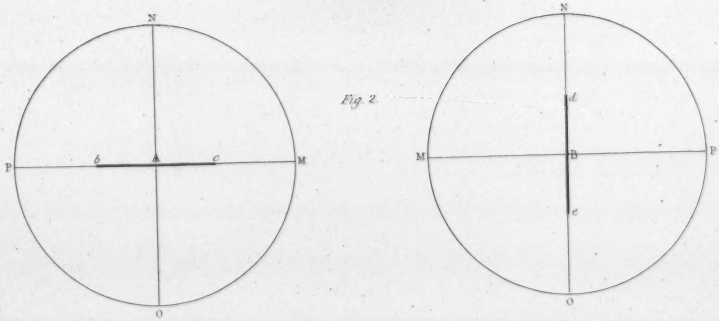


Fig. 2.

VI. *On the Anatomical and Optical Structure of the Crystalline Lenses of Animals. Continued from a former Paper* (*Phil. Trans.* 1833, *p.* 332.). *By Sir* David Brewster, *K.H. LL.D. F.R.S. &c. &c.*

Received November 26, 1835,—Read January 21, 1836.

§ 2. *On the Anatomical and Optical Structure of the Crystalline Lenses of Animals, particularly those of the Hare and the Salmon.*

In describing the various structures which exist in the crystalline lenses of animals, I shall proceed from the most simple to the most complex combination of fibres. In the paper which I have already submitted to the Society, I took the lens of the cod as the type of the *first* or simplest structure, in which the fibres, like the meridians of a globe, converge to two opposite points of a spheroidal or lenticular solid; and I shall now proceed to describe the *second* or next simplest structure, as exemplified in the lenses of the salmon and the hare.

This structure is shown in Plate IV. fig. 1, where A A is the anterior and B B the posterior hemisphere of the spheroidal lens of the salmon. A lens thus constituted is said to have *two septa* at each pole, A and B (fig. 2.), namely, the septa A *b*, A *c*, and B *d*, B *e*, in different points of which all the fibres have their origin and termination. One of the fibres, for example, which has its origin in A, passes over N, and terminates in *d*; while the other passes from A to O, and terminates in *e*. In like manner, the fibre which begins at *b* passes over P, and ends in B; while the fibre which begins at *c* passes over M, and terminates in B. The different parts of these four fibres lie in one plane, like the meridians of a globe. All the other fibres, which have their origin between A and B, have their termination between *d* and B; and all those which have their origin between A and *c* have their termination between *e* and B. All these fibres, or every fibre in each lamina except *four*, have their different parts lying in different planes, or form curves of contrary flexure. In order to understand this structure, and appreciate its beauty, it is necessary to draw the fibres upon a globular surface. A perspective view of such a globe is given in fig. 3, where A *b* is one of the anterior septa, and B *c* one of the posterior ones. The two curves which go from the poles A and B to the ends of the septa *c*, *b*, are ellipses, while all the rest are curves of contrary flexure.

The length of the septa A *b*, A *c*, &c. varies in different fishes; and when they are very short, as they must necessarily be in small lenses, we are apt to mistake the structure which they indicate for that of a diffused polarity round the two extremities

of the axis of the lens. When such an ambiguity, however, presents itself, we must observe carefully if the diffusion is more elongated in one direction than another. If it is not, and especially if there is seen a small circular dimple or depression at the poles when we follow the light reflected from the surface of the lens, there can be no doubt that the fibres converge to two opposite poles, like the meridians of a globe. But if the diffused polarity be of an *oval* form, and if the greater axis of the oval on one side of the lens be perpendicular to the same axis on the other side, we may then safely infer that the fibres are related, as in the lens of the salmon, to *two* septa at each extremity of the axis of vision. When these methods fail, the ambiguity may sometimes be removed by boiling the lenses, and observing the manner in which they crack at the poles.

From a too hasty generalization of a small number of facts, Leeuwenhoek maintained that all fishes and birds have two short septa; and M. Sattig* has advanced the same opinion in reference to fishes, and committed the additional mistake of making every part of the fibres lie in the same plane. Dr. Thomas Young maintains that there are no septa in the lenses either of birds or fishes†; and in pursuing the chimæra of a muscular lens, he not only renounced discoveries of his own, after Leeuwenhoek, but adopted grave errors which have no foundation whatever either in observation or analogy.

It is a very remarkable circumstance, that the hare and the rabbit should be the only quadrupeds whose lenses have two septa, like that of the salmon. This fact was observed by Leeuwenhoek‡ and also by Sattig, who remarks that the septa are larger in the lenses of these animals than in those of fishes.

The fibres of the lenses of the hare and the rabbit are curves of contrary flexure, like those of the salmon, and they are distinctly toothed, though the teeth are much smaller than those of the salmon and the cod, of which I have given a representation in a former paper.

Although I have stated that the hare and the rabbit are the only quadrupeds whose lenses are known to have the same structure as that of the salmon, yet there is a rare marsupial quadruped, the *Perameles nasuta* of Geoffroy, whose lens will probably be found to have two septa on each of its surfaces. Professor Grant was so good as to send me a single lens of this curious animal; but as one of its faces was much injured, I was able only to discover the two septa on the side of the lens which was uninjured. It is highly probable that the fibres will have a similar arrangement on the other side of the lens; but until this is actually determined, it is possible that

* Lentis Crystallinæ Structura Fibrosa. Preside Reil Defendit Samuel Godofredius Sattig Silesius. Halæ, 1794. It is not surprising that the author of this thesis should have followed Leeuwenhoek in this mistake, as he seems to have examined the lenses *only* of the carp and the perch, in which the two septa are most distinctly developed.

† Elements of Natural Philosophy, vol. ii. p. 599. col. 1. and Plate XII. fig. 100.

‡ Opera, vol. ii. p. 66. Lugd. Batav. 1722.

the *Perameles nasuta* may have its lens formed according to another class of structures which will afterwards be described. The fibres of the lens of this animal are extremely small, and the teeth upon them, though very minute, are distinctly seen with a high magnifying power.

The structure indicated by two septa is perceived very distinctly in the lens of the *Cobra Capella* and the *Lacerta Gecko*, and indistinctly in that of the *Stellio Gecko* and the *Frog*. The lenses of the *Cobra Capella* and the *Lacerta Gecko* are nearly spherical; and the laminæ of the lens of the *Stellio Gecko* are composed of a fibrous tissue, and not of fibres united by teeth.

In the following Table I have given the names of the different animals whose lenses have the structure shown in fig. 1.

	QUADRUPEDS.	
Hare.	Rabbit.	*Perameles nasuta* (one side).
	REPTILES.	
Cobra Capella.	*Lacerta Gecko.*	*Stellio Gecko* (probably).
Frog (probably).	Alligator.	
	FISHES.	
Salmon.	Tench.	Hickory Chad.
Dolphin.	Carp.	Cavala (Georgia).
Shark.	Perch.	Stingarie.
Porpoise.	Sturgeon.	Skip-jack.
Skate.	Gudgeon.	Chad.
Thornback.	Cat-fish.	Black-fish (Georgia).
Boneto.	Par.	Fish from Singapore.
Dog-fish.	Red Trout.	Sheep-head.
Sword-fish.	River Trout, common.	

1. *Hare.*—I have examined the lenses both of the common hare and the blue mountain hare, which have the same structure. In observing their action upon polarized light, I find that they depolarize *two series* of luminous sectors, the inner sectors having the *negative* structure like *calcareous spar*, and the outer sectors the *positive* structure like *zircon*. In order to perceive the inner sectors, the lens must be taken out of the eye with great care, and subjected to no pressure, and the polarized light must be transmitted through its axis.

2. *Rabbit.*—The lens of the rabbit resembles that of the hare in its general properties. The cornea of a rabbit depolarized faint sectors of light, and its polarizing structure was *negative*.

3. *Perameles nasuta.*—When the lens of this animal was taken out of the spirits which preserved it, and the outer coat removed, its fibres were crossed perpendicularly by irregular and slightly serrated lines, much more distinct than the fibres. As

these lines are not now visible in the indurated lens, I cannot even form a conjecture respecting their origin.

4. *Stellio Gecko* and *Frog*.—The lenses of these animals require to be re-examined.

5. *Salmon*.—The length of the septa in the lens of the salmon is less than those in the hare and rabbit, and the teeth upon the fibres are extremely distinct. The lens depolarizes *three* series of luminous sectors, the *inner* and outer series being *negative* and the intermediate series *positive**. The polarizing structure of the cornea was *negative*, and it depolarized very high tints at its junction with the sclerotic coat. The structure of the *sclerotic coat* is very remarkable. In the eye which I examined, the thickness of the sclerotic was about the fifteenth of an inch, and with a sharp knife it could be cut like a piece of cheese. It had a milky transparency like some opals. When a slice with parallel faces, nearly perpendicular to the surface, was exposed to polarized light, it exhibited the system of biaxal rectilinear fringes†, exactly like those in a plate of glass heated by boiling water or oil, and in the act of rapid cooling. The same structure exists in the sclerotic coat of the *Cod*.

6. *Dolphin*.—The lens of the dolphin is decidedly an oblate spheroid, the axis of which is that of vision. In an indurated lens the axis of the spheroid is 0·254 of an inch, and the equatorial diameter 0·307. The teeth of the fibres are small and irregular, like those of quadrupeds.

7. *Shark*.—The lenses of the common and blue-eyed shark have the same structure. The sclerotic coat has the remarkable property (when cut in the manner already described in the case of the salmon) of depolarizing light like *a plate of bent glass*‡; but, what is very curious, the *concave* side of the sclerotic has the same action upon light as the *convex* side of the bent glass.

8. *Alligator*.—The lens of the alligator is nearly spherical: the teeth of the fibres, as in the dolphin, are shorter than those in fishes.

9. *Skate*.—The lens of this fish depolarizes *three* series of luminous sectors, but the inner series is not so distinct and near the axis as those of the cod. The inner and outer series are *negative*, and the intermediate series *positive*. The horny sclerotic of the skate has the very same polarizing structure as that of the salmon. The teeth of the fibres of the lens are exceedingly small.

10. *Thornback*.—The fibres are very delicate, and their teeth small but distinct. The lens depolarizes luminous sectors; but from the imperfection of the lens I could not observe their character.

11. *Boneto*.—The lens is an oblate spheroid, whose axis is that of vision. The teeth of the fibres are very distinct§.

12. *Sword-fish*.—The diameter of the lens of this fish which I examined was 1·10th of an inch. The teeth of the fibres were so distinct that *three* circles of the secondary colours produced by them were distinctly seen‖.

* Philosophical Transactions, 1816. † Ibid., 1816. ‡ Ibid., 1816.
§ Ibid., 1833, p. 332. ‖ Ibid., 1833, p. 327.

13. *Tench.*—This fish has a very small eye, and a small lens. Owing to the faintness of the polarized tint I have observed only one series of luminous sectors, whose character is *positive*.

14. *Perch.*—The lens of this fish, like that of the tench, exhibits only one set of luminous sectors by polarized light, and their character is also *positive*.

15. *Gudgeon.*—The teeth of the fibres are very distinct. I have observed only one series of luminous sectors, which are *positive*.

16. *Cat-fish.*—This is a poisonous fish from Georgia. The teeth of the fibres are very fine.

17. *Red Trout.*—Its lens was nearly spherical, and 0·244 of an inch in diameter. It distinctly depolarized *three* series of luminous sectors, the inner and outer ones being *negative*, and the intermediate one *positive*. The teeth of the fibres are very distinct.

18. *Hickory Chad.*—This fish is from Georgia. The fibres are distinctly seen.

19. *Cavala.*—This fish is from Georgia. On one side of the lens there are two short septa. The secondary colours produced by the teeth of the fibres are very distinct.

20. *Stingarie.*—This is a poisonous fish from Georgia. The lens is of an oblong form when viewed in the direction of the axis of vision. The two septa on one side are very long.

21. *Skip-jack.*—The teeth of the fibres are short, like those of quadrupeds.

22. *Black-fish from Georgia.*—The fibres of its lens are large, and the colours which they produce very finely seen near the septa.

23. *Sheep-head from Georgia.*—The fibres of its lens are unusually large*.

24. *Fish from Singapore.*—Mr. George Swinton sent me a number of the eyes of this fish, but the name has been lost. Its lens has the form of an oblate spheroid, the axis of which is the axis of vision. This axis is 0·60 of an inch, and the equatorial diameter 0·70. The secondary colours produced by the teeth are finely displayed.

§ 3. *On the Anatomical and Optical Structure of the Crystalline Lenses of Animals, particularly those of the Lion, Tiger, Horse, Ox, and other Quadrupeds.*

In the two preceding sections I have described the two simplest combinations of fibres which characterize the structure of the crystalline lens in animals. The lenses of birds possess the *first* or simplest structure; the lenses of fishes in general exhibit either the *first* or the *second* structure; and we shall now show that the lenses of the Mammalia in general, with the exception of the *hare* and the *rabbit*, and other quadrupeds of peculiar habits, are characterized by a *third* and a more complex structure, in which *three* septa diverge from each pole of the lens at angles of 120°, the

* See Philosophical Transactions, 1833, p. 329.

septa of the posterior surface bisecting the angles formed by the septa of the anterior surface.

This beautiful structure is shown in Plate V. figg. 1. and 2; fig. 1. representing the anterior, and fig. 2. the posterior, surface of the lens, which in quadrupeds is a real lens or lenticular solid, the curvature of the posterior surface having a shorter radius than that of the anterior surface.

The progress of the fibres round the edges of the lens in their passage from the one surface to the other is shown in fig. 3, where the lens is supposed to be transparent, the dark continuous lines representing the *three septa* and the fibres of the anterior surface, and the dotted lines the three septa and the fibres of the posterior surface. From this representation it will be seen that there are *three* fibres having their origin in the anterior pole, and terminating at the extremities of the posterior septa; and other three having their origin in the posterior pole, and terminating in the extremities of the anterior septa, which have their parts all lying in one plane, while every other fibre of the lens forms a curve of contrary flexure, in order to carry it to its proper termination in the opposite septum. Hence it follows, that with the exception of the six fibres originating in the poles, the parts of all the other fibres which constitute the margin or rim of the lens are not parallel to its axis.

The arrangement of fibres shown in figg. 1. and 2. may frequently be seen, particularly in the lenses of old animals when they are large, by examining them in their entire and transparent state within their capsule. In some cases I have seen them distinctly by looking down upon the surface of the lens; but when they are visible in this way they may be seen to most advantage by looking through the surfaces with a small magnifier, when the lens is placed in a fluid of nearly the same refractive power.

In tracing the fibres to their termination in three *septa*, I employed the optical method already described in a preceding paper; but in general the superficial coloured images are not so distinct as those in the lenses of fishes, though, like them, they may be transferred to wax, and are like the colours of mother-of-pearl.

The fibres of the lenses of quadrupeds gradually diminish in size from the equator or margin of the lens, where they are a maximum, to their termination in the anterior and posterior septa. They are united together by small teeth, like those of fishes; but generally speaking the teeth are smaller and less distinctly pronounced; and in some lenses I have found it extremely difficult to exhibit them with the finest microscopes. As the teeth can only be seen in the indurated state of the fibres, it is probable that their form may be in a great measure altered or obliterated by the process of induration, especially when we consider that the lenses of quadrupeds are very much softer than those of fishes, and that the evaporation of the aqueous portion must produce a greater change upon the indurated albumen when it is most abundant.

The existence of three septa was observed by Leeuwenhoek, and afterwards by Sattig, in the lenses of the ox, the horse, the sheep, the goat, the fox, the dog, and

N° III.

Fig. 1.

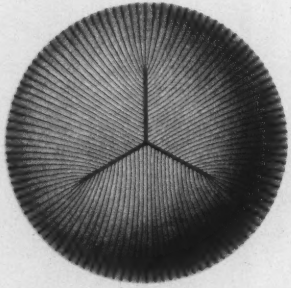


Fig. 2.

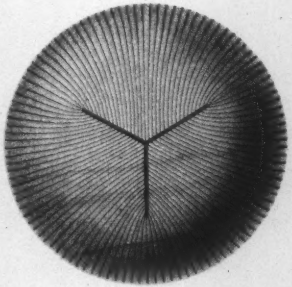
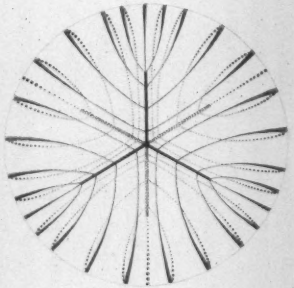


Fig. 3.



N° IV.

Fig. 1.

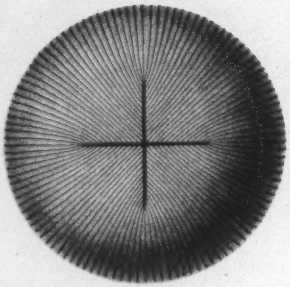
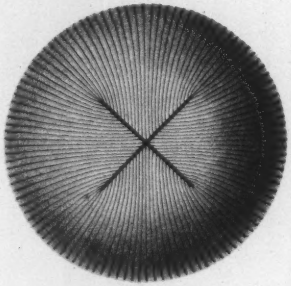


Fig. 2.



the cat; but LEEUWENHOEK committed the strange mistake of supposing that each coat or lamina of the lens consists of circumvolutions of a single fibre, whereas each fibre has a distinct termination in the septum at both its extremities. Dr. THOMAS YOUNG seems to have been occupied with the examination of the crystalline lens of the ox in 1792 or 1793, previous to the publication of SATTIG's thesis; and in his observations on Vision, read before the Royal Society on the 30th of May, 1793, he has given a drawing and description of the result of his observations*. This description is exactly the same as that previously given by LEEUWENHOEK, excepting that Dr. YOUNG erroneously maintains that each of the coats of the lens "consists of fibres intermixed with a gelatinous substance," which we presume he considered to be necessary for cementing the fibres into a compact body.

In Dr. YOUNG's subsequent and more elaborate paper on the Mechanism of the Eye†, he renounces as erroneous the description which he had given of the arrangement of the fibres in the crystalline of the ox; and he substitutes, in place of his former correct drawing, another, which is altogether visionary. "In man," says he, "and in the most common quadrupeds, the structure of the lens is *nearly similar*. The number of the radiations is of little consequence; but I find that, sometimes at least, in the human crystalline there are *ten* on each side, not *three*, as I once, perhaps from a too hasty generalization, concluded. Those who find any difficulty in discovering the fibres must have a sight very ill adapted to microscopical researches." Notwithstanding the assertion in this passage that the structure of the lens in man and the most common quadrupeds is *nearly similar*, yet fig. 93, in which he represents "the order of the fibres of the human crystalline," is essentially different from fig. 95, in which he shows the "ramifications (of the fibres) from the margin of the crystalline lens" in quadrupeds. In the first of these figures there are no ramifications of the fibres from the margin of the lens, whereas in the second there are no fewer than six, placed without any symmetrical relation either to the septa or to one another.

The following Table contains the names of the different animals in whose lenses I have found the structure shown in Plate V. figg. 1, 2, and 3. These animals are all quadrupeds, with the exception of the anonymous fish caught near the Azores, to which I shall have occasion again to refer‡.

Lion.	Ox.	Goat.
Tiger.	Cow.	Sow.
Horse.	Sheep.	Deer, Fallow.

* Elements of Natural Philosophy, vol. ii. p. 525. "In examining," says he, "the crystalline from an ox, I discovered a structure which appears to remove all the difficulties with which this branch of optics has long been obscured."

† Elements of Natural Philosophy, vol. ii. p. 597.

‡ For the lenses of several of the rarer animals contained in the following Table, I have been indebted to the liberality of the Zoological Society of London.

Deer, Roe.	Dog.	Baboon.
—— Indian.	Cat.	Monkey, Douroucouli.
—— Moose.	Otter.	——— Black Spider.
Buffalo, Wild.	Rat.	——— Entellus.
Nilgao, India.	Mouse.	——— Green.
Llama.	Opossum.	——— Lesser White-nosed.
Puma.	Squirrel.	——— Jacketed.
Cheetah.	Lemur, Black.	Vicugua.
Antelope, Melampus.	——— Red-fronted.	Capybara.
——— Pygmy.	——— Nocturnal.	Chinchilla.
Fox, Common.	Coati, Brown.	Ichneumon.
—— Black, from America.	Suricate.	Fish caught near the Azores.

1. *Lion.*—The posterior surface of the lens is the most convex. The fibres which compose the lens are exceedingly distinct, and the teeth smaller and sharper than those of fishes.

2. *Tiger.*—The fibres and the teeth which unite them are very distinct, and like those of the lion. In a lens preserved in spirits, and deprived of its external lamina, the lamina shone with all the incommunicable colours of mother-of-pearl. When the lens was dried these colours disappeared.

3. *Horse.*—When the lens of the horse is thrown for a second or two into boiling water, and is allowed to dry slowly, it splits in such a manner as to show the septa and the general structure of the lens very satisfactorily. The fibres are well seen, and the teeth distinct and small like those of the lion and the tiger.

4. *Llama, Puma, Capybara,* &c.—The fibres and teeth are exceedingly distinct in the lenses of these animals, and equally or less so in all the other animals in the Table.

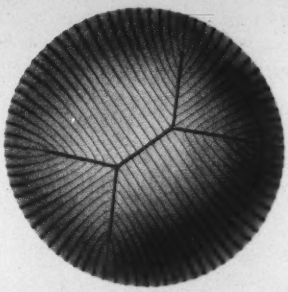
5. *Cat.*—In order to observe if the fibrous structure was the same in the young animal, I took the eyes of three kittens, six in number, about eighteen hours after birth; and what was very remarkable, I found that the lenses in all the six were white and opake, with a perfectly transparent rim. The three septa were distinctly seen in all the lenses. It will be interesting to ascertain, if in all animals which are born blind, the crystalline is opake at their birth, and gradually becomes transparent from the margin to the centre during the period in which the eye is closed against the admission of light.

6. *Chinchilla.*—The lens of the chinchilla seems to fill the whole ball, and the cornea is exceedingly large, forming almost a hemisphere.

7. *Suricate.*—The cornea of the suricate is nearly of the same size as that of the chinchilla.

8. *Unknown Fish from the Azores.*—The lens of this fish, which was sent to me without a name by Mrs. Green of Cumberland Island, is in every way remarkable. It is the only lens of a fish which has the structure belonging to quadrupeds; and it is equally peculiar in having the form of a prolate spheroid, the axis of which coin-

Fig. 3.



Nº. IV.

Fig. 4.

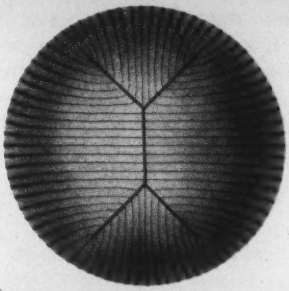


Fig. 5.

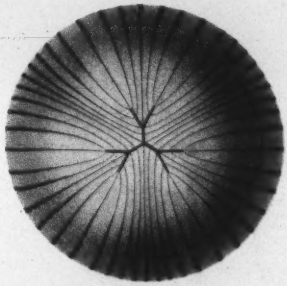


Fig. 6.

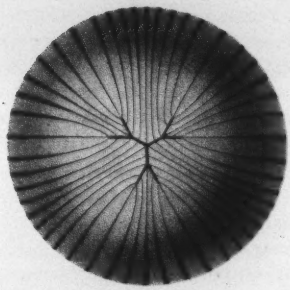
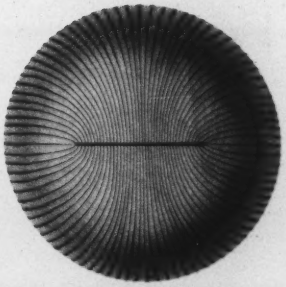
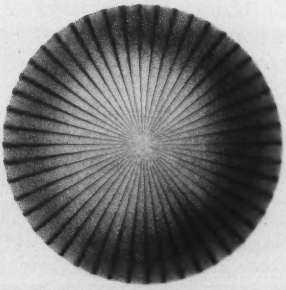


Fig. 1.



Nº. V.

Fig. 2.



cides with that of the eye. The length of the axis is 0·327, and its equatorial diameter 0·273 of an inch.

In some quadrupeds I have observed an irregularity in the septa, which may have arisen either from an original malconformation of the lens, or from some accidental injury. In the lens of a horse I observed a spurious septum, one of the three being double. The same fact will be more particularly noticed in describing the lens of the elephant, in which it is more common.

§ 4. *On the Anatomical and Optical Structure of the Crystalline Lenses of Animals, particularly those of the Whale, the Seal, the Bear, and the Elephant.*

From the lenses of quadrupeds in which the fibres are related to three septa, I shall now proceed to describe the fourth and last class of symmetrical structures, in which the fibres are related to four septa, placed at right angles to one another. This combination of fibres is of rare occurrence, and I have found it only in the lenses of the *whale*, the *seal*, and the *bear*.

The character of this structure will be understood from Plate V. figg. 1 and 2, where fig. 1 represents the anterior surface, and fig. 2 the posterior surface, of the lens. The septa on the posterior surface are inclined 45° to those on the anterior surface, so that if the lens were transparent, the septa when seen at the same time would appear like the eight radii of an octagon, inclined 45° to one another. In this structure there are eight fibres, all the parts of which lie in one plane, passing through the axis of the lens; namely, *four* extending from the extremities of the four anterior septa and terminating in the posterior pole, and other *four* extending from the extremities of the *four* posterior septa and terminating in the anterior pole. All the other fibres of the lens, except these *eight*, are curves of contrary flexure, which necessarily change their direction in passing from one septum in the one face to another septum in the other inclined 45° to it.

The structure which I have now described is exhibited in the crystalline lenses of the *whale* and the *seal*. I found it distinctly developed in the lens of a whale, forty-six feet long, caught by Captain Ross in his first voyage in the Arctic regions, and also in a specimen of the great seal, or *Phoca barbata*, which Lieutenant Robertson of the Isabella brought home from Baffin's Bay in the same year.

In the lenses of other whales and seals, however, I have found a different structure, which is represented in Plate VI. figg. 3 and 4. This combination of fibres differs from that in figg. 1 and 2, in having two centres of divergence in place of one in each surface of the lens; but if we conceive these two points to coincide, the two structures become identical. In both, the principal septa are at right angles to each other on the same face, and inclined 45° to those on the opposite face; so that the general character of the two structures is the same.

In the lenses of one of the bears killed by Captain Ross during his first voyage, I found the structure shown in figg. 2 and 3. The distance between the centres of

divergence was greater on one side of the lens than on the other, in both lenses; and in one it was so small as to show very nearly the structure in figg. 1 and 2.

In the lenses of other whales, one of which was thirty-five feet long, I found the structure shown in figg. 1, 2, 3, and 4, in which there are *four* radiations of fibres, or vortices, as I believe LEEUWENHOEK calls them, on each surface of the lens; but I have found in one lens a spurious structure, in which there are *five** radiations on one face and *three* on the other face of the lens.

When the lens of the whale has been preserved in spirits, the coats have often a brilliant pearly lustre, a phenomenon which I have seen with equal beauty in the lenses of some quadrupeds, especially in that of the tiger. The fibres of the lens of the whale are extremely distinct, and the teeth upon them, which are visible with high powers, resemble those in the fibres of quadrupeds.

The crystalline lens of the *elephant* possesses many remarkable peculiarities. The following are the dimensions of the eyeball of an elephant, which Mr. GEORGE SWINTON was so kind as to send me from Bengal.

Eyeball.

	Inches.
Longest diameter	1·82
Shortest diameter	1·10

Cornea.

Longest diameter	1·35
Shortest diameter	0·92

Crystalline Lens.

Longest diameter	0·700
Shortest diameter	0·627
Thickness	0·400

Ratio of the two diameters, 1 to 1·1125.

In another eye the dimensions of the lens were as follow:

Crystalline Lens.

	Inch.
Longest diameter	0·784
Shortest diameter	0·700
Thickness	0·450

Ratio of the two diameters, 1 to 1·116.

* LEEUWENHOEK says that there are *five* septa in the lens of the whale; but I have not his paper beside me in order to ascertain whether he means by this five radii inclined 72° to each other, or *five* radiations such as I have found in the spurious structure here mentioned.

Hence it is obvious that the crystalline lens of the elephant differs from that of most other animals, in being of an *elliptical* form, the horizontal diameter of the ellipse being the longest.

When the lens has been preserved in spirits, and the outer coats are removed, it resembles a piece of the finest amber. In this condition, when dry, it does not crack, like other lenses, but exfoliates in thin scales, which give it the appearance of a flat pearl. When these scales are rubbed off it resumes its appearance of amber, and when well dried, after two or three exfoliations, it becomes as permanent in its appearance and colour as that substance, and almost as hard and durable.

This peculiar property of the elephant's lens arises from the peculiar structure of its coats or laminæ. These coats are not, properly speaking, composed of fibres, but are of a fibrous tissue, the elementary fibres of which are not united mechanically by teeth, but by some other process, probably that of agglutination, which I cannot discover by the finest microscopes which I possess *.

Owing to this structure of the laminæ, the superficial colours are not displayed, as in other lenses, and it is very difficult to trace the elementary fibres into the septa, to which they are related. I have succeeded, however, in determining that there are *six* radiations of fibres and *three* centres of divergence on each surface of the lens.

This structure is shown in Plate VI. figg. 5 and 6, in which there are *three* septa diverging from the poles of the lens, as in quadrupeds, and from the extremity of each *two* additional septa, which are the real septa, to which the fibrous radiations are principally related. The three central septa are inclined 120° to each other, and the two additional septa seem to be inclined at an angle of 60° to each of the central ones; but these measures are of course only rude estimates of the inclination of lines, which in animal and vegetable organizations, and even in those of the mineral world, approximate only to the mathematical type of their characteristic structure.

In the combination of fibres shown in figg. 5 and 6, there are twelve fibres whose parts all lie in the same plane, all the rest forming curves of contrary flexure.

In some lenses of the elephant, I have found the three septa which meet in the poles of the lens exceedingly small, and approaching to evanescence; and I have no doubt that, as happens in the case of four septa, these three central septa will in some lenses be wanting; so that the other six septa will diverge from the poles at angles of 60°, like the radii of a hexagon. Such a lens will bear the same relation to the structure shown in figg. 5 and 6 as the structure in figg. 1 and 2 bears to that in figg. 3 and 4.

* In extremely thin and highly-dried fibres, the fibres are better seen with the microscope; and I have observed something like a mechanical union of them.

§ 5. *On the Structure of the Crystalline Lens of the Turtle and other animals in which the Fibres are differently combined in the anterior and posterior Surfaces.*

In the various structures of the crystalline lens which have been described in preceding communications, the fibres are similarly arranged on both the surfaces of the lens, whether these two surfaces are similar, or have different degrees of convexity. In this respect they resemble the artificial lenses of the optician, in which there is no other deviation from symmetry but in the curvature of their surfaces. I have discovered, however, in the lens of the turtle, and in that of several fishes, a new combination of fibres, in which they are differently arranged in the anterior and posterior faces of the lens.

Ever rich in her forms and fertile in her resources, Nature thus presents to us in the crystalline lens four singular properties, which the most skilful optician, even if he knew their design, is not likely ever to attempt to imitate. But the study of these properties is not on this account the less interesting; for though we may never be able to produce the same effect, either by similar or analogous means, yet we may be led to discover some other principle within the sphere of art by which the desired result may be obtained. The four properties to which I refer are the increase of density from the surface to the centre of the lens; the alternations of negative and positive structures, as exhibited by the action of the lens on polarized light; the arrangement of the fibres in reference to different numbers of septa; and the defect of symmetry in this arrangement in the turtle and a few fishes. The first of these properties, namely, a variation of density, is no doubt intended to correct spherical aberration, an effect which may be produced by the union of several spherical surfaces, or by hyperbolical or elliptical surfaces, or by surfaces of contrary flexure; but the design of the other three properties has not even excited the ingenuity of conjecture, and will probably remain among the numerous problems which will exercise the sagacity of another age.

When I first observed a defect of symmetry in the arrangement of the fibres in the two halves of the lens of the turtle, I was extremely doubtful of the accuracy of my observations, and was therefore at peculiar pains to confirm the result by examining several lenses of the turtle. In every lens, however, I found the deviation from symmetry was clearly indicated, though it did not possess the same character in every lens which came under my notice.

In the eye of the *turtle* which I first examined, the ball was one inch in diameter, and the diameter of the lens only 0·200 of an inch. The lens was nearly spherical; and it had on its anterior face *two* septa, like the hare and the salmon, as shown in Plate VI. fig. 1; but on its posterior face the fibres converged to a single pole, as shown in fig. 2. In this structure there are only four fibres in each lamina, which have their different parts lying in the same plane. All the other fibres are concave towards a plane passing through the two anterior septa, and of course convex towards a plane

at right angles to it, so that in this structure the fibres exhibit no contrariety of flexure, as in all the other lenses with septa at each pole. The very same structure appeared in the other lens of the same turtle; and was found likewise in both the lenses of another turtle, in which the diameter of the eyeball was 1·2 of an inch, and that of the lens 9·23.

The fibres and their teeth are nearly the same in the turtle as in the lenses of quadrupeds, the teeth being very short, though perfectly distinct.

The structure represented by figg. 1 and 2 (Plate VI.) is possessed by the lenses of the following fishes:

The Drune.	Crocer.	Grey Gurnard.
Bass.	Angel Fish.	Red Gurnard.
Whiting from Georgia.	Mullet? from Georgia.	

1. The *Drune*, or *Drum*, from Georgia.—The lenses of this fish, along with those of the other fishes in the above table, except those of the gurnard, were sent to me with their common names by Mrs. Green of Cumberland Island. In both the lenses of the drune the different structures were very distinctly seen. The fibres varied greatly in diameter as they approached the pole; and though the secondary colours were not visible, yet the teeth of the fibres were beautiful and distinct.

2. The *Bass* from Georgia.—In this lens the double arrangement of the fibres is distinctly seen.

3. The *Whiting* from Georgia.—The same structure is clearly seen in both the lenses of this fish.

4. The *Crocer* from Georgia.—In both the lenses two short septa are seen in the anterior face, and a single pole in the posterior one. The teeth of the fibres are very close, but distinct.

5. The *Angel Fish* from Georgia.—In the lenses of this fish the two structures are very distinct, and also the teeth of the fibres.

6. *Mullet* from Georgia.—I am in some doubt respecting the existence of the double structure in the lens of this fish. The observation which I made many years ago is thus recorded: "A sort of diffused polarity round two poles, as if there were two septa on each side, like those of the hare, or on one, like that of the turtle." As a diffused polarity was the only fact actually observed, I find that I have placed the mullet in the list of fishes without septa.

7, 8. *Red* and *Grey Gurnard*.—In the lenses of both these fishes there were two short septa on one side and none on the other.

In examining the lens of a very large turtle, in which the eyeball was 1·4 of an inch in diameter, and that of the lens 0·25 of an inch, I found to my great surprise that there were three septa in front, as in quadrupeds, and *four* septa, or rather *six*, as in the seal. The following is the account of the observation which I recorded at the time: "I saw distinctly the above septa on both sides when the lens was fresh and transparent, and plunged in oil of almonds, and I confirmed this result by boil-

ing the lens and removing the laminæ. The four septa require nice observation to be seen. One of the *three* was inclined about 45° to one of the *four* septa. The other lens of this turtle has most distinctly four septa, but *only two on the other side.*" A similar variation of structure has already been referred to as existing in the lenses of the horse and the whale.

Belleville by Kingusie,
November 14th, 1835.

VII. *On an Artificial Substance resembling Shell: by* Leonard Horner, *Esq. F.R.SS. Lond. & Edinb. With an Account of an Examination of the same: by Sir* David Brewster, *LL.D. F.R.S. &c.*

Received October 5, 1835,—Read February 25, 1836.

WHILE I was, some time ago, officially inspecting the cotton-factory of Messrs. J. Finlay and Co., at Catrine, in the county of Ayr, on going over the bleaching-establishment attached to it, I was struck with an unusual appearance of a part of the machinery, which, at a distance, looked as if it were made of brass. On a closer examination, I found that it was a large circular wooden box coated with an incrustation of a brown compact substance, having a highly polished surface, a metallic lustre, in some places beautifully iridescent, and when broken exhibiting a foliated texture *. This resemblance in structure and pearly lustre to some species of shells, such as the *Meleagrina*, *Malleus*, *Avicula*, *Ostrea*, *Pinna*, and others, induced me to examine the substance more closely, conceiving that it might possibly throw some light on the formation of shell.

The part of the machinery on which I observed the incrustation is called a Dash-wheel, and consists of a circular box, about seven feet in diameter and three feet in width, revolving upon a horizontal axis, and having its interior divided into four compartments, into each of which there is a circular opening on one side. The purpose of this wheel is to wash or rinse the cloth in pure water, after it has been boiled or steeped in the bleaching-liquors. It makes twenty-two revolutions in a minute, which is found to be the proper degree of speed, in order that the cloth may be tossed about and *dashed* against the sides as the wheel turns; a greater velocity causing it to keep at the circumference without shifting its position.

I was told that the incrustation was a deposit of carbonate of lime, and the source of the lime was mentioned. But whence the brown colour, and the metallic nacreous lustre? If the substance were analogous to shell, it ought to contain animal matter; and whence could that be derived? It was necessary to trace the operations from the beginning.

The cotton cloth is brought to the bleach-field in the state in which it is taken from the weaver's loom. The first process is to steep it in water for several hours, after which it is immersed in cream of lime. This is made in the following manner: fresh-burned lime is slaked and passed through a fine sieve, and added to water in the proportion

* Specimens are deposited at the British Museum.

of 38 lbs. of dry lime to 1000 lbs. of cloth. The cloth is boiled in this liquor from four to six hours, the lime acting as an alkali; and it is used only from being considerably cheaper than potash or soda. After this boiling, the cloth is taken to the dash-wheel to be thoroughly cleared of the lime, which is effected by its being tossed about for ten minutes in clear water in the interior of one of the compartments into which the wheel is divided. Here, then, is the source of the calcareous matter of the incrustation; and we have the lime dissolved or suspended in the water in a state of extremely minute division, and from which it is deposited, most probably, by a partial evaporation. It is difficult to say whether the deposit takes place while the wheel is revolving, by the water being broken into a kind of spray, and so presenting a greater surface for evaporation, or during the night, when the wheel is still: some of the properties, to be afterwards described, render the latter supposition the most probable. But in whatever way it takes place, the operation is an exceedingly gradual one; for the wheel had been in constant use for ten years, and the coating in the interior did not exceed one tenth of an inch in thickness. It had been in operation about two years before any perceptible deposit showed itself in the inside; but it had not been going half a year before an incrustation began to be formed on the outside of the wheel. I remarked that the deposit was in greatest quantity around the orifice where the cloth is put in and taken out. The deposit in the interior, and which coated the whole surface of the compartment, was of a darker brown colour, and was as smooth and splendent as a lining of highly polished bronze would have been. The high polish is no doubt partly produced by friction; and I observed that it was highest on that part of the outside nearest the opening.

So far we have *calcareous*, but no *animal*, matter; but in going a little further back in the history of the process to which the cotton had been subjected, before it came to the bleach-field, I discovered that animal matter might be contained in the incrustation. I learned that the cloth had been woven in power-looms; and on making inquiry as to the composition of the dressing or paste used to smooth and stiffen the warp before it is put into the loom, I was told that in the factory from whence the cloth had come, it is the practice to mix glue with the wheaten flour, generally in equal proportions by weight.

We have thus lime and gelatine, the same materials which are employed by the molluscous animal in the formation of its covering, and apparently in the same degree of minute division as that in which they are exuded from its mantle.

Chemical examination of the Substance.

1. *The external deposit.*—Exposed to the flame of a wax candle, it blackens, and gives out the usual smell of burning animal matter, the thin laminæ of which it is composed separating and curling up like films of horn; appearances similar to those exhibited by membranous shells when heated. When the flame is urged by the blow-pipe, the laminæ separate still more, and are changed into an extremely light and

brittle enamel, pure white, and having a pearly lustre. A fragment moistened on the back of the hand gives a sensation of heat, as quicklime does when so treated. The substance, when thrown into dilute muriatic acid, is entirely dissolved; the fluid is tinged yellow, and the effervescence produces a froth, like beer. When the acid is very much diluted, and a portion of the substance is suspended in it, the solution takes place gradually, minute flocculi of animal matter being separated, and floating in the fluid.

2. *The internal deposit.*—This is separable into extremely thin laminæ, and these, when in small fragments, are hardly distinguishable from scales of brown mica, showing also the most beautiful play of colours. The action of heat produces the same effect as on the external deposit, except that the separated laminæ are thinner. The action of muriatic acid is the same, but the yellow tinge is deeper, and the froth is more permanent, indicating a larger proportion of animal matter than in the other. The nacreous lustre is also much more conspicuous in this.

Mr. Gray, in his paper on the Structure of the Shells of Molluscous Animals, observes that the pearly or iridescent lustre appears to be confined to shells of the concretionary structure, which when broken exhibit a nearly uniform texture, but separate when heated into numerous thicker or thinner laminæ; and he adverts to the observation of Mr. Hatchett, that when they are digested in weak muriatic acid, the lime is dissolved, leaving a great number of thin plates of animal matter, which retain the original shape of the shell. He adds, "This variety of structure is found to constitute the whole shell of the *Anomiæ* and *Placunæ*, and to form the inner coat of those shells which have pearly insides, as the *Turbines*, *Haliotides*, *Uniones*, &c., as well as the laminar portion of the *Pinnæ* and Mother-of-pearl shells *."

Besides the laminated structure, there is, in the case of the Pinna and some other shells, a prismatic crystalline arrangement of the particles perpendicular to, and passing uninterruptedly through, the laminæ; but I have not discovered such an arrangement in any portion of the incrustation, even when examined by the microscope.

I felt very desirous that this singular deposit should be examined by Sir David Brewster; the more especially as he had long since directed his attention to the peculiar structure of mother-of-pearl†. On showing him the specimens in my possession he observed, that it was one of the most remarkable artificial productions he had seen; and he readily undertook to examine it carefully. He shortly afterwards sent me the particulars of that examination, which had afforded some curious and interesting results. Having subsequently visited Catrine, I procured more perfect specimens; and I sent these to Sir David Brewster, in order to ascertain whether they might not afford something new, in addition to the results he had obtained from the fragments he had formerly examined. They did so, and I now subjoin the very interesting account which Sir David has given me of the properties he has discovered in this new substance.

* Philosophical Transactions, 1833, p. 794. † Id., 1814.

My dear Sir, Belleville, January 1st, 1836.

In the communication which I had the pleasure of addressing to you on the 20th of January 1835, I gave a brief account of the observations I had made on the highly interesting substance which you had put into my hands; but as the specimens which you sent me a few days ago are so much superior to those with which I made my former experiments, and have led me to some new and I think rather extraordinary results, I shall include in the present letter all my former observations.

The substance in question does not resemble in its general aspect any natural or artificial production which I have seen. It is, generally speaking, brown where the surface is not iridescent, and in very thin plates: it is almost perfectly transparent, with a slight yellowish brown tinge like plates of glue or lac of the same thickness. The laminæ of which it is composed are sometimes separated by vacant spaces, at other times slightly coherent, but generally adhering to each other with a force greater than that of the laminæ of sulphate of lime or mica, and less than those of calcareous spar. When the adhering plates are separated, the separated surfaces are sometimes colourless, especially when these surfaces are corrugated or uneven; but they are almost always covered with an iridescent film of the most brilliant, and, generally, uniform tint, which exhibits all the variety of colours displayed by thin plates or polarizing laminæ.

The substance is of intermediate hardness between calcareous spar and sulphate of lime. It scratches the latter easily, and is not scratched by mother-of-pearl. Its specific gravity is shown in the following Table, which indicates its relation to analogous substances.

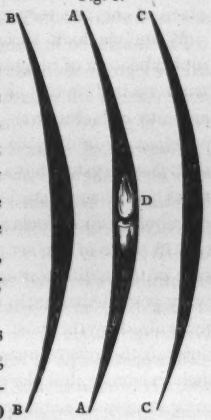
Calcareous spar	2·72
Oriental pearls	2·68
New substance	2·44
Mother-of-pearl	2·19
Oyster-shell	2·02

The new substance has the property of refracting light doubly, like most crystallized bodies; and, as in agate, mother-of-pearl, &c., one of the two images is perfectly distinct, while the other contains a considerable portion of nebulous light, varying with the thickness of the plate and the inclination of the refracted ray. It has one axis of double refraction, like calcareous spar, which is negative, as in that mineral, and, like it also, it gives a beautiful system of coloured rings by polarized light. The double refraction of the substance is very considerable, though greatly less than that of calcareous spar. A plate, one seventy-fifth of an inch thick, makes the first red ring of the system *eight* inches in diameter at a distance of twenty-six inches from the eye. The substance belongs to the rhombohedral system, and, as in the *Chaux carbonatée basée* of Hauy, the axis of the rhombohedron, or that of double refraction, is perpendicular to the surface of the thin plates. As mother-of-pearl has two axes of

double refraction like aragonite, this new substance may be considered as having the same optical relation to calcareous spar that mother-of-pearl has to aragonite.

When we look through a plate of this substance perpendicularly to its surface, or along the axis of double refraction, the flame of a candle is seen encircled with a nebulous haze like a halo. By the slightest inclination of the plate in any azimuth whatever, three elongated and curved nebulous images are seen, as in fig. 1., the central one, A A, having a distinct image, D, of the candle in the middle of it, and the other two, B B and C C, being equidistant from A A. These elongated images are parallel and concave towards the end of the plate nearest the eye. In the direction of the axis of double refraction, when all the nebulous light is in one mass, the distinct image, D, is redder than in any other direction; and by slightly inclining the plate the red light disappears, and the distinct image becomes brighter and whiter. All the three images, A A, B B, and C C, are united into a mass round D, at a perpendicular incidence, but they separate upon inclining the plate, and their distance increases with the inclination.

Fig. 1.



If we examine the nature of the light of which these images are composed, we shall find that the nebulous images, A A, B B, are wholly polarized in a plane passing through the direction of their length, while C C and the greater part of D are polarized in an opposite plane. As the thickness of the plate increases, more and more of the distinct image, D, is polarized in the same plane, as in mother-of-pearl*, till at a certain thickness the whole of it is thus polarized. In this case all the doubly refracted light which forms the nebulous image, A A, and the bright one, D, consists of two oppositely polarized pencils, the one forming the nebulous and the other the distinct image.

In investigating the cause of these phenomena, we must take as our guide the analogous facts presented by certain composite crystals of calcareous spar. Having long ago described this class of phenomena very fully†, I shall only state at present the general fact. Let A B C D, fig. 2., be a section of a rhomb of calcareous spar having its axis perpendicular to the faces A B, C D, and let E F be another crystal, or vein of the same substance crossing it according to the law of crystallographic composition. If we now look at a candle through this compound crystal, it will appear single in the direction of the axis of A B C D; but if we incline the plate in a plane passing through A B, we shall see two images together as at A and D, fig. 3., and other two, namely one at B

Fig. 2.

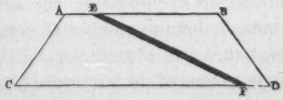


Fig. 3.

* See Philosophical Transactions, 1814.

† Id., 1815. Edinburgh Encyclopedia, art. Optics.

and the other at C. These images B, C, separate by the inclination of the plate exactly like those in fig. 1., and all the four, A, B, C, and D, have the same absolute and relative polarization as the four analogous images seen through the new substance, with this difference only, that none of them are nebulous.

If we conceive the vein E F to consist, as in fig. 4., of a great number of small crystals, *a*, *b*, *c*, *d*, &c. in place of one, the very same effects will be produced.

Fig. 4.

When we look through the new substance, the multiplication of images takes place *in whatever azimuth* we incline the plate, the elongated images being always perpendicular to the azimuth of inclination. Hence it follows, that these images are produced by *numbers of minute crystals* lying in or near the azimuth in which the plate is inclined; and that these crystals have their axes all inclined to that of the plate which contains them, at the same angle as the vein E F, figg. 2. & 4., is inclined to the axis of the rhombohedron of Iceland spar. But the remarkable result of these observations is, that in place of one set of crystals, or sometimes three sets, which occur in calcareous spar in three different azimuths, we have here *an infinite number of them* lying in every possible azimuth, and these so small in their dimensions that they cannot be recognised by the most powerful microscopes, except as dark specks disseminated through the general mass; and yet they indicate by their action on light, not only their existence, but the position of their axes, and their doubly refracting and polarizing structure, as unequivocally as if we could handle them, and cleave them, and place them upon the goniometer.

It may now be asked why the images are nebulous, and not distinct as in calcareous spar. The reason is, that the substance is imperfectly crystallized like the agate, mother-of-pearl and other bodies in which the doubly refracting force separates the incident light into two oppositely polarized pencils, which are not perfectly equal and similar, but which differ from each other, sometimes in the intensity of their light, sometimes in the distinctness of the image, sometimes in the nature or brightness of the colour, and sometimes in more than one of these characters. But though the new substance resembles the crystals above mentioned in giving dissimilar pencils of doubly refracted light, it stands unique among all bodies with which I am acquainted in possessing the extraordinary system of composite crystallization, in which an infinite number of crystals are disseminated equally in every possible azimuth through a larger crystalline plate, having their axes all inclined at the same angle to that of the larger plate, and producing similar phenomena in every direction, and through every portion of the plate; or we may describe this remarkable structure by saying that the minute elementary crystals form the surfaces of an infinite number of cones whose axes pass perpendicularly through every point of the larger plate*.

* A rude idea of this structure is given by the beautiful cones, or rather pyramids of microscopic crystals of titanium which I have somewhere described as existing within the pyramids of many crystals of amethyst from the Brazils.

The iridescent phenomena exhibited by the new substance are extremely interesting, and I have been at much pains to examine them in a great number of specimens. The plates into which the substance is divisible have been formed in succession, and certain intervals of time have elapsed between their formation. In general every two contiguous laminæ are separated by a thin iridescent film, varying from the three to the fifty millionth part of an inch in thickness, and producing all the various colours of thin plates which correspond to intermediate thicknesses. Between some of the laminæ no such film exists, probably in consequence of the interval of time between their formation being too short; and between others the film has been formed of unequal thickness, as happens in the oxidations upon steel when they are formed upon or around hard parts of the metal called *pins* by the workmen.

There can be no doubt that these iridescent films are formed when the dash-wheel is at rest during the night, and that when no film exists between two laminæ, an interval too short for its formation (arising perhaps from the stopping of the work during the day,) has elapsed during the drying or induration of the one lamina and the deposition of the other.

That these iridescent films are not thin films of the substance itself, may be inferred from the fact that light is reflected from their surfaces when they firmly adhere to the laminæ which inclose them. If, for example, we remove or raise up from a piece of mica a thin film which gives a bright green tint, and press it again into optical contact with the surface from which it was separated, it will then cease to exhibit any colour, because no light is reflected from its posterior surface; but if we press it into optical contact with another surface which has a different refractive power, its green colour will still be exhibited. It is owing to this cause that the colours of the oxidations on steel are so distinctly visible, and that the analogous oxidations are seen upon glass even before the film has begun to separate into coloured scales.

The iridescent films in the new substance possess another source of interest, in so far as they promise to throw a new light on the origin of the *incommunicable* colours of mother-of-pearl, which arise from the interior structure of the shell, and which cannot therefore be communicated to wax. These colours have frequently occupied my attention since the year 1814, when I described the phenomena of the colours communicable to wax*; but though I have devoted much time to the inquiry, I never could obtain a single result worthy of being communicated to the public. I took plates of mother-of-pearl that exhibited different bright colours through different parts of their surface, and by getting the mother-of-pearl ground away in different places by the seal-engraver's wheel, I endeavoured to discover the thicknesses at which the colours were produced, and the cause of the capricious variation of tints which arose from every inclination of the plate: but all my experiments were fruitless, and I abandoned the subject as beyond my reach. The phenomena, however, presented by the new substance seem to me to disclose the secret of which I was in

* Philosophical Transactions, 1814.

quest. The layers of mother-of-pearl are deposited in succession like those which are formed upon the dash-wheel; and there can be no doubt that the animal whose mucous secretions form the shell that incloses it, rests occasionally from its toils, and affords a sufficient interval for the formation of an iridescent film upon the surface of the plate of shell which it daily deposits. Owing to the firm adhesion of the successive layers of the shell, we cannot, as in the more imperfectly formed new substance, separate each stratum in order to see the iridescent film upon their surfaces; but we can easily determine what phenomena would be produced if the layers of the new substance were as transparent as those of mother-of-pearl. If this were the case, we should see, both by reflected and transmitted light, the combined colours of all the iridescent films in the plate. When these films are numerous and flat, and of various thicknesses, the union of all their colours would form a pearly whiteness by reflected light, and when films of a particular colour predominate, both the reflected and the transmitted light would exhibit that prevailing colour; but if their surfaces are undulated as in mother-of-pearl, from the form of the shell and other causes,—if the iridescent films vary in thickness, and consequently in colour,—if they are wanting in some parts of the shell, and abound in others,—and if films of equal thicknesses occur in several laminæ in succession, and films of other thicknesses in other laminæ, which must necessarily take place from the varying and remitting action of the animal agent, then we shall have the very structure which is necessary for the production of the incommunicable colours of mother-of-pearl.

I have no doubt that this is the true cause of the phenomena which had so long perplexed me; and the results which I formerly obtained, though I could then attach no meaning to them, are in perfect unison with the preceding views. In order, however, to obtain something like an experimental confirmation of this opinion, I have examined the *fracture* of a mother-of-pearl shell where the laminæ have been all deposited with considerable regularity, and where their overlying edges are exhibited, and I find distinct and positive proofs of the existence of iridescent films, sometimes green, and sometimes red in several successive strata.

I am, my dear Sir,

Ever most truly yours,

D. Brewster.

To Leonard Horner, Esq.

Diurnal inequality in the height of high water at Liverpoo

Upper Transits ... *P. M.*
Do. ... *A. M.*
Lower Interpolated Transits ... *A. M.*
Do. ... *P. M.*

The letters A.M., P.M. refer to the time o
The continuous lines have been laid down t
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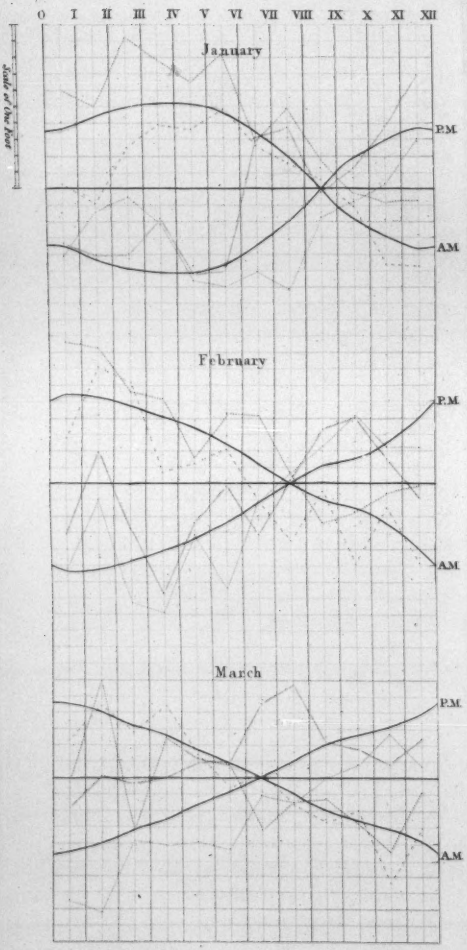
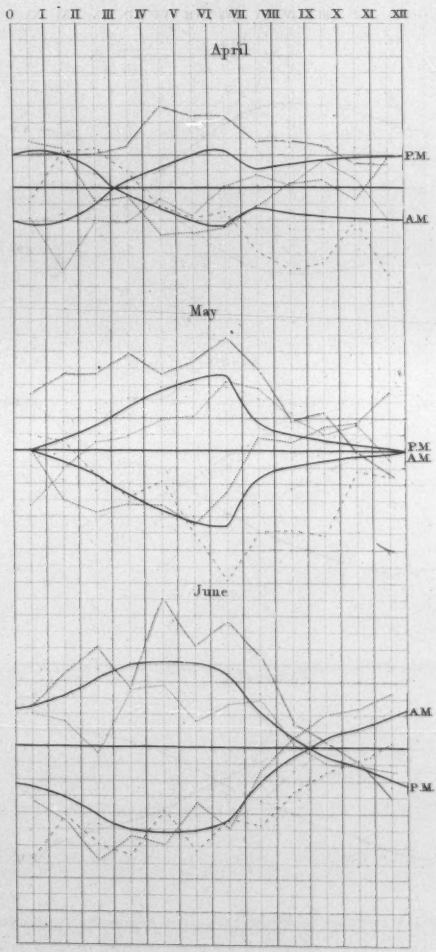
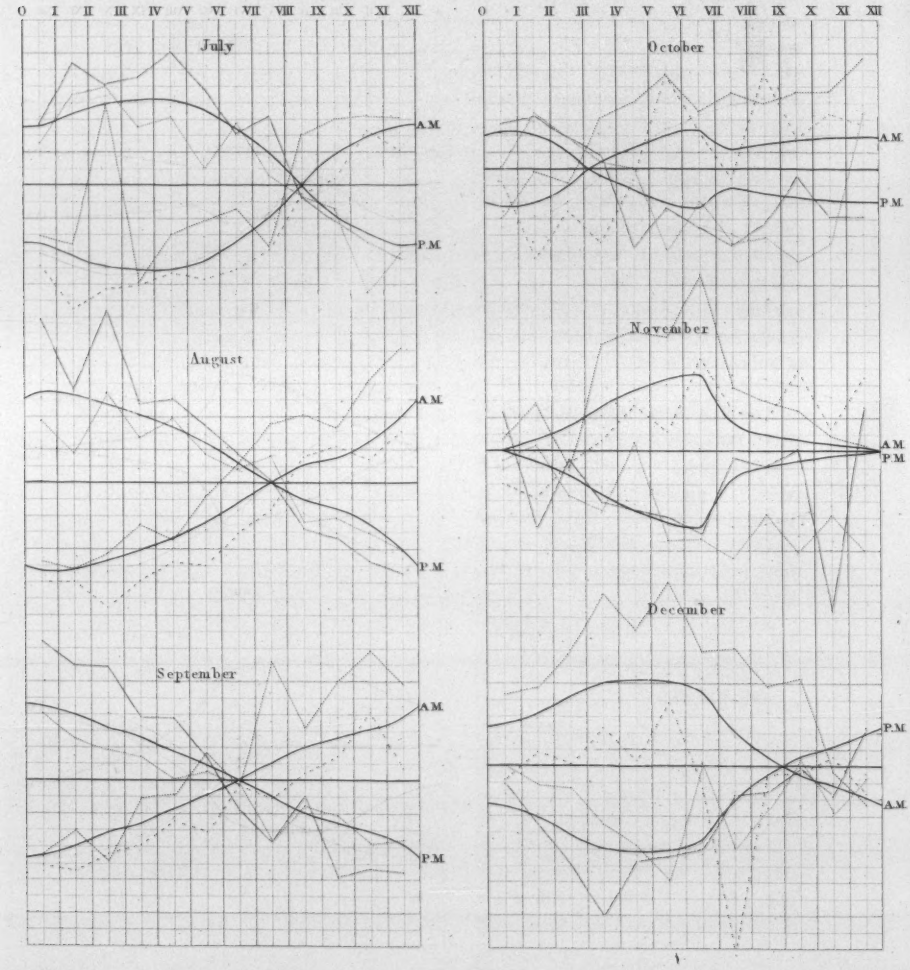




ool from 13,327 observations. — See Tables XXVIII and XXIX.

e of the moon's transit preceding the time of high water.
from Table XXIX. which is conjectural and formed by arbitrary alterations from Table XXVIII. The interval
the difference between the height of the morning and evening Tide for the middle of each month



VIII. *Discussion of Tide Observations made at Liverpool.* By J. W. Lubbock, *Esq. F.R.S.*

Received and Read January 28th, 1836.

I AM enabled, through the indefatigable perseverance of M. Dessiou, to present to the Society other Tables, in continuation of those published in the Philosophical Transactions for 1835, Part II., founded upon the observations instituted by Mr. Hutchinson at Liverpool.

The chief intention of the Tables now offered is to exhibit the diurnal inequality in the height of high water, which is, I believe, insensible in the river Thames, but which at Liverpool amounts to more than a foot. So that, for example, in January, when the moon is in quadrature, (neap tides,) the evening tide may be a foot higher than the morning tide.

Table XXVII. gives the results as immediately deduced from observation.

Table XXVIII. was formed by reducing the argument, (moon's transit,) by interpolation, to the even half-hours, and then taking the differences between the numbers so found and those in Table III.*

The results exhibited in Table XXVIII. are extremely irregular: these irregularities were arbitrarily removed in forming Table XXIX., which is intended to be used in predicting the phenomena. As the question of the diurnal inequality of the tides is important, and as the numbers in Table XXVIII. are so irregular, I have thought it desirable to exhibit them in a diagram, together with the inequalities definitively adopted in Table XXIX., in order that the nature and extent of the alterations we have introduced may be perceived. The diurnal inequality in the *interval* appears to be insensible.

Bernoulli's Theory of the Tides leads to the expressions

$$h = D + E\{A\cos(2\mu - 2\alpha) + \cos(2\mu - 2\alpha')\}$$

$$\tan 2\psi = \frac{A\sin 2\phi}{1 + A\cos 2\phi} \qquad A = \frac{m\cos^3\delta P^3}{m'\cos^3\delta' P'^3} \qquad E = C m'\cos^2\delta\ P'^3$$

$$h = D + E\{\cos 2\psi + A\cos(2\varphi - 2\psi)\},$$

in which expressions α denotes right ascension, μ sidereal time, δ declination, m the mass of the luminary, P the sine of the horizontal parallax, C a constant depending upon geographical latitude, and D a constant depending only on the zero line, from which the heights are reckoned. The unaccented quantities refer to the sun, and those which are accented to the moon. h is the height of the water above any given line, and ψ is a small variable angle to be added with a certain constant to the time of the moon's transit, in order to obtain the time of high water.

* Philosophical Transactions, 1835, p. 283.

The constant A has the same value for London and Liverpool, and I find for both places the mean value of $\log A = 9{\cdot}5784858$. The following Tables, calculated by Mr. Jones, will assist in comparing results deduced from observation with Bernoulli's expressions.

	Semimenstrual Inequality.				Semimenstrual Inequality.		
ϕ.	ψ in Time.	Height of Tide.	ϕ.	ϕ.	ψ in Time.	Height of Tide.	ϕ.
°	+ m	feet.	°	°	+ m	feet.	°
0	0	22·81	180	45	42	21·44	135
5	6	22·80	175	50	43	21·15	130
10	11	22·75	170	55	44	20·86	125
15	16	22·65	165	60	44	20·57	120
20	21	22·52	160	65	41	20·28	115
25	26	22·36	155	70	37	20·02	110
30	31	22·17	150	75	32	19·79	105
35	36	21·95	145	80	24	19·61	100
40	40	21·71	140	85	13	19·49	95
45	42	21·44	135	90	0	19·44	90
	−				−		

The preceding Table has been calculated with $\log A = 9{\cdot}5784858$ and $\log E = 0{\cdot}6481648$.

Sun's Declination.

d ψ in Time.

	Decl. 0°.	Decl. 3°.	Decl. 6°.	Decl. 9°.	Decl. 12°.	Decl. 15°.	Decl. 18°.	Decl. 21°.	Decl. 24°.	
ϕ.			+					−		ϕ.
°	m	m	m	m	m	m	m	m	m	°
0	0	0	0	0	0	0	0	0	0	180
15	1	1	1	1	1	0	0	1	1	165
30	2	1	1	1	0	0	1	2	3	150
45	2	2	1	1	0	0	2	3	5	135
60	4	3	2	2	1	0	2	3	5	120
75	3	2	2	1	1	0	2	4	5	105
90	0	0	0	0	0	0	0	0	0	90
			−					+		

Moon's Parallax.

d ψ in Time.

	H. P. 54′.	H. P. 55′.	H. P. 56′.	H. P. 57′.	H. P. 58′.	H. P. 59′.	H. P. 60′.	H. P. 61′.	
ϕ.		+				−			ϕ.
°	m	m	m	m	m	m	m	m	°
0	0	0	0	0	0	0	0	0	180
15	2	2	1	0	0	1	2	2	165
30	4	3	1	0	1	3	4	5	150
45	6	4	2	0	2	4	6	8	135
60	9	6	3	0	2	5	7	9	120
75	8	5	2	0	3	5	6	8	105
90	0	0	0	0	0	0	0	0	90
		−				+			

Moon's Declination.

d ☽ in Time.

φ.	Decl. 0°.	Decl. 3°.	Decl. 6°.	Decl. 9°.	Decl. 12°.	Decl. 15°.	Decl. 18°.	Decl. 21°.	Decl. 24°.	Decl. 27°.	Decl. 30°.	φ.
	−						+					
°	m	m	m	m	m	m	m	m	m	m	m	°
0	0	0	0	0	0	0	0	0	0	0	0	180
15	1	1	0	0	0	0	1	1	2	2	3	165
30	2	2	2	1	1	0	1	2	3	4	6	150
45	3	3	3	2	1	0	1	2	4	6	8	135
60	3	3	3	2	1	0	2	4	6	9	12	120
75	3	3	3	2	2	0	1	3	5	8	11	105
90	0	0	0	0	0	0	0	0	0	0	0	90
	+						−					

Sun's Declination.

d *h*.

φ.	Decl. 0°.	Decl. 3°.	Decl. 6°.	Decl. 9°.	Decl. 12°.	Decl. 15°.	Decl. 18°.	Decl. 21°.	Decl. 24°.	φ.
°	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	°
0	+·12	+·11	+·10	+·08	+·04	·00	−·05	−·11	−·18	180
15	·11	·10	·09	·07	·04	·00	·05	·11	·17	165
30	·09	·08	·07	·05	·03	·00	·04	·08	·13	150
45	+·04	+·04	+·03	+·02	+·01	·00	−·02	−·04	−·06	135
60	−·02	−·02	−·02	−·01	−·01	·00	+·01	+·02	+·03	120
75	·09	·09	·09	·06	·03	·00	·04	·08	·12	105
90	−·12	−·11	−·10	−·07	−·04	·00	+·06	+·12	+·18	90

Moon's Parallax.

d *h*.

φ.	H. P. 54′.	H. P. 55′.	H. P. 56′.	H. P. 57′.	H. P. 58′.	H. P. 59′.	H. P. 60′.	H. P. 61′.	φ.
°	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	°
0	−·66	−·45	−·23	·00	+·24	+·49	+·75	+1·01	180
15	·66	·45	·23	·00	·24	·48	·73	0·99	165
30	·64	·44	·23	·00	·23	·47	·72	0·97	150
45	·62	·42	·21	·00	·22	·45	·70	0·95	135
60	·61	·42	·21	·00	·22	·45	·69	0·94	120
75	·64	·44	·22	·00	·23	·46	·71	0·97	105
90	−·66	−·45	−·23	·00	+·24	+·49	+·75	+1·01	90

Moon's Declination.

d *h*.

φ.	Decl. 0°.	Decl. 3°.	Decl. 6°.	Decl. 9°.	Decl. 12°.	Decl. 15°.	Decl. 18°.	Decl. 21°.	Decl. 24°.	Decl. 27°.	Decl. 30°.	φ.
°	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	°
0	+·32	+·31	+·27	+·21	+·12	·00	−·13	−·29	−·47	−·66	−·87	180
15	·31	·30	·26	·20	·11	·00	·14	·29	·47	·66	·87	165
30	·31	·30	·26	·20	·11	·00	·13	·28	·45	·63	·84	150
45	·30	·29	·25	·19	·10	·00	·13	·27	·44	·62	·81	135
60	·30	·29	·25	·19	·11	·00	·13	·27	·43	·60	·79	120
75	·30	·29	·25	·19	·11	·00	·13	·28	·45	·61	·79	105
90	+·31	+·31	+·27	+·21	+·12	·00	−·13	−·29	−·47	−·66	−·87	90

I have ascertained that Bernoulli's expressions present a very remarkable accordance with observation. But it must be recollected, that the phenomena of the tides at London and Liverpool cannot be considered as depending (mechanically) upon the coordinates of the sun and moon at the transit immediately preceding the times of high water; and if I had taken in my discussions a transit more remote, the law of the intervals would not have been the same. This difference in the law of the intervals depends upon the difference in the intervals between the successive transits of the moon. Hence the law of the intervals, when the discussion is instituted with reference to the transit immediately preceding the time of high water, whether at London, Liverpool, or Brest, depends partly upon the phenomena as deducible from Bernoulli's expressions, and partly upon the law of the intervals between the moon's successive transits, which latter interval may be considered roughly as depending upon her parallax and declination. For practical tables intended to serve in predicting the phenomena, this is of no consequence; but in order safely to compare the results of theory with those of observation, it is absolutely necessary first to obtain the law of the changes in the moon's motion, which may easily be done. This consideration does not apply to the heights.

I have little doubt from comparisons which I have made, and which I mean to extend, that the results of observation present a very remarkable agreement with Bernoulli's theory, and with the formulæ in p. 57, by which this theory is approximately represented. In this comparison, however, it cannot be expected that the phenomena at any given place are the same (as Laplace seems to imply*) as would belong to that geographical latitude, if the figure of the ocean were that of a perfect spheroid, and were not intersected by continents. I apprehend that the inequalities of the tides at Brest are produced *mechanically* some time previous, and are not due to the position of the luminaries (in a certain sense) at the time of high water.

Mr. Deacon has furnished me with a continuation of Tables A, B, C, D, for the last six months of the year 1835, which serve to show the degree of congruity in the observations at the London and St. Katherine Docks with each other and with the predicted tides in the British Almanac.

* "Si le port a une latitude, ces pleines mers pourraient être fort différentes; et quand la déclinaison des astres est égale à l'obliquité de l'ecliptique, la marée du soir à Brest serait environ huit fois plus grande que celle du matin."—*Mécanique Celeste*, tom. v. p. 148.

Table A.

Showing a comparison of the observations of the Times of High Water made at the London Docks, increased by five minutes, and those at the St. Katherine Docks. The observations marked with an * appear doubtful.

Date. 1835.	July.			August.			September.			October.			November.			December.		
	London Docks. +5 min.	St. Kath. Docks.	Difference.	London Docks. +5 min.	St. Kath. Docks.	Difference.	London Docks. +5 min.	St. Kath. Docks.	Difference.	London Docks. +5 min.	St. Kath. Docks.	Difference.	London Docks. +5 min.	St. Kath. Docks.	Difference.	London Docks. +5 min.	St. Kath. Docks.	Difference.
	h m	h m	m	h m	h m	m	h m	h m	m	h m	h m	m	h m	h m	m	h m	h m	m
1.	5 30	5 27	+ 3	6 35	6 42	− 7	7 50	7 56	− 6	8 35	8 41	− 6	11 15	11 21	− 6	11 15	11 18	− 3
	5 45	5 42	+ 3	6 45	6 53	− 8	8 20	8 16	+ 4	9 20	9 46	−26	.....	.....	.....	11 45	11 58	−13
2.	6 5	6 8	− 3	7 25	7 28	− 3	9 5	9 12	− 7	10 15	10 21	− 6	12 15	12 3	+12	.....	.....	.....
	6 20	6 23	− 3	7 35	7 48	−13	9 45	9 42	+ 3	11 5	11 12	− 7	12 20	12 23	− 3	12 10	12 7	+ 3
3.	7 0	6 57	+ 3	8 20	8 35	−15	10 45	10 47	− 2	11 25	11 38	−13	12 50	12 58	− 8	12 45	12 48	− 3
	7 15	7 7	+ 8	8 50	8 48	+ 2	11 30	11 27	+ 3	.....	.....	.....	12 50	12 58	− 8	12 55	12 54	+ 1
4.	8 0	7 57	+ 3	9 25	9 37	−12	12 5	12 12	− 7	12 15	12 22	− 7	1 25	1 32	− 7	1 20	1 24	− 4
	8 15	8 7	+ 8	10 5	10 4	+ 1	.....	.....	.....	12 40	12 48	− 8	1 30	1 42	−12	1 35	1 32	+ 3
5.	9 0	8 21	+39	10 45	10 53	− 8	12 35	12 42	− 7	1 15	1 13	+ 2	2 0	2 8	− 8	1 55	1 55	0
	9 20	8 57	+23	11 15	11 25	−10	1 0	1 3	− 3	1 20	1 25	5	2 10	2 12	− 2	2 10	2 9	+ 1
6.	10 5	10 12	− 7	12 5	12 12	− 7	1 25	1 32	− 7	1 50	2 2	−12	2 25	2 38	−13	2 25	2 28	− 3
	10 25	10 27	− 2	.....	.....	.....	1 50	1 42	+ 8	1 55	2 3	− 8	2 35	2 41	− 6	2 45	2 53	− 8
7.	11 5	11 13	− 8	12 25	12 31	− 6	2 25	2 22	+ 3	2 35	2 32	+ 3	2 50	3 2	−12	3 10	3 5	+ 5
	11 35	11 37	− 2	12 50	1 8	−18	2 30	2 28	+ 2	2 40	2 42	− 2	3 5	3 13	− 8	3 30	3 23	+ 7
8.	12 5	12 25	−20	1 25	1 33	− 8	2 50	3 8	−18	3 5	3 7	− 2	3 35	3 33	+ 2	3 45	3 37	+ 8
	.....	.....	.....	1 45	1 58	−13	3 5	3 13	− 8	3 5	3 13	− 8	3 35	3 47	−12	4 5	3 57	+ 8
9.	12 35	12 36	− 1	2 15	2 27	−12	3 30	3 33	− 3	3 30	3 32	− 2	3 55	3 57	− 2	4 20	4 13	+ 7
	1 5	1 8	− 3	2 35	2 46	−11	*3 30	3 34	+ 1	3 35	3 41	− 6	4 15	4 17	− 2	4 35	4 33	+ 2
10.	1 30	1 42	−12	3 25	3 18	+ 7	3 50	3 57	− 7	3 55	3 57	− 2	4 25	4 32	− 7	4 50	4 48	+ 2
	2 10	2 15	− 5	3 30	3 31	− 1	3 50	4 2	−12	4 0	3 52	+ 8	4 40	4 39	+ 1	5 10	5 16	− 6
11.	2 25	2 33	− 8	3 55	4 3	− 8	4 25	4 17	+ 8	4 25	4 21	+ 4	5 0	4 54	+ 6	5 25	5 32	− 7
	2 45	2 57	−12	4 10	4 8	+ 2	4 30	4 33	− 3	4 25	4 32	− 7	5 10	5 17	− 7	5 50	6 8	−18
12.	3 15	3 32	−17	4 35	4 32	+ 3	4 55	5 2	− 7	4 55	4 48	+ 7	5 35	5 23	+12	6 15	6 17	− 2
	3 35	3 47	−12	4 35	4 44	− 9	5 5	5 7	− 2	4 55	4 52	+ 3	5 50	5 57	− 7	6 35	6 27	+ 8
13.	4 15	4 22	− 7	5 5	5 14	− 9	5 20	5 26	− 6	5 20	5 22	− 2	6 15	6 13	+ 2	7 5	6 58	+ 7
	4 20	4 26	− 6	5 15	5 15	0	5 35	5 41	− 6	5 30	5 26	+ 4	6 55	7 2	− 7	7 40	7 38	+ 2
14.	4 50	4 55	− 5	5 45	5 48	− 3	6 0	6 6	− 6	5 55	6 2	− 7	7 25	7 28	− 3	8 5	8 9	− 4
	5 5	5 7	− 2	5 50	5 58	− 8	6 10	6 12	− 2	6 0	5 58	+ 2	8 5	8 7	− 2	8 55	8 57	− 2
15.	5 40	5 47	− 7	6 20	6 27	− 7	6 35	6 37	− 2	6 40	6 45	− 5	8 35	8 47	−12	9 20	9 27	− 7
	5 45	5 47	− 2	6 10	6 27	−17	6 45	6 52	− 7	7 15	7 21	− 6	9 40	9 37	+ 3	10 15	10 21	− 6
16.	6 20	6 31	−11	6 50	7 0	−10	7 20	7 22	− 2	7 50	8 8	−18	10 15	10 17	− 2	10 30	10 42	−12
	6 25	6 27	− 2	6 50	7 3	−13	7 20	7 42	−22	8 40	8 52	−12	10 50	10 53	− 3	11 15	11 13	+ 2
17.	7 0	7 7	− 7	7 20	7 31	−11	8 35	8 41	− 6	9 25	9 49	−24	11 20	11 23	− 3	11 35	12 40	− 5
	7 5	7 11	− 6	7 45	7 42	+ 3	9 20	9 34	−14	10 25	10 36	−11	11 45	11 50	− 5	12 5	12 11	− 6
18.	7 55	8 7	−12	8 20	8 45	−25	10 20	10 23	− 3	11 5	11 12	− 7	12 5	12 15	−10	.....	.....	.....
	7 45	7 57	−12	8 55	8 58	− 3	11 20	11 7	+13	11 55	11 41	+14	.....	.....	.....	12 25	12 36	−11
19.	8 45	8 47	− 2	9 35	10 2	−27	11 35	11 47	−12	.....	.....	.....	12 55	1 6	−11	1 0	12 58	+ 2
	8 45	8 46	− 1	10 20	10 19	+ 1	12 5	12 3	+ 2	12 10	12 15	− 5	12 45	12 43	+ 2	1 30	1 17	+13
20.	9 45	9 57	−12	11 15	11 19	− 4	.....	.....	.....	12 40	12 44	− 4	1 25	1 26	− 1	1 55	1 57	− 2
	10 5	10 7	− 2	11 50	11 47	+ 3	12 25	12 27	− 2	12 55	1 1	− 6	1 40	1 49	− 9	2 15	2 15	0
21.	10 40	10 42	− 2	.....	.....	.....	1 10	1 12	− 2	1 15	1 14	+ 1	2 15	2 13	+ 2	2 45	2 43	+ 2
	11 15	11 12	+ 3	12 15	12 17	− 2	1 20	1 22	− 2	1 30	1 30	0	2 25	2 29	− 4	3 5	3 8	− 3
22.	11 50	12 10	−20	12 50	12 42	+ 8	1 45	1 50	− 5	1 55	1 57	− 2	2 45	2 44	+ 1	3 30	3 25	+ 5
	.....	.....	.....	1 0	1 7	− 7	2 5	2 2	+ 3	2 15	2 14	+ 1	3 5	3 18	−13	3 50	3 51	− 1
23.	12 15	12 11	+ 4	1 30	1 32	− 2	2 30	2 27	+ 3	2 35	2 37	− 2	3 40	3 48	− 8	4 15	4 15	− 0
	12 45	12 52	− 7	1 40	1 47	− 7	2 45	2 43	+ 2	2 50	2 51	− 1	3 55	3 57	− 2	4 40	4 43	− 3
24.	1 10	1 11	− 1	2 5	2 15	−10	3 5	3 9	− 4	3 10	3 17	− 7	4 25	4 28	− 3	5 0	5 5	− 5
	1 30	1 42	−12	2 20	2 28	− 8	3 20	3 15	+ 5	3 35	3 38	− 3	4 45	4 58	−13	5 35	5 36	− 1
25.	1 55	1 52	+ 3	2 40	2 53	−13	3 45	3 43	+ 2	4 0	4 3	− 3	5 10	5 7	+ 3	5 55	5 51	+ 4
	2 20	2 17	+ 3	3 0	3 2	− 2	3 50	4 3	−13	4 10	4 13	− 3	5 45	5 53	− 8	6 30	6 26	+ 4
26.	2 45	2 32	+13	3 25	3 33	− 8	4 20	4 26	− 6	4 30	4 33	− 3	5 55	5 48	+ 7	6 55	6 32	+23
	2 55	2 58	− 3	3 45	3 43	+ 2	4 35	4 32	+ 3	4 50	4 57	− 7	6 45	6 50	− 5	7 5	7 22	−17
27.	3 15	3 11	+ 4	4 10	4 15	− 5	4 55	5 8	−13	5 15	5 13	+ 2	6 40	6 44	− 4	7 25	7 27	− 2
	3 30	3 25	+ 5	4 20	4 26	− 6	5 10	5 13	− 3	5 30	5 37	− 7	7 50	7 50	0	8 10	8 8	+ 2
28.	3 45	3 46	− 1	4 45	4 48	− 3	5 40	5 43	− 3	5 50	6 8	−18	7 45	7 44	+ 1	8 25	8 26	− 1
	4 0	4 3	− 3	5 5	5 1	+ 4	5 55	5 57	− 2	6 30	6 53	−23	8 45	8 38	+ 7	9 5	9 17	−12
29.	4 25	4 28	− 3	5 30	5 27	+ 3	6 25	6 15	+10	7 0	7 10	−10	9 5	9 2	+ 3	8 5	8 13	− 5
	4 40	4 41	− 1	5 40	5 41	− 1	6 45	6 56	−11	8 5	8 14	− 9	10 10	9 57	+13	10 5	10 45	−40
30.	5 5	4 42	+23	6 10	6 9	+ 1	7 15	7 11	+ 4	8 15	8 23	− 8	10 20	10 27	− 7	10 45	11 22	−37
	5 25	5 22	+ 3	6 20	6 19	+ 1	7 55	8 2	− 7	9 25	9 37	−12	11 5	11 18	−13	11 5	11 46	−41
31.	5 50	5 53	− 3	6 50	7 2	−12	.....	.....	.....	10 0	10 12	−12	.....	.....	.....	11 30	.....	.....
	6 5	6 3	+ 2	7 10	7 12	− 2	.....	.....	.....	10 50	11 5	−15	.....	.....	.....	12 5	12 21	−16

Table B.

Showing a comparison of the observations of the Heights of High Water made at the London Docks, increased by five feet, and those at the St. Katherine Docks. The observations marked with an * appear doubtful.

Date. 1835.	July.			August.			September.			October.			November.			December.		
	London Docks. + 5 ft.	St. Kath. Docks.	Difference.	London Docks. + 5 ft.	St. Kath. Docks.	Difference.	London Docks. + 5 ft.	St. Kath. Docks.	Difference.	London Docks. + 5 ft.	St. Kath. Docks.	Difference.	London Docks. + 5 ft.	St. Kath. Docks.	Difference.	London Docks. + 5 ft.	St. Kath. Docks.	Difference.
	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.
1.	26 6	26 6	0	26 7	26 8	− 1	25 1	25 1	0	23 7	23 8	− 1	25 2	25 2	0	26 8	26 10	− 2
	26 8	26 9	− 1	26 4	26 5	− 1	25 4	25 3	+ 1	24 8	24 10	− 2	.....	.....	.....	27 3	27 2	+ 1
2.	26 9	26 9	0	26 3	26 3	0	24 3	24 4	− 1	23 7	23 6	+ 1	26 6	26 7	− 1	.....	.....	.....
	26 1	26 7	− 6	26 6	26 1	+ 5	25 0	25 0	0	25 8	25 9	− 1	26 1	26 2	− 1	26 5	26 5	0
3.	26 1	26 8	− 7	26 3	26 4	− 1	24 7	24 6	+ 1	26 5	26 5	0	27 0	27 0	0	27 7	27 8	− 1
	25 4	25 5	− 1	25 6	25 7	− 1	25 6	25 5	+ 1	.....	.....	.....	26 6	26 7	− 1	27 5	27 6	− 1
4.	25 9	25 9	0	25 3	25 2	+ 1	25 1	25 1	0	27 6	27 9	− 3	26 4	26 5	− 1	27 3	27 4	− 1
	25 5	25 5	0	25 4	25 5	− 1	.....	.....	.....	26 1	26 1	0	26 4	26 6	− 2	26 9	26 10	− 1
5.	26 0	25 2	+10	25 4	25 5	− 1	26 6	26 6	0	27 5	27 4	+ 1	26 10	26 9	+ 1	27 1	27 0	+ 1
	25 2	25 11	− 9	26 0	25 9	+ 3	26 6	26 6	0	27 0	27 1	− 1	27 3	27 4	− 1	27 8	27 8	0
6.	25 8	25 8	0	25 4	25 6	− 2	27 8	27 9	− 1	28 1	28 2	− 1	27 6	27 7	− 1	27 2	27 4	− 2
	26 0	26 1	− 1	.....	.....	.....	27 0	27 0	0	28 1	28 1	0	27 6	27 7	− 1	27 3	27 3	0
7.	26 4	24 6	− 2	26 1	26 2	− 1	27 3	27 4	− 1	27 11	28 0	− 1	26 8	26 8	0	27 6	27 6	0
	26 1	26 3	− 2	26 0	26 7	− 7	27 5	27 6	− 1	28 1	28 2	− 1	26 1	26 1	0	27 2	27 4	− 2
8.	26 0	26 1	− 1	27 10	27 8	+ 2	27 10	27 5	+ 5	27 11	27 11	0	26 1	26 2	− 1	27 1	27 1	0
	.....	.....	.....	27 11	28 0	+ 1	27 8	27 10	− 2	27 10	27 9	+ 1	26 7	26 7	0	26 7	26 6	+ 1
9.	27 3	27 4	− 1	28 2	28 3	− 1	27 4	27 5	− 1	27 1	27 1	0	26 10	27 0	− 2	26 0	26 2	− 2
	26 11	26 10	+ 1	27 9	27 9	0	27 8	27 9	− 1	27 4	27 4	0	27 2	27 2	0	27 7	27 8	− 1
10.	27 2	27 3	− 1	27 9	27 8	+ 1	28 1	28 1	0	28 6	28 6	0	26 8	26 7	+ 1	25 3	25 4	− 1
	27 2	27 1	+ 1	27 6	27 5	+ 1	26 6	26 7	− 1	28 4	28 4	0	26 4	26 3	+ 1	25 5	25 6	− 1
11.	28 1	28 1	0	27 9	27 8	+ 1	25 6	25 7	− 1	27 0	27 3	− 3	25 1	25 1	0	24 6	24 6	0
	28 1	28 0	+ 1	27 8	27 10	− 2	26 8	26 8	0	27 3	27 4	− 1	25 8	25 3	+ 5	24 3	25 6	−15
12.	27 9	27 10	− 1	27 5	27 6	− 1	26 6	26 7	− 1	28 1	28 1	0	24 9	24 8	+ 1	24 6	24 8	− 2
	27 0	27 0	0	27 5	27 4	+ 1	26 5	26 5	0	25 10	25 9	+ 1	25 3	25 5	− 2	24 6	24 7	− 1
13.	27 4	27 7	− 3	27 6	27 7	− 1	26 2	26 4	− 2	25 9	25 9	0	23 2	23 2	0	24 0	24 2	− 2
	27 3	27 3	0	27 4	27 4	0	26 3	26 3	0	25 8	25 8	0	23 9	23 10	− 1	24 7	24 10	− 3
14.	27 6	27 7	− 1	26 5	26 5	0	25 3	25 3	0	25 8	25 7	+ 1	22 7	22 7	0	24 0	24 1	− 1
	27 2	27 2	0	26 6	26 7	− 1	25 1	25 4	− 3	25 9	25 8	+ 1	23 11	23 11	0	25 2	25 4	− 2
15.	27 0	27 1	− 1	25 6	25 6	0	24 1	24 3	− 2	23 2	23 1	+ 1	22 9	23 10	− 1	24 4	24 3	+ 1
	26 3	26 5	− 2	25 10	25 10	0	24 2	24 2	0	23 3	23 3	0	24 5	24 6	− 1	25 0	25 1	− 1
16.	26 5	26 6	− 1	24 10	24 10	0	23 4	23 4	0	22 3	22 3	0	24 9	24 7	+ 2	25 0	25 1	− 1
	26 5	26 6	− 1	24 11	24 10	+ 1	23 10	23 8	+ 2	23 3	23 4	− 1	25 7	25 6	+ 1	25 8	25 5	+ 3
17.	25 7	25 8	− 1	24 6	24 5	+ 1	22 11	22 10	+ 1	22 11	22 11	0	25 2	25 1	+ 1	25 3	25 3	0
	25 0	24 11	+ 1	24 1	24 0	+ 1	23 6	23 6	0	24 2	24 2	0	26 8	26 8	0	26 10	26 9	+ 1
18.	25 2	25 2	0	23 6	23 7	− 1	23 3	23 3	0	24 0	24 0	0	26 3	26 4	− 1	.....	.....	.....
	24 7	24 8	− 1	23 11	23 5	+ 6	24 3	24 3	0	25 0	25 4	− 4	.....	.....	.....	26 8	26 9	− 1
19.	24 3	24 0	+ 3	23 3	23 3	0	23 3	23 3	0	.....	.....	.....	28 11	28 11	0	29 1	29 1	0
	24 3	24 3	0	24 0	23 10	+ 2	25 11	25 10	+ 1	25 0	25 1	− 1	28 3	28 4	− 1	29 4	29 3	+ 1
20.	23 4	23 4	0	24 3	24 3	0	.....	.....	.....	26 3	26 4	− 1	27 11	27 11	0	29 4	29 3	+ 1
	24 5	24 5	0	24 4	24 6	− 2	25 3	25 5	− 2	26 4	26 6	− 2	27 3	27 4	− 1	29 0	29 1	− 1
21.	24 11	24 11	0	.....	.....	.....	26 2	26 2	0	28 0	28 0	0	27 11	27 10	+ 1	28 4	28 5	− 1
	24 7	24 8	− 1	25 0	25 1	− 1	26 8	26 8	0	27 2	27 2	0	27 8	27 9	− 1	28 9	28 10	− 1
22.	25 7	25 7	0	25 11	25 9	+ 2	28 0	28 1	− 1	28 2	28 1	+ 1	28 3	28 1	+ 2	28 3	28 2	+ 1
	.....	.....	.....	24 9	25 9	−12	27 7	27 7	0	28 1	28 1	0	28 0	28 0	0	28 3	28 4	− 1
23.	25 2	25 2	0	26 0	26 0	0	27 7	27 7	0	28 2	28 3	− 1	27 8	28 0	− 4	27 2	27 4	− 2
	25 8	25 7	+ 1	26 2	26 1	+ 1	26 7	26 9	− 2	28 0	28 1	− 1	29 0	29 0	0	27 11	27 11	0
24.	25 5	25 6	− 1	27 1	27 1	0	28 5	28 5	0	28 2	28 3	− 1	27 11	27 11	0	26 11	26 9	+ 2
	26 3	26 3	0	27 1	27 2	− 1	28 6	28 7	− 1	28 2	28 1	+ 1	27 5	27 6	− 1	27 0	27 1	− 1
25.	26 1	26 1	0	27 11	27 11	0	29 2	29 2	0	27 8	27 10	− 2	26 8	26 9	− 1	26 4	26 4	0
	26 11	27 0	− 1	27 8	27 9	− 1	28 4	28 5	− 1	28 10	28 11	− 1	27 1	27 2	− 1	26 9	26 10	− 1
26.	26 9	26 11	− 2	27 4	27 5	− 1	27 8	27 11	− 3	26 4	26 4	0	26 7	26 7	0	25 8	26 5	− 9
	27 4	27 5	− 1	27 4	27 4	0	28 3	28 2	+ 1	27 3	27 3	0	26 11	26 11	0	25 6	25 5	+ 1
27.	27 2	27 3	− 1	27 8	27 8	0	28 6	28 8	− 2	26 7	26 8	− 1	25 7	25 7	0	24 8	24 7	+ 1
	27 3	27 4	− 1	27 10	28 1	− 3	28 2	28 2	0	27 7	27 7	0	25 8	25 5	+ 3	25 4	25 5	− 1
28.	27 4	27 5	− 1	27 10	27 10	0	27 0	27 0	0	25 8	25 9	− 1	26 3	26 5	− 2	24 3	24 3	0
	27 6	27 8	− 2	28 0	28 0	0	27 7	27 8	− 1	26 0	26 2	− 2	26 8	26 10	− 2	22 8	22 8	0
29.	27 11	28 0	− 1	27 5	27 4	+ 1	24 6	26 5	−23	24 9	24 8	+ 1	25 2	25 3	− 1	24 10	24 8	+ 2
	27 1	27 2	− 1	27 6	27 6	0	25 2	25 1	+ 1	24 5	24 6	− 1	26 8	26 10	− 2	26 0	25 9	+ 3
30.	27 4	27 1	+ 3	27 0	27 0	0	26 1	26 1	0	24 10	24 10	0	26 2	26 1	+ 1	24 8	24 11	− 3
	27 2	27 2	0	27 3	27 4	− 1	25 3	25 5	− 2	25 6	25 8	− 2	26 5	26 3	+ 2	25 0	24 7	+ 5
31.	27 7	27 7	0	26 4	26 4	0	.....	.....	.....	24 5	24 4	+ 1	.....	.....	.....	24 6	.....	.....
	27 7	27 7	0	26 2	26 0	+ 2	.....	.....	.....	26 8	26 8	0	.....	.....	.....	24 10	24 9	+ 1

Table C.

Showing a comparison of the observed Times of High Water at the St. Katherine Docks, increased by five minutes, with the predicted Times given in the British Almanac. The observations marked with an * appear doubtful.

Date. 1835.	July.			August.			September.			October.			November.			December.		
	British Alman.	St. Kath. Docks. +5 min.	Error of Prediction.	British Alman.	St. Kath. Docks. +5 min.	Error of Prediction.	British Alman.	St. Kath. Docks. +5 min.	Error of Prediction.	British Alman.	St. Kath. Docks. +5 min.	Error of Prediction.	British Alman.	St. Kath. Docks. +5 min.	Error of Prediction.	British Alman.	St. Kath. Docks. +5 min.	Error of Prediction.
	h m	h m	m	h m	h m	m	h m	h m	m	h m	h m	m	h m	h m	m	h m	h m	m
1.	5 16	5 32	−16	6 25	6 47	−22	7 31	8 1*	−30	8 56	8 46	+10	11 29	11 26	+ 3	11 38	11 23	+15
	5 38	5 47	− 9	6 48	6 58	−10	8 12	8 21	− 9	9 46	9 51	− 5	.....	.....	.....	12 2	12 3	− 1
2.	6 0	6 13	−13	7 10	7 33	−23	8 58	9 17	−19	10 37	10 26	+11	11 50	12 8	−18	.....	.....	.....
	6 26	6 28	− 2	7 34	7 53	−19	9 46	9 47	− 1	11 17	11 17	0	12 26	12 28	− 2	12 22	12 12	+10
3.	6 57	7 2	− 5	8 2	8 40*	−38	10 35	10 52	−17	11 51	11 43	+ 8	12 47	1 3	−16	12 44	12 53	− 9
	7 22	7 12	+10	8 35	8 53	−18	11 20	11 32	−12	.....	.....	.....	1 4	1 3	+ 1	1 2	12 59	+ 3
4.	7 46	8 2	−16	9 12	9 42	−30	11 59	12 17	−18	12 21	12 27	− 6	1 20	1 37	−17	1 19	1 29	−10
	8 9	8 12	− 3	9 56	10 9	−13	.....	.....	.....	12 46	12 53	− 7	1 36	1 47	−11	1 37	1 37	0
5.	8 41	8 26	+15	10 39	10 58	−19	12 32	12 47	−15	1 11	1 18	− 7	1 51	2 13	−22	1 51	2 0	− 9
	9 13	9 2	+11	11 21	11 30	− 9	1 2	1 8	− 6	1 32	1 30	+ 2	2 8	2 17	− 9	2 10	2 14	− 4
6.	9 49	10 17	−28	12 1	12 17	−16	1 27	1 37	10	1 52	2 7	−15	2 23	2 43	−20	2 28	2 33	− 5
	10 23	10 32	− 9	.....	.....	.....	1 50	1 47	+ 3	2 9	2 8	+ 1	2 36	2 46	−10	2 44	2 58	−14
7.	11 0	11 18	−18	12 35	12 36	− 1	2 9	2 27	−18	2 24	2 37	−13	2 51	3 7	−16	3 2	3 10	− 8
	11 32	11 42	−10	1 6	1 13	− 7	2 29	2 33	− 4	2 38	2 47	− 9	3 5	3 18	−13	3 20	3 28	− 8
8.	.....	.....	.....	1 34	1 38	− 4	2 48	3 13	−25	2 53	3 12	−19	3 21	3 38	−17	3 40	3 42	− 2
	12 7	12 30	−23	1 59	2 3	− 4	3 7	3 18	−11	3 9	3 18	− 9	3 37	3 52	−15	3 54	4 2	− 8
9.	12 41	12 41	0	2 24	2 32	− 8	3 25	3 38	−13	3 24	3 37	−13	3 54	4 2	− 8	4 13	4 18	− 5
	1 9	1 13	− 4	2 45	2 51	− 6	3 40	3 39	+ 1	3 41	3 46	− 5	4 10	4 22	−12	4 30	4 38	− 8
10.	1 38	1 47	− 9	3 7	3 23	−16	3 55	4 2	− 7	3 52	4 2	−10	4 25	4 37	−12	4 47	4 53	− 6
	2 5	2 20	−15	3 25	3 36	−11	4 9	4 7	+ 2	4 5	3 57	+ 8	4 41	4 44	− 3	5 7	5 21	−14
11.	2 40	2 38	+ 2	3 43	4 8	−25	4 24	4 22	+ 2	4 20	4 26	− 6	4 58	4 59	− 1	5 27	5 37	−10
	2 54	3 2	− 8	4 3	4 13	−10	4 37	4 38	− 1	4 33	4 37	− 4	5 17	5 22	− 5	5 50	6 13	−23
12.	3 14	3 37	−23	4 22	4 37	−15	4 53	5 7	−14	4 45	4 53	− 8	5 40	5 28	+12	6 15	6 22	− 7
	3 37	3 52	−15	4 42	4 49	− 7	5 9	5 12	− 3	5 2	4 57	+ 5	6 4	6 2	+ 2	6 44	6 32	+12
13.	4 1	4 27	−26	4 58	5 19	−21	5 23	5 31	− 8	5 20	5 27	− 7	6 29	6 18	+11	7 12	7 3	+ 9
	4 23	4 31	− 8	5 13	5 20	− 7	5 41	5 46	− 5	5 39	5 31	+ 8	7 1	7 7	− 6	7 43	7 43	0
14.	4 45	5 0	− 5	5 30	5 53	−23	5 59	6 11	−12	5 55	6 7	−12	7 34	7 33	+ 1	8 18	8 14	+ 4
	5 4	5 12	− 8	5 47	6 3	−16	6 13	6 17	− 4	6 20	6 3	+17	8 12	8 12	0	8 51	9 2	−11
15.	5 26	5 52	−26	6 2	6 32	−30	6 31	6 42	−11	6 48	6 50	− 2	8 54	8 52	+ 2	9 21	9 32	−11
	5 48	5 52	− 4	6 19	6 32	−13	6 54	6 57	− 3	7 18	7 26	− 8	9 33	9 42	− 9	9 51	10 26	−35
16.	6 13	6 36	−23	6 37	7 5	−28	7 22	7 27	− 5	8 0	8 13	−13	10 10	10 22	−12	10 21	10 47	−26
	6 33	6 32	+ 1	6 57	7 8	−11	7 54	7 47	+ 7	8 46	8 57	−11	10 48	10 38	−10	10 52	11 18	−26
17.	6 50	7 12	−22	7 17	7 36	−19	8 36	8 46	−10	9 29	9 54	−25	11 20	10 28	− 8	11 24	12 45*	−81
	7 10	7 16	− 6	7 41	7 47	− 6	9 25	9 39	−16	10 14	10 41	−27	11 45	11 55	−10	11 55	12 16	−21
18.	7 32	8 12	−40	8 10	8 50	−40	10 11	10 28	−17	10 53	11 17	−24	12 8	12 20	−12	.....	.....	.....
	7 54	8 2	− 8	8 45	9 3	−18	10 56	11 12	−16	11 24	11 46	−22	.....	.....	.....	12 25	12 41	−16
19.	8 19	8 52*	−33	9 24	10 7	−43	11 29	11 53*	−23	.....	.....	.....	12 29	1 11	−42	12 51	1 3	−12
	8 49	8 51	− 2	10 6	10 24	−18	12 2	12 8	− 6	11 56	12 17	−21	12 54	12 48	+ 6	1 18	1 22	− 4
20.	9 19	10 2*	−43	10 47	11 24	−37	.....	.....	.....	12 23	12 49	−26	1 14	1 31	−17	1 42	2 2	−20
	9 51	10 12	−21	11 25	11 52	−27	12 28	12 32	− 4	12 49	1 6	−17	1 36	1 54	−18	2 10	2 20	−10
21.	10 24	10 47	−23	.....	.....	.....	12 55	1 17	−22	1 10	1 19	− 9	1 58	2 18	−20	2 36	2 48	−12
	10 59	12 17	−18	11 58	12 22	−24	1 17	1 27	−10	1 28	1 35	− 7	2 20	2 34	−14	3 2	3 13	−11
22.	11 31	12 15	−44	12 27	12 47	−20	1 37	1 55	−18	1 44	2 2	−18	2 46	2 49	− 3	3 28	3 30	− 2
	.....	.....	.....	12 55	1 12	−17	1 58	2 7	− 9	2 4	2 19	−15	3 10	3 23	−13	3 54	3 56	− 2
23.	11 59	12 16	−17	1 20	1 37	−17	2 17	2 32	−15	2 25	2 42	−17	3 36	3 53	−17	4 19	4 20	− 1
	12 28	12 57	−29	1 44	1 52	− 8	2 35	2 48	−13	2 47	2 56	− 9	4 1	4 2	− 1	4 42	4 48	− 6
24.	12 53	1 16	−23	2 3	2 20	−17	2 54	3 14	−20	3 5	3 22	−17	4 23	4 33	−10	5 7	5 10	− 3
	1 17	1 47	−30	2 23	2 33	−10	3 12	3 20	− 8	3 29	3 43	−14	4 49	5 3	−14	5 29	5 41	−12
25.	1 40	1 57	−17	2 42	2 58	−16	3 32	3 48	−16	3 51	4 8	−17	5 15	5 12	+ 3	5 52	5 56	− 4
	1 59	2 22	−23	3 3	3 7	− 4	3 51	4 8	−17	4 12	4 18	− 6	5 41	5 58	−17	6 15	6 31	−16
26.	2 18	2 37	−19	3 22	3 38	−16	4 11	4 31	−20	4 32	4 38	− 6	6 8	5 53	+15	6 43	6 37	+ 6
	2 38	3 3	−25	3 40	3 48	− 8	4 29	4 37	− 8	4 56	5 2	− 6	6 36	6 55	−19	7 12	7 37	−15
27.	3 0	3 16	−16	3 59	4 20	−21	4 49	5 13	−24	5 20	5 18	+ 2	7 6	6 49	+17	7 37	7 32	+ 5
	3 20	3 30	−10	4 15	4 31	−16	5 13	5 18	− 5	5 46	5 42	+ 4	7 40	7 55	−15	8 3	8 13	−10
28.	3 38	3 51	−13	4 33	4 53	−20	5 35	5 48	−13	6 13	6 13	0	8 17	7 49*	+28	8 27	8 31	− 4
	3 57	4 8	−11	4 51	5 6	−15	6 0	6 2	− 2	6 42	6 58	−16	8 54	8 43	+11	8 58	9 22	−28
29.	4 17	4 33	−16	5 13	5 32	−19	6 23	6 20	+ 3	7 16	7 15	+ 1	9 32	9 7	+25	9 28	8 18*	+70
	4 37	4 46	− 9	5 37	5 46	− 9	6 49	7 1	− 2	7 59	8 19	−20	10 10	10 2	+ 8	9 58	10 50*	−52
30.	4 58	4 47	+11	5 58	6 14	−16	7 20	7 16	+ 4	8 47	8 28	+19	10 45	10 32	+13	10 30	11 27*	−57
	5 20	5 27	− 7	6 20	6 24	− 4	8 5	8 7	− 2	9 30	9 42	−12	11 13	11 23	−10	11 3	11 51*	−48
31.	5 45	5 58	−13	6 40	7 7	−27	.....	.....	.....	10 14	10 17	− 3	.....	.....	.....	11 32	0 0	
	6 5	6 8	− 3	7 2	7 17	−15	.....	.....	.....	10 56	11 10	−14	.....	.....	.....	11 59	12 26	−27

Table D.

Showing a comparison of the observed Heights of High Water at the St. Katherine Docks, with the predicted Heights given in the British Almanac, increased by five feet. The observations marked with an * appear doubtful.

Date. 1835.	July.			August.			September.			October.			November.			December.		
	British Alman. + 5 ft.	St. Kath. Docks.	Error of Prediction.	British Alman. + 5 ft.	St. Kath. Docks.	Error of Prediction.	British Alman. + 5 ft.	St. Kath. Docks.	Error of Prediction.	British Alman. + 5 ft.	St. Kath. Docks.	Error of Prediction.	British Alman. + 5 ft.	St. Kath. Docks.	Error of Prediction.	British Alman. + 5 ft.	St. Kath. Docks.	Error of Prediction.
	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.
1.	27 1	26 6	+ 7	26 8	26 8	0	24 10	25 1	− 3	24 3	23 8	+ 7	25 4	25 2	+ 2	25 1	26 10	−21
	26 10	26 9	+ 1	26 4	26 5	− 1	24 6	25 3	− 9	24 6	24 10	− 4	.....	.....	.....	25 3	27 2	−23
2.	26 7	26 9	− 2	25 10	26 3	− 5	24 5	24 4	+ 1	24 10	23 6*	+16	25 8	26 7	−11	.....	.....	.....
	26 5	26 7	− 2	25 5	26 1	− 8	24 7	25 0	− 5	25 3	25 9	− 6	25 11	26 2	− 3	25 5	26 5	−12
3.	26 2	26 8	− 6	25 1	26 4	−15	25 0	24 6	+ 6	25 7	26 5	−10	26 3	27 0	− 9	25 9	27 8	−23
	25 10	25 5	+ 5	24 9	25 7	−10	25 5	25 5	0	.....	.....	.....	26 6	26 7	− 1	26 1	27 6	−17
4.	25 7	25 9	− 2	24 8	25 2	− 6	25 10	25 1	+ 9	26 0	27 9*	−21	26 8	26 5	+ 3	26 4	27 4	−12
	25 4	25 5	− 1	24 10	25 5	− 7	.....	.....	.....	26 4	26 1	+ 3	26 9	26 6	+ 3	26 6	26 10	− 4
5.	25 2	25 2	0	25 2	25 5	− 3	26 3	26 6	− 3	26 8	27 4	− 8	26 11	26 9	+ 2	26 9	27 0	− 3
	25 1	25 11	−10	25 7	25 9	− 2	26 8	26 6	+ 2	26 11	27 1	− 2	27 1	27 4	− 3	26 11	27 8	− 9
6.	25 2	25 8	− 6	26 0	25 6	+ 6	27 0	27 9	− 9	27 2	28 2	−12	27 2	27 7	− 5	27 1	27 4	− 3
	25 4	26 1	− 9	.....	.....	.....	27 2	27 0	+ 2	27 5	28 1	− 8	27 3	27 7	− 4	27 2	27 3	− 1
7.	25 8	24 6	+14	26 6	26 2	+ 4	27 5	27 4	+ 1	27 6	28 0	− 6	27 3	26 8	+ 7	27 2	27 6	− 4
	26 0	26 3	− 3	26 11	26 7	+ 4	27 7	27 6	+ 1	27 6	28 2	− 8	27 3	26 1	+14	27 3	27 4	− 1
8.	26 5	26 1	+ 4	27 2	27 8	− 6	27 8	27 5	+ 3	27 5	27 11	− 6	27 2	26 2	+12	27 3	27 1	+ 2
	.....	.....	.....	27 6	28 0	− 6	27 8	27 10	− 2	27 4	27 9	− 5	27 1	26 7	+ 6	27 1	26 6	+ 7
9.	26 11	27 4	− 5	27 8	28 3	− 7	27 8	27 5	+ 3	27 4	27 1	+ 3	27 0	27 0	0	27 0	26 2	+10
	27 3	26 10	+ 5	27 9	27 9	0	27 7	27 9	− 2	27 3	27 4	− 1	26 10	27 2	− 4	26 10	27 8	−10
10.	27 6	27 3	+ 3	27 9	27 8	+ 1	27 6	28 1	− 7	27 2	28 6	−16	26 8	26 7	+ 1	26 8	25 4	+16
	27 9	27 1	+ 8	27 9	27 5	+ 4	27 4	26 7	+ 9	27 0	28 4	−16	26 5	26 3	+ 2	26 6	25 6	+12
11.	27 11	28 1	− 2	27 9	27 8	+ 1	27 1	25 7	+18	26 10	27 3	− 5	26 2	25 1	+13	26 3	24 6	+21
	27 11	28 0	− 1	27 7	27 10	− 3	26 11	26 8	+ 3	26 7	27 4	− 9	25 10	25 3	+ 7	26 0	25 6	+ 6
12.	28 0	27 10	+ 2	27 5	27 6	− 1	26 8	26 7	+ 1	26 4	28 1*	−21	25 7	24 8	+11	25 9	24 8	+13
	28 0	27 0	+12	27 2	27 4	− 2	26 5	26 5	− 0	26 1	25 9	+ 4	25 3	25 5	− 2	25 6	24 7	+11
13.	27 10	27 7	+ 3	27 0	27 7	− 7	26 2	26 4	− 2	25 9	25 9	0	24 11	23 2	+21	25 3	24 2	+13
	27 7	27 3	+ 4	26 10	27 4	− 6	25 11	26 3	− 4	25 5	27 9*	−28	24 8	23 10	+10	25 1	24 10	+ 3
14.	27 4	27 7	− 3	26 6	26 5	+ 1	25 5	25 3	+ 2	25 1	25 7	− 6	24 6	22 7*	+23	25 1	24 1	+12
	27 1	27 2	− 1	26 1	26 7	− 6	25 1	25 4	− 3	24 8	25 8	−12	24 6	23 11	+ 7	25 2	25 4	− 2
15.	26 9	27 1	− 4	25 9	25 6	+ 3	24 8	24 3	+ 5	24 8	23 1	+19	24 8	23 4	+16	25 3	24 3	+12
	26 4	26 5	− 1	25 5	25 10	− 5	24 3	24 2	+ 1	24 2	23 3	+11	25 0	24 6	+ 6	25 4	25 1	+ 3
16.	26 0	26 6	− 6	25 1	24 10	+ 3	24 0	23 4	+ 8	24 0	22 3	+21	25 5	24 7	+10	25 7	25 1	+ 6
	25 8	26 6	−10	24 9	24 10	− 1	23 10	23 8	+ 2	24 1	23 4	+ 9	22 11	25 6	+ 5	25 10	25 5	+ 5
17.	25 4	25 8	− 4	24 4	24 5	− 1	23 9	22 10	+11	24 5	22 11	+18	26 5	25 1	+16	26 3	25 3	+12
	24 11	24 11	0	24 0	24 0	0	23 10	23 6	+ 4	24 11	24 2	+9	26 8	26 8	0	26 6	26 9	− 3
18.	24 7	25 2	− 7	23 9	23 7	+ 2	24 4	23 3	+13	25 5	24 0	+17	26 11	26 4	+ 7	.....	.....	.....
	24 3	24 8	− 5	23 8	23 5	+ 3	24 10	24 3	+ 7	26 0	25 4	+ 8	.....	.....	.....	26 11	26 9	+ 2
19.	24 0	24 0	− 0	23 8	23 3	+ 5	25 4	23 3*	+25	.....	.....	.....	27 3	28 11*	−20	27 4	29 1	−21
	23 10	24 3	− 5	23 11	23 10	+ 1	25 10	25 10	0	26 5	25 1	+16	27 6	28 4	−10	27 7	29 3	−16
20.	23 9	23 4	+ 5	24 4	24 3	+ 1	.....	.....	.....	26 11	26 4	+ 7	27 9	27 11	− 2	27 11	29 3	−16
	23 9	24 5	− 8	24 8	24 6	+ 2	26 5	25 5	+12	27 5	26 6	+11	28 0	27 4	+ 8	28 2	29 1	−11
21.	23 11	24 11	−12	.....	.....	.....	26 11	26 2	+ 9	27 9	28 0	− 3	28 2	27 10	+ 4	28 3	28 5	− 2
	24 2	24 8	− 6	25 3	25 1	+ 2	27 4	26 8	+ 8	27 11	27 2	+ 9	28 3	27 9	+ 6	28 4	28 10	− 6
22.	24 7	25 7	−12	25 9	25 9	0	27 8	28 1	− 5	28 1	28 1	0	28 4	28 1	+ 3	28 4	28 2	+ 2
	.....	.....	.....	26 4	25 9	+ 7	28 0	27 7	+ 5	28 3	28 1	+ 2	28 5	28 0	+ 5	28 2	28 4	− 2
23.	25 0	25 7	− 2	26 9	26 0	+ 9	28 2	27 7	+ 7	28 4	28 3	+ 1	28 4	28 0	+ 4	27 11	27 4	+ 7
	25 5	25 7	− 2	27 1	26 1	+12	28 4	26 9	+19	28 5	28 1	+ 4	28 2	29 0	−10	27 8	27 11	− 3
24.	25 11	25 6	+ 5	27 6	27 1	+ 5	28 4	28 5	− 1	28 6	28 3	+ 3	27 10	27 11	− 1	27 3	26 9	+ 6
	26 4	26 3	+ 1	27 9	27 2	+ 7	28 5	28 7	− 2	28 5	28 1	+ 4	27 5	27 6	− 1	26 10	27 1	− 3
25.	26 8	26 1	+ 7	27 11	27 11	0	28 5	29 2	− 9	28 4	27 10	+ 6	26 11	26 9	+ 2	26 5	26 4	+ 1
	27 1	27 0	+ 1	28 1	27 9	+ 4	28 4	28 5	− 1	28 1	28 11	−10	26 5	27 2	− 9	26 0	26 10	−10
26.	27 5	26 11	+ 6	28 2	27 5	+ 9	28 3	27 11	+ 4	27 10	26 4	+18	25 11	26 7	− 8	25 7	26 5	−10
	27 7	27 5	+ 2	28 3	27 4	+10	28 1	28 2	− 1	27 4	27 3	+ 1	25 5	26 11	−18	25 2	25 5	− 3
27.	27 9	27 3	+ 6	28 3	27 8	+ 7	27 9	28 8	−11	26 9	26 8	+ 1	24 11	25 7	− 8	24 10	24 7	+ 3
	27 10	27 4	+ 6	28 2	28 1	+ 1	27 4	28 2	−10	26 3	27 7	−16	24 7	25 5	−10	24 7	25 5	−10
28.	27 11	27 5	+ 6	28 1	27 10	+ 3	26 10	27 0	− 2	25 7	25 9	− 2	24 5	25 6	− 1	24 3	24 3	0
	27 10	27 8	+ 2	27 11	28 0	− 1	26 4	27 8	−16	25 1	26 2	−13	24 4	26 10*	−30	24 0	22 8*	+16
29.	27 10	28 0	− 2	27 7	27 4	+ 3	25 8	26 5	− 9	24 7	24 8	− 1	24 6	25 2	− 8	23 11	24 8	− 9
	27 9	27 2	+ 7	27 3	27 6	− 3	25 1	25 1	0	24 4	24 6	− 2	24 8	26 10*	−26	23 11	25 9*	−22
30.	27 8	27 1	+ 7	26 10	27 0	− 2	24 8	26 1	−17	24 3	24 10	− 7	24 9	26 2	−17	24 1	24 11	−10
	27 6	27 2	+ 4	26 5	27 4	−11	24 8	25 5	− 9	24 5	25 8	−13	24 11	26 3	−16	24 3	24 7	− 4
31.	27 3	27 7	− 4	25 11	26 4	− 5	.....	.....	.....	24 8	24 4	+ 4	.....	.....	.....	24 6	.....	.....
	27 0	27 7	− 7	25 4	26 0	− 8	.....	.....	.....	25 0	26 8	−20	.....	.....	.....	24 10	24 9	+ 1

TABLE XXVII.

January.

Upper Transits, P.M.

Moon's Transit.	Moon's Parallax.	Interval.	Height.	Moon's Declination.	Equation of Time.
h m		h m	ft. in.	°	m
31·3	57·5	11 12	17 0·8	S. 18·6	+10·1
1 30·5	58	10 58·6	17 7·2	S. 15	+10·1
2 27·6	57·2	10 46·1	16 6	S. 10	+10·3
3 31	57·1	10 37·6	15 10·1	S. 3·6	+ 9·8
4 29·3	56·9	10 31·8	14 7·2	N. 2·8	+10·5
5 30·6	56·5	10 34·1	13 1	N. 7·6	+ 9·9
6 33·8	56·7	10 50·1	12 1	N. 14	+10·2
7 30·3	56·9	11 18	13 0	N. 17·7	+ 9·6
8 32·9	56·6	11 39·3	13 5·1	N. 21·1	+10·1
9 34·1	57·4	11 44·8	14 11·6	N. 22·8	+10·1
10 34·2	57·3	11 46·4	16 1	N. 22·5	+10·3
11 33·5	57·7	11 26·2	17 1·1	N. 21·6	+10

Lower (Interpolated) Transits, A.M.

Moon's Transit.	Moon's Parallax.	Interval.	Height.	Moon's Declination.	Equation of Time.
h m		h m	ft. in.	°	m
30·5	58	11 10·5	18 1·2	S. 19·1	+ 9·8
1 32·6	57	10 58·6	18 3	S. 14·6	+10
2 32·9	57·5	10 44·4	18 5·6	S. 9·9	+ 9·9
3 29·6	57	10 36·6	16 10·3	S. 3·6	+ 9·8
4 27·5	56·5	10 31·3	15 10	N. 2·0	+10
5 29·2	56·6	10 33·2	14 6·4	N. 7·1	+ 9·7
6 26·8	56·6	10 48·1	12 10·9	N. 13·7	+ 9·8
7 29·5	56·6	11 16·8	12 10·2	N. 18·2	+10·2
8 29	56·8	11 40·7	13 8·6	N. 20·3	+ 9·7
9 23·8	57·4	11 46·8	14 9·3	N. 22·5	+ 9·9
10 25·7	57·2	11 41·4	15 10·6	N. 22·3	+10·2
11 32	57·5	11 28·5	16 8·5	N. 22	+ 9·8

Upper Transits, A.M.

30·4	57·3	11 14	17 6·1	N. 18·3	+10·2
1 32·1	56·9	11 0	17 7·9	N. 14·9	+10
2 30·7	57·3	10 44·8	17 10·5	N. 10·4	+ 9·9
3 29	57	10 37·1	16 5·8	N. 4·7	+10·1
4 30·4	56·6	10 30·5	15 6	S. 4·0	+10·4
5 29·8	57·2	10 33	14 2·1	S. 8·1	+ 9·8
6 28·9	56·8	10 48·6	12 10·5	S. 13·7	+ 9·6
7 33·1	56·8	11 18·4	12 6·7	S. 18·3	+10
8 35	57·4	11 39·5	13 8·8	S. 21	+ 9·9
9 32	57·8	11 44·2	14 10·5	S. 22·3	+10·1
10 29·6	57·5	11 40·3	15 6·2	S. 22·5	+ 9·8
11 27·3	57·5	11 28	16 2·8	S. 22	+ 9·7

Lower (Interpolated) Transits, P.M.

32·4	57·4	11 14·2	17 1·6	N. 18·1	+10·2
1 31·8	57·4	10 59·2	17 3·8	N. 15·1	+10·1
2 30·5	57·2	10 48·2	16 11·1	N. 9·4	+10·3
3 30·6	57·1	10 37·2	15 10·8	N. 4·8	+10
4 29·8	56·8	10 31·6	14 7·5	S. 3·3	+10
5 30·3	56·6	10 35·1	13 2·3	S. 8·7	+10·1
6 30·1	56·7	10 50·1	12 11·1	S. 14	+10·2
7 25·9	57·3	11 16·2	12 8·7	S. 18	+10·1
8 26·7	57	11 38·5	13 5·6	S. 20·5	+10·1
9 31·9	57·5	11 44·8	15 1·6	S. 22·8	+ 9·9
10 34·4	57·9	11 36·7	16 5·3	S. 22·8	+10·3
11 33·6	58·1	11 24·8	17 5·8	S. 21·6	+10·3

February.

Upper Transits, P.M.

33·1	56·8	11 18·9	17 6	S. 10·1	+14·2
1 34·3	57·7	11 2·8	17 11·7	S. 2·8	+14·1
2 31·5	57	10 51	16 11·3	N. 2·7	+14·2
3 30·8	56·9	10 37·8	15 11·2	N. 8·6	+14·2
4 30·1	56·9	10 28	14 10·6	N. 13·5	+14·1
5 28·7	56·4	10 27·1	12 10·6	N. 18·2	+14·1
6 29·9	56·6	10 39·6	12 1	N. 21·3	+14·2
7 32·2	56·7	11 11·2	11 10·6	N. 22·5	+14
8 34·1	57	11 38·8	13 2·7	N. 22·8	+14·1
9 31·8	57·1	11 49·3	14 11·2	N. 21	+14·2
10 32·8	57·2	11 43·4	16 3·1	N. 19·1	+14·2
11 30·8	57	11 32·6	17 7·3	N. 14·5	+14·3

Lower (Interpolated) Transits, A.M.

31·6	57·7	11 14·5	18 11·2	S. 10·1	+14·2
1 27	57·1	11 3·8	18 10·9	S. 3·5	+14·2
2 28·6	57·3	10 49·9	18 3·2	N. 2·1	+14·3
3 27·6	57·2	10 36·7	17 3·7	N. 8·5	+14·2
4 29·8	56·8	10 28·6	15 4·3	N. 13·6	+14·1
5 32·3	56·2	10 26·7	13 9·2	N. 18·8	+14·2
6 30·7	56·8	10 35·7	12 4·5	N. 21·1	+14·1
7 29·5	56·7	11 7·4	11 10·2	N. 21·9	+14·1
8 28	56·6	11 39	12 6·1	N. 23·4	+14·1
9 28	57	11 48·2	14 2·9	N. 21·6	+14·2
10 27·3	57·4	11 45·5	15 11·4	N. 18·7	+14·2
11 29·4	57·3	11 31·7	17 4·2	N. 15	+14·3

Upper Transits, A.M.

28·8	57·6	11 16·7	18 3·8	N. 9·8	+14·2
1 30·4	57	11 4·5	18 9	N. 3·6	+14·2
2 34	57·6	10 48·2	18 3·3	S. 2·4	+14·3
3 30	57·3	10 37·3	16 9·6	S. 7·4	+14·3
4 29·9	56·8	10 28·9	15 4	S. 13·8	+14·2
5 32·2	57	10 24·3	13 6·4	S. 18·7	+14·1
6 28·2	57	10 32	11 10·4	S. 20·8	+14·1
7 29·6	57·2	11 4·3	11 7·3	S. 22·9	+14·1
8 31·5	57·3	11 36·2	12 9·1	S. 23	+14·2
9 32·6	57	11 49·5	14 0·6	S. 21·7	+14·2
10 33·1	57·6	11 42·1	15 10·7	S. 18·2	+14·2
11 29·8	57·8	11 29·9	16 11	S. 15	+14·3

Lower (Interpolated) Transits, P.M.

33·1	57·1	11 18·5	17 9·2	N. 9·3	+14·2
1 30·2	57·6	11 5	18 2·8	N. 3·9	+14·2
2 29·4	57·1	10 49·6	17 5·1	S. 2·5	+14·2
3 28·7	57·1	10 39	16 1	S. 7·2	+14·2
4 28·5	57·2	10 27·8	15 0	S. 13·9	+14·2
5 26·1	57·3	10 26·4	13 5·8	S. 18·2	+14·2
6 30·3	56·8	10 37·7	11 7·8	S. 21·2	+14·2
7 33·2	57·1	11 10·4	11 9·9	S. 23·3	+14·1
8 28·5	57·2	11 36·7	13 1·1	S. 22·8	+14
9 26·5	57·5	11 45·6	14 9	S. 22	+14·2
10 28·7	57·2	11 42·6	16 1	S. 19·5	+14·2
11 31·8	57·3	11 31	17 4	S. 14·1	+14·1

Table XXVII. (Continued.)

March.

Upper Transits, P.M.

Moon's Transit.	Moon's Parallax.	Interval.	Height.	Moon's Declination.	Equation of Time.
h m		h m	ft. in.	°	m
31·8	57·7	11 17·5	17 7·6	N. 1·5	+ 8·5
1 32·2	57·1	11 4·5	17 8	N. 8·5	+ 8·2
2 31·3	57·5	10 48·3	17 2·6	N. 12·6	+ 8·5
3 30·1	56·7	10 33·5	15 10·9	N. 17·4	+ 9
4 31·4	56·7	10 20·7	14 0·1	N. 21·2	+ 8·5
5 29·9	56·5	10 16·2	12 7	N. 22·6	+ 8·4
6 31·8	56·4	10 31·9	11 3·4	N. 22·5	+ 8·1
7 30·8	56·7	11 7·4	11 2	N. 22·6	+ 8·5
8 31·5	56·6	11 43·3	12 9·3	N. 19·9	+ 8·6
9 32·3	56·9	11 53·1	14 10	N. 14·8	+ 8
10 29·5	57·5	11 45	16 9·4	N. 10·3	+ 8·3
11 29·4	57·2	11 35·6	17 8·8	N. 5·0	+ 8·8

Lower (Interpolated) Transits, A.M.

Moon's Transit.	Moon's Parallax.	Interval.	Height.	Moon's Declination.	Equation of Time.
h m		h m	ft. in.	°	m
31	57·4	11 19·1	18 4·1	N. 1·3	+ 8·5
1 29·8	57·9	11 3	19 0·8	N. 7·1	+ 8·8
2 31·6	56·8	10 50·8	17 2·9	N. 13·2	+ 8·4
3 31	57	10 32·2	16 6·2	N. 18·2	+ 8·4
4 28·3	56·6	10 21·9	14 8·3	N. 20·5	+ 8·7
5 30·6	56·3	10 17·2	13 0·5	N. 22·2	+ 8·8
6 33	56·4	10 31·6	11 0·1	N. 22·4	+ 8·6
7 32·1	56·6	11 8·8	11 2·3	N. 21·8	+ 8·4
8 30·2	57	11 40·4	12 6·7	N. 19·5	+ 8·5
9 32·8	57·2	11 49	14 6·1	N. 15	+ 8·3
10 35	57	11 47·4	16 3·1	N. 10	+ 8·6
11 31·5	57·8	11 32·2	17 7·6	N. 4·7	+ 9

Upper Transits, A.M.

32·8	58·1	11 18·1	18 8·3	S. 1·9	+ 8·6
1 30·2	57·6	11 4·1	18 11·2	S. 6·8	+ 8·9
2 31·9	57·4	10 48·5	17 10·1	S. 13·1	+ 8·7
3 35	57·2	10 32·6	16 5·8	S. 18·2	+ 8·8
4 30	56·9	10 19·4	14 7·4	S. 20·8	+ 8·9
5 30·5	56·7	10 16·1	12 11·3	S. 22·3	+ 8·6
6 32·4	56·5	10 28·4	11 4·6	S. 23·2	+ 8·8
7 31·3	57	11 4·7	11 1·8	S. 21·6	+ 8·4
8 32·5	56·8	11 42·5	12 6·6	S. 19·4	+ 8
9 27·8	57·5	11 50·7	14 3·7	S. 15·6	+ 8·5
10 26·9	57·2	11 43·9	15 9	S. 11·2	+ 9
11 29·6	57·9	11 32·9	17 4·1	S. 5·4	+ 8·4

Lower (Interpolated) Transits, P.M.

28·8	57·5	11 21·3	18 2	S. 1·3	+ 8·8
1 31	57·9	11 3·3	18 6·2	S. 8	+ 8·4
2 30·2	57·8	10 48·2	17 7·2	S. 12·7	+ 8·6
3 29·6	57·2	10 33·8	16 3·9	S. 17·6	+ 9
4 30·4	56·6	10 19·3	14 7	S. 21·1	+ 8·3
5 30·2	57	10 15·5	13 1·2	S. 22·2	+ 8·7
6 29·8	56·6	10 30·6	11 10·5	S. 22·9	+ 8·1
7 27·9	57	11 5·7	11 9·9	S. 21·7	+ 8·3
8 29·4	56·8	11 39·6	12 10·5	S. 20·1	+ 8
9 32·9	57·3	11 49·5	14 11·6	S. 14·8	+ 8·3
10 33·1	57·5	11 45·5	16 9·2	S. 9·8	+ 8·1
11 28·5	57·9	11 30·4	17 10·5	S. 5·0	+ 8·6

April.

Upper Transits, P.M.

31·6	57·5	11 18	17 10·1	N. 13·1	− ·01
1 31·4	57·1	11 2	17 4·5	N. 16·8	+ ·03
2 31	57·5	10 42·3	16 9·7	N. 20·6	0
3 33·1	56·9	10 25·6	15 8·6	N. 22·7	− ·03
4 34	56·8	10 13·3	14 3·3	N. 22·7	·02
5 34·7	56·6	10 12·2	12 7·9	N. 22·1	0
6 31	56·6	10 32	11 5·4	N. 20·2	− ·01
7 32·6	56·7	11 17·2	11 10	N. 15·6	− ·03
8 32	56·9	11 46	13 4·4	N. 10·8	− ·03
9 27	57·1	11 53·2	15 1·7	N. 5·7	0
10 27·5	57	11 48·6	16 7·6	S. 0·7	− ·03
11 28·1	57·9	11 33	17 4·8	S. 7	− ·01

Lower (Interpolated) Transits, A.M.

26·8	57·2	11 20·7	18 3·7	N. 12·1	+ ·04
1 32·7	57·5	11 1	18 1·1	N. 17·1	0
2 30	57	10 45	16 11·2	N. 20·3	+ ·01
3 27·3	57	10 29·3	15 11·9	N. 22	− ·01
4 28·4	56·7	10 15	14 2·2	N. 22·9	0
5 28	57	10 12·2	12 8·1	N. 22·1	+ ·02
6 30·3	56·6	10 35	11 2·5	N. 18·9	− ·03
7 31·4	56·9	11 18·4	11 7·6	N. 16·2	+ ·01
8 30·3	57	11 43·3	13 3·8	N. 12	− ·01
9 29·6	57·1	11 54·7	15 1·4	N. 5·1	− ·03
10 32·2	57·2	11 48·4	16 7·1	S. 0·4	− ·02
11 32·1	57·3	11 36	17 9·6	S. 7·5	− ·03

Upper Transits, A.M.

28	57·1	11 21·2	17 11	S. 11·5	+ ·05
1 29·1	57·9	11 0·5	18 1·2	S. 17·4	− ·02
2 28·4	57·8	10 42·6	17 3·3	S. 20·7	0
3 31·1	57·3	10 26·9	16 0·5	S. 22	+ ·03
4 31·6	57·3	10 14·1	14 3·3	S. 23·4	− ·02
5 31·8	57·1	10 12·5	12 8·1	S. 22·1	0
6 32·5	56·7	10 33·6	11 2·8	S. 19·9	− ·01
7 31·4	57·1	11 11·9	11 4·2	S. 16	− ·01
8 27·2	57·1	11 46·2	12 8·5	S. 12	+ ·02
9 30·8	56·8	11 56·1	14 7·6	S. 6	− ·03
10 28	57·5	11 47·2	16 4·5	N. 0·6	0
11 30	56·6	11 38	17 0·1	N. 6·3	+ ·03

Lower (Interpolated) Transits, P.M.

32	58	11 16·5	18 2·8	S. 13	+ ·01
1 28·5	57·7	11 0·5	18 0·9	S. 16·5	+ ·02
2 30·7	57·4	10 43·8	17 2·6	S. 20·7	0
3 32·5	57·3	10 24·6	16 2·3	S. 22·6	+ ·03
4 30	57·2	10 13·9	14 11·2	S. 22·6	− ·03
5 25·7	57	10 11·8	13 5·2	S. 22·2	− ·01
6 27·3	56·8	10 26·9	11 10·8	S. 20·5	0
7 27	57·2	11 9·7	11 11·3	S. 16·3	− ·02
8 29·3	57	11 43·4	13 6·5	S. 10·8	− ·02
9 30·2	57·2	11 52·5	15 5	S. 5·1	− ·01
10 29·1	57	11 48·8	16 8·9	N. 0·8	− ·01
11 28·7	57	11 37	17 8·8	N. 7·0	− ·01

Table XXVII. (Continued.)

May.

Upper Transits, P.M.					Lower (Interpolated) Transits, A.M.				
Moon's Transit.	Hor. Par.	Interval.	Height.	Moon's Declination.	Equation of Time.	Moon's Transit.	Hor. Par.	Interval.	Height.	Moon's Declination.	Equation of Time.
h m	′	h m	ft. in.	°	m	h m	′	h m	ft. in.	°	m
28·6	56·9	11 17·2	17 0	N. 20·9	− 3·6	32·3	57·3	11 16·4	17 4	N. 20·4	− 3·7
1 31·2	57·1	10 53	17 0·4	N. 22·2	− 3·7	1 32	56·9	10 58	16 10	N. 21·9	− 3·6
2 31·3	57·1	10 39	16 7·3	N. 22·7	− 3·7	2 33·5	56·9	10 38·6	16 1·8	N. 23	− 3·7
3 28	57	10 26·4	15 10·5	N. 21·9	− 3·7	3 33·1	57	10 25·6	15 4·5	N. 22·1	− 3·7
4 27·8	57	10 15·7	14 8·5	N. 20·4	− 3·7	4 30·9	56·9	10 16·1	14 1·5	N. 19·6	− 3·6
5 30	56·9	10 18·6	13 6·6	N. 16	− 3·6	5 27·6	56·9	10 19·8	12 11·1	N. 16	− 3·7
6 29·2	57	10 40·4	12 8	N. 11·6	− 3·6	6 28·8	56·8	10 42·5	12 1·2	N. 12·6	− 3·6
7 27·4	57·1	11 18	12 10·1	N. 5·5	− 3·7	7 30·1	57	11 20·6	12 8	N. 5·5	− 3·7
8 29	56·9	11 47·4	13 11·2	S. 0·5	− 3·7	8 29·5	57·1	11 47·8	13 9·6	N. 0·5	− 3·7
9 29·9	57·6	11 50·8	15 3·8	S. 6·9	− 3·7	9 28·2	57	11 54·2	15 3·8	S. 7·3	− 3·7
10 31·1	57·1	11 46·7	16 1·7	S. 11·8	− 3·7	10 32·8	57·7	11 45·5	16 6·1	S. 11·9	− 3·7
11 32·5	57·6	11 29·9	16 11·7	S. 17·5	− 3·6	11 26·2	57·5	11 33·1	17 4	S. 16·4	− 3·6
Upper Transits, A.M.						**Lower (Interpolated) Transits, P.M.**					
30	57·5	11 15·8	17 5·1	S. 19·8	− 3·7	27·4	57·2	11 16·4	17 8	S. 20·3	− 3·7
1 27·3	57·3	10 59·5	17 2·8	S. 22	− 3·7	1 30	57·9	10 58·8	17 7·3	S. 22·3	− 3·7
2 27·5	57·6	10 39·2	16 5·6	S. 22·7	− 3·6	2 30·1	57·6	10 38·5	17 0·2	S. 22·6	− 3·7
3 32·8	57·3	10 24·8	15 5·1	S. 22·1	− 3·7	3 27·9	57·5	10 23·0	16 4·5	S. 22	− 3·7
4 33·4	57·5	10 17·3	14 2·6	S. 19·7	− 3·7	4 28·9	57·2	10 14·3	14 11·6	S. 20·3	− 3·7
5 30·3	57·5	10 18·3	12 9·9	S. 16·6	− 3·6	5 31·4	57·2	10 18·5	13 10·2	S. 15·5	− 3·7
6 27	56·9	10 40·4	11 6·7	S. 12·3	− 3·6	6 32·2	57·2	10 42·7	13 0	S. 11·6	− 3·6
7 27·8	57	11 22·4	12 0·8	S. 6·0	− 3·7	7 33·5	57·2	11 21·1	13 0·9	S. 5·5	− 3·7
8 30	57	11 45·8	13 3·2	N. 0·5	− 3·6	8 31·6	56·8	11 47·3	13 11·6	N. 0·9	− 3·7
9 32·5	56·7	11 53·7	14 9·1	N. 7·0	− 3·6	9 29·1	57·2	11 50·9	15 5·1	N. 6·3	− 3·7
10 33·7	57	11 47·4	16 2·6	N. 11·8	− 3·7	10 28·2	56·3	11 46·7	16 3	N. 11·9	− 3·6
11 29·5	57·2	11 32·9	16 10·2	N. 16·5	− 3·7	11 31·5	57·1	11 31·9	16 10·8	N. 17·0	− 3·6

June.

Upper Transits, P.M.						Lower (Interpolated) Transits, A.M.					
32·3	57·2	11 11·7	16 11·3	N. 22·8	+ ·1	29·7	56·9	11 13·8	16 4·7	N. 23	0
1 29	57·1	10 55·7	17 0·1	N. 22·6	− ·1	1 33·8	57	10 56·1	16 4·5	N. 21·7	+ ·3
2 30·1	56·9	10 40	16 8·4	N. 19·4	+ ·2	2 32·7	57·1	10 39·6	16 0·3	N. 19·4	+ ·2
3 31·6	56·7	10 29·6	16 1·8	N. 15·4	+ ·3	3 26·9	56·9	10 31·3	15 3·8	N. 16·3	0
4 30·9	57	10 24·5	15 0·6	N. 11·1	− ·1	4 30·7	56·7	10 25·1	14 1·1	N. 11·8	− ·1
5 29·3	56·8	10 32	13 11·1	N. 5·1	+ ·8	5 32·4	56·8	10 31·8	13 4·9	N. 4·7	+ ·6
6 29·5	57	10 52·9	13 4·1	S. 0·7	+ ·2	6 28·1	56·7	10 53·4	12 7	S. 0·7	+ ·1
7 31·7	57·2	11 24	13 2·1	S. 7·5	+ ·5	7 28·9	57	11 24·5	12 11·4	S. 7·1	+ ·2
8 33·3	57·2	11 49·7	13 11·2	S. 11·3	+ ·2	8 27·9	57·6	11 42	13 10·8	S. 12·0	0
9 34·9	57·5	11 48·6	14 10·8	S. 18·0	+ ·7	9 29·1	57·2	11 49·4	15 1·5	S. 16·6	+ ·2
10 30·1	58·4	11 40·8	15 9·4	S. 20·1	+ ·1	10 31·8	57·7	11 40·7	16 2·2	S. 20	+ ·4
11 32	57·8	11 26·7	16 5·4	S. 22·4	+ ·3	11 31·4	57·6	11 28	16 11·1	S. 22·3	+ ·3
Upper Transits, A.M.						**Lower (Interpolated) Transits, P.M.**					
30	56·8	11 12·2	16 0·6	S. 23·8	− ·1	30·1	57·8	11 11·5	17 5·8	S. 22·9	+ ·1
1 28·5	57·4	10 55·4	16 5·1	S. 22	0	1 29·4	57·4	10 54·9	17 4·8	S. 22·5	0
2 28·9	57·7	10 41	16 1·9	S. 18·9	+ ·4	2 28·5	57·3	10 37·3	17 1·9	S. 19·7	+ ·1
3 28·3	57	10 28·9	15 2·3	S. 17·1	− ·1	3 31·9	57·2	10 28·9	16 5·2	S. 15·6	+ ·4
4 27·7	57·2	10 25·2	14 4	S. 10·6	+ ·5	4 32·2	57·2	10 24·5	15 7·1	S. 11	+ ·2
5 27·4	56·9	10 31·1	13 1·9	S. 6·0	+ ·1	5 29·7	56·7	10 31·1	14 4·5	S. 5·3	+ ·1
6 31	57·1	10 54	12 7·6	N. 0·9	+ ·1	6 29·3	56·9	10 53·5	13 10	N. 0·9	+ ·3
7 33·8	56·9	11 25·9	12 7·6	N. 6·9	+ ·1	7 26·7	57·1	11 21	13 7·8	N. 7·4	+ ·4
8 30·5	56·9	11 45·2	13 7·2	N. 12·4	+ ·2	8 28·8	56·9	11 45	13 11·8	N. 12·1	+ ·2
9 28·8	57	11 50	14 9	N. 16·8	0	9 32·2	57·7	11 47·9	15 0	N. 17·6	+ ·3
10 31	57	11 42·6	15 10·2	N. 20·7	+ ·2	10 30·8	57·2	11 42·7	15 10	N. 20·2	+ ·4
11 32·7	57	11 28	16 7·6	N. 22·0	+ ·3	11 29·2	56·9	11 31·2	16 3	N. 22·2	0

Table XXVII. (Continued.)

July.

Upper Transits, P.M.

Moon's Transit.	Hor. Par.	Interval.	Height.	Moon's Declination.	Equation of Time.
h m	′	h m	ft. in.	°	m
35·3	56·9	11 13·3	17 4·8	N. 18·9	+ 5·3
1 34·5	57·2	10 55·5	17 8·6	N. 15·3	+ 5·2
2 31·3	57	10 46	17 5·2	N. 10·8	+ 5·3
3 31·4	56·7	10 37·3	16 7·4	N. 4·8	+ 5·3
4 32·1	56·9	10 32·5	15 4·1	S. 1·1	+ 5·1
5 28·6	57·1	10 35·0	13 11·6	S. 7·6	+ 5·3
6 28·8	56·8	10 50·6	13 3	S. 13·3	+ 5·4
7 25·6	57·2	11 13·3	12 9·5	S. 17·7	+ 5·3
8 25·1	57·2	11 36·9	13 3·1	S. 20·3	+ 5·2
9 28·6	58	11 43	14 5·6	S. 22·4	+ 5·3
10 34·5	57·7	11 38·3	15 0·4	S. 22·7	+ 5·4
11 37·8	58	11 24·4	16 3·1	S. 21·8	+ 5·3

Lower (Interpolated) Transits, A.M.

Moon's Transit.	Hor. Par.	Interval.	Height.	Moon's Declination.	Equation of Time.
h m	′	h m	h m	°	m
30·1	57	11 12·1	16 9·2	N. 19·9	+ 5·6
1 30·1	56·8	11 1	16 9·3	N. 16·2	+ 5·2
2 32·4	57	10 47·3	17 3·9	N. 10·3	+ 5·2
3 31·4	56·9	10 38·4	15 8·1	N. 4·6	+ 5·2
4 31·4	56·8	10 32·8	14 7·7	S. 1·5	+ 5·2
5 33	56·9	10 35·2	13 6·8	S. 7·6	+ 5·3
6 30·4	57·1	10 50·4	12 8·8	S. 12·6	+ 5·2
7 32·8	57	11 19·6	12 4·3	S. 17·6	+ 5·2
8 35·2	57·3	11 37·8	13 9·7	S. 21·3	+ 5·4
9 30·9	57·8	11 42·1	15 0·7	S. 22	+ 5·3
10 26·4	57·8	11 39·9	15 11·7	S. 23·3	+ 5·2
11 27·2	58	11 25·1	16 11·5	S. 22·4	+ 5·1

Upper Transits, A.M.

29·7	58	11 13·6	16 7·1	S. 19·7	+ 5·2
1 28	57·2	10 58·3	16 4·8	S. 16·5	+ 5·1
2 30·8	57·2	10 47·8	16 2·6	S. 10·9	+ 5·3
3 31·4	57·1	10 38·3	15 8·1	S. 5·0	+ 5·2
4 30·1	56·7	10 33·7	14 5·3	N. 1·7	+ 5·2
5 30·5	56·7	10 37·7	13 3	N. 6·7	+ 5·2
6 29·7	56·8	10 52·6	12 3·1	N. 12·4	+ 5·2
7 29·7	56·8	11 18·5	12 4	N. 17·3	+ 5·2
8 28·4	57	11 41·5	13 3·7	N. 19·5	+ 5·2
9 28·2	57·3	11 46·5	14 8	N. 22·1	+ 5·3
10 30·3	57·5	11 34	15 11·4	N. 22·6	+ 5·2
11 32·9	57·2	11 27·8	16 9	N. 22·1	+ 5·1

Lower (Interpolated) Transits, P.M.

32·2	57·7	11 10·5	17 5·8	S. 19·2	+ 5·3
1 33·7	57·2	10 59·1	17 10·8	S. 15·7	+ 5·2
2 29·2	57·2	10 46·5	17 5·5	S. 11·2	+ 5·1
3 31·8	57	10 37	16 11·1	S. 4·2	+ 5·4
4 31	57	10 33	15 9	N. 1·8	+ 5·4
5 26·8	56·7	10 36	14 5·8	N. 7·0	+ 5·3
6 29·2	56·7	10 51·1	13 2·3	N. 13·3	+ 5·3
7 32·7	57	11 16·7	13 1·8	N. 17	+ 5·2
8 33·6	56·9	11 41·1	13 4	N. 21·4	+ 5·3
9 33·1	57·3	11 45·5	14 5·7	N. 22·2	+ 5·4
10 31·2	57·2	11 41	15 3·3	N. 22·9	+ 5·2
11 31·7	57	11 27·6	16 1·5	N. 21·4	+ 5·3

August.

Upper Transits, P.M.

32	57·4	11 16·7	18 0·8	N. 11·2	+ 3·5
1 32·4	57·3	11 4·3	18 1·6	N. 4·2	+ 3·3
2 29·6	57·3	10 51·3	17 11	S. 1·0	+ 3·6
3 27·4	56·6	10 40·2	16 8·2	S. 7·1	+ 3·7
4 28·5	57	10 30	15 4·3	S. 13	+ 3·3
5 24·5	56·7	10 26·4	13 9·4	S. 17	+ 3·6
6 24·9	56·6	10 37·9	12 1·7	S. 20·4	+ 3·7
7 26·3	57·1	11 5·4	11 11·6	S. 22·5	+ 3·3
8 30·1	56·8	11 38	12 7	S. 23	+ 3·2
9 29·8	57·7	11 44·7	14 1·5	S. 21·7	+ 3·4
10 26·6	57·8	11 42	15 3·1	S. 19·8	+ 3·3
11 25·9	57·9	11 30·7	16 6·2	S. 15·7	+ 3·4

Lower (Interpolated) Transits, A.M.

30·5	57·4	11 17·5	17 2·6	N. 10·4	+ 3·4
1 24·7	57·6	11 6	17 5·7	N. 5·4	+ 3·4
2 26·8	57	10 52·5	16 11·7	S. 1·2	+ 3·5
3 31·9	57·1	10 39·3	16 2	S. 6·8	+ 4·0
4 33	56·8	10 31·3	14 7·4	S. 13	+ 3·8
5 35·3	56·8	10 28·7	13 3·3	S. 18·1	+ 3·4
6 34	56·9	10 42·2	12 1	S. 21·3	+ 3·5
7 30·9	57·3	11 7·7	12 2·1	S. 22·6	+ 3·5
8 30·2	57	11 37·5	13 2·8	S. 22·6	+ 3·5
9 33·8	57·5	11 45·8	14 9·4	S. 22·2	+ 3·7
10 37	57·6	11 40·8	16 7·6	S. 18·4	+ 3·2
11 34·9	58·1	11 29·1	17 10·1	S. 15	+ 3·3

Upper Transits, P.M.

31·3	57·7	11 17·2	17 0·8	S. 10·7	+ 3·3
1 30·3	57·7	11 3·9	17 4·3	S. 5·2	+ 3·7
2 27·7	57	10 52·7	16 7·8	N. 0·6	+ 3·7
3 30·3	57	10 38·1	15 10·4	N. 7·2	+ 3·5
4 26·9	56·8	10 30·7	14 7·5	N. 12·4	+ 4·0
5 29·5	56·4	10 28·4	12 11·5	N. 17·9	+ 3·4
6 29·9	56·5	10 40·7	11 8·3	N. 20·5	+ 3·5
7 26·5	56·5	11 8·9	11 7·2	N. 21·6	+ 3·7
8 27	56·9	11 35·2	12 10·9	N. 23·1	+ 3·7
9 27·4	57	11 47·6	14 6	N. 21·9	+ 3·3
10 29·6	57	11 42·9	15 11·8	N. 19·5	+ 3·3
11 30·7	57·7	11 30·3	17 3·7	N. 15·3	+ 3·5

Lower (Interpolated) Transits, P.M.

29·6	57·8	11 17	18 8	S. 10·3	+ 3·5
1 29·6	57	11 4·5	18 6·2	S. 5·4	+ 4·0
2 30·6	57·8	10 50	18 4·8	N. 1·5	+ 3·5
3 28·6	56·7	10 39·5	16 11	N. 6·7	+ 3·8
4 29·6	56·8	10 29·2	15 6·2	N. 13·4	+ 3·0
5 30·2	56·7	10 27·7	13 9·7	N. 16·8	+ 3·7
6 31	56·7	10 41·2	12 1·9	N. 20·8	+ 3·6
7 31·5	56·5	11 11·1	11 9·8	N. 22·3	+ 3·4
8 32·1	56·8	11 38·6	12 6·9	N. 23·1	+ 3·4
9 31	56·7	11 49·4	14 0·3	N. 21	+ 3·0
10 29·9	57·2	11 43·2	15 4·5	N. 19·2	+ 3·5
11 30·7	57·4	11 30·3	16 5·6	N. 16	+ 3·5

TABLE XXVII. (Continued.)

September.

Upper Transits, P.M.

Moon's Transit.	Hor. Par.	Interval.	Height.	Moon's Declination.	Equation of Time.
h m	′	h m	ft. in.	°	m
30	57·8	11 19·7	18 8·5	S. 0·8	−5·4
1 29·8	57·4	11 4·5	18 7·7	S. 6·2	−5·3
2 30·4	57·2	10 46·8	18 0·8	S. 12·5	−5·7
3 26·2	57·5	10 34·4	16 8·9	S. 17	−4·9
4 25·8	57·	10 22·2	15 0·9	S. 19·6	−4·6
5 23·8	57·1	10 15·3	13 5	S. 22·3	−5·1
6 26·7	56·7	10 27·3	11 7·8	S. 23·2	−5·4
7 28·4	57·2	11 1·6	11 4·8	S. 22.1	−5·3
8 27	57·1	11 38·1	12 7·9	S. 19·9	−5·3
9 30·3	57·6	11 48	14 6·7	S. 16·1	−5·3
10 31·5	57·2	11 46·8	16 0·4	S. 11	−5·8
11 32·2	57·9	11 33·1	17 4·6	S. 5·1	−5·8

Lower (Interpolated) Transits, A.M.

Moon's Transit.	Hor. Par.	Interval.	Height.	Moon's Declination.	Equation of Time.
h m	′	h m	ft. in.	°	m
30·6	57·4	11 21·4	17 10·6	0	− 5·2
1 34·4	57·7	11 1·3	18 1·3	S. 7·3	− 5·5
2 31·9	57·2	10 49·1	17 4·7	S. 11·5	− 4·6
3 31·7	56·8	10 35	16 5·2	S. 17·2	− 4·7
4 33·2	57	10 20·5	14 9·3	S. 20·4	− 5·2
5 35·8	56·8	10 16·6	13 2·3	S. 22·4	− 5
6 34·5	57	10 30·6	11 6	S. 22·8	− 4·8
7 34·2	57·2	11 3·3	12 4·6	S. 22·3	− 5
8 33·9	57	11 40·7	13 3·5	S. 20·4	− 5·3
9 36·1	57·4	11 49·7	15 7	S. 15·5	− 5·5
10 30·9	57·6	11 45·6	17 2·2	S. 11·1	− 5·5
11 30·5	57·3	11 35·3	18 3·4	S. 6·2	− 5·1

Upper Transits, A.M.

32·8	57·1	11 20·7	17 9·4	N. 0·8	− 5·2
1 29·1	57·4	11 6·2	17 10·2	N. 6·3	− 5·5
2 27·4	57·7	10 50·8	17 5·9	N. 12	− 4·9
3 29·5	56·8	10 34·5	16 1·4	N. 16·9	− 4·9
4 29	56·8	10 21·4	14 8·7	N. 20·1	− 5
5 25·6	56·4	10 17·3	13 0	N. 21·5	− 5
6 31·8	56·6	10 31·4	11 7·1	N. 22·2	− 4·7
7 27·6	56·9	11 5·5	11 7·2	N. 22	− 5·1
8 26·7	56·7	11 39·5	12 10·5	N. 20·5	− 5·1
9 29·1	56·7	11 51·8	14 10·8	N. 15·5	− 5·6
10 28·3	57·5	11 45·2	16 9	N. 11·3	− 5·8
11 30	56·9	11 37·3	17 9·1	N. 5·9	− 5·4

Lower (Interpolated) Transits, P.M.

30·9	57·7	11 18·5	19 1·8	N. 1·6	− 5·8
1 31·2	57·3	11 4·2	19 0·8	N. 6·9	− 5·1
2 31·9	57·6	10 48·2	18 6·5	N. 12·9	− 5·6
3 33·2	57	10 32·4	16 9·5	N. 16·7	− 5·2
4 33	56·9	10 21·1	15 2·7	N. 20·2	− 5·6
5 30·3	56·7	10 18·1	13 3·9	N. 22·1	− 5·3
6 31	56·4	10 29	11 5·3	N. 23·3	− 5·2
7 32·4	56·6	11 9·2	11 4·8	N. 22·1	− 5·3
8 32·4	56·7	11 42·2	12 9·8	N. 19·8	− 5·4
9 30·1	56·9	11 52·2	14 2·4	N. 16·8	− 5·1
10 31·5	56·9	11 47·1	15 10·5	N. 11·1	− 6
11 31·6	57·6	11 35·2	17 3·5	N. 5	− 5·6

October.

Upper Transits, P.M.

28·6	57	11 20·6	18 5·8	S. 11·5	−14·4
1 26·5	57·7	11 2	18 7·7	S. 16·5	−14·2
2 24	57·4	10 43·6	17 10·8	S. 19·8	−14
3 22·5	57·6	10 28	16 8·5	S. 22	−14·1
4 26·1	57·2	10 16	14 10·8	S. 23	−14
5 30·3	57·4	10 11·6	12 10·3	S. 22·2	−14
6 27·7	57	10 27	11 5·5	S. 20·3	−14·3
7 27·3	57·3	11 6·6	11 5·7	S. 16·9	−13·9
8 31·3	57	11 45·7	12 11·8	S. 12·8	−14·1
9 30·5	57·1	11 54	14 7	S. 5·9	−14·7
10 28·5	57·4	11 47·9	16 1·8	S. 0·5	−14
11 27·8	57	11 37·3	17 3·7	N. 5·0	−14·1

Lower (Interpolated) Transits, A.M.

30·6	57·4	11 18·3	18 2·7	S. 11·7	−14
1 27·6	57·5	11 3·2	18 3·1	S. 15·8	−13·8
2 30·7	57·3	10 41·1	17 6·4	S. 20·5	−14
3 33·1	57·7	10 25·8	16 8·6	S. 22·4	−13·7
4 34·3	57·4	10 14·7	15 0·6	S. 22·7	−13·8
5 33·3	57·4	10 11·6	13 7·8	S. 21·9	−13·7
6 34·2	56·5	10 30·7	11 11·6	S. 20·7	−13·8
7 36·4	57·2	11 11·5	12 6·2	S. 16·1	−14·2
8 34·5	57	11 44·9	13 10·6	S. 12·8	−13·9
9 33·3	57	11 54·4	15 8·2	S. 5·9	−14·1
10 29·5	57	11 49·6	17 3·4	S. 0·5	−14·3
11 30·6	57·7	11 32·9	18 7·1	N. 5·2	−14·1

Upper Transits, A.M.

29·7	57·7	11 18·4	18 5·4	N. 11·1	−14·2
1 30·5	56·9	11 2·3	17 8·9	N. 16·8	−14
2 30·5	57·3	10 43	17 4·2	N. 20·5	−14·2
3 27·8	57	10 27·3	16 0·9	N. 21·5	−13·9
4 26·8	57·4	10 14·4	14 10·8	N. 23·2	−13·8
5 27·8	56·8	10 11·9	13 2·1	N. 22·2	−13·6
6 26	56·6	10 27·3	11 11·7	N. 20·6	−13·9
7 29·5	56·7	11 12·4	11 10·7	N. 16·9	−14·1
8 30·3	57	11 44·9	13 11·3	N. 12·6	−13·7
9 28·8	57	11 55·3	15 3·3	N. 7·1	−14
10 29·2	57·2	11 49·1	17 1·8	0	−12·2
11 29	57·3	11 35·4	18 2·2	S. 5·5	−13·8

Lower (Interpolated) Transits, P.M.

28·9	56·9	11 19·9	18 9·5	N. 11·2	−14
1 31·8	57·3	11 0·2	18 7·2	N. 16·5	−14·4
2 32·7	57·2	10 41·3	17 9·3	N. 20·6	−14·1
3 32·4	57·2	10 25·8	16 5·9	N. 21·9	−14
4 35·2	57	10 13·1	14 1·6	N. 23·4	−14·1
5 36·5	56·5	10 13·5	12 9·1	N. 21·6	−14
6 36·3	56·8	10 32·5	11 3·3	N. 20·1	−14·2
7 31·8	56·8	11 14	11 6	N. 17·1	−14
8 30·4	56·6	11 47·9	13 0·4	N. 12·6	−14
9 31·4	57·2	11 54·9	15 1·4	N. 6·3	−14·2
10 32·3	57·1	11 48·9	16 7·1	0	−14
11 32	57·3	11 35	17 8	S. 6·4	−14·4

Table XXVII. (Continued.)

November.

Upper Transits, P.M.

Moon's Transit.	Hor. Par.	Interval.	Height.	Moon's Declination.	Equation of Time.
h m	′	h m	ft. in.	°	m
31·5	56·9	11 14·9	17 11	S. 19·9	−14·5
1 28·9	57·2	11 0·3	17 9·2	S. 21·9	−14·5
2 28·3	57·3	10 38·2	16 9·1	S. 23	−14·1
3 29·8	57·3	10 22·8	15 8	S. 22·1	−14
4 28·4	57·3	10 14·6	14 7	S. 20·3	−14·4
5 27·2	57·2	10 16·8	12 10·3	S. 16·6	−14·2
6 29·4	56·9	10 41	11 9·1	S. 12·1	−14·2
7 29·4	57	11 16·3	11 11·8	S. 6·4	−14·3
8 27·2	57·4	11 46	13 6·1	S. 1·2	−14·5
9 29·7	57·1	11 53·3	15 0	N. 5·6	−14·3
10 33·2	57·7	11 44·3	16 4·5	N. 10·7	−14·3
11 31·5	57·1	11 33·6	16 11·1	N. 17·2	−14·3

Lower (Interpolated) Transits, A.M.

Moon's Transit.	Hor. Par.	Interval.	Height.	Moon's Declination.	Equation of Time.
h m	′	h m	ft. in.	°	m
33·3	57·6	11 12·7	18 0·5	S. 20·4	−14·5
1 31·7	57·5	10 55·5	18 0·5	S. 22·2	−14·2
2 29·1	57·5	10 38·1	16 10·9	S. 23	−14·4
3 28·2	57·2	10 23·7	16 8·2	S. 22·6	−14·2
4 31·1	57·1	10 15·1	15 2·4	S. 20·4	−14·5
5 36·9	57·1	10 18·3	13 10·8	S. 16·2	−14·3
6 32·1	57·2	10 40·4	13 4·2	S. 12·5	−14·5
7 29·3	57	11 19·6	13 0	S. 6·5	−14
8 30·1	56·9	11 46·2	14 3	S. 0·7	−14·3
9 27·3	57·4	11 51·9	15 9·7	N. 4·6	−14·6
10 26·7	57·2	11 47·1	16 9·3	N. 10·6	−14·5
11 29·9	57·2	11 32·6	17 6·5	N. 16·5	−14·4

Upper Transits, A.M.

35·6	57·1	11 14·7	17 8·8	N. 19·8	−14·4
1 33·1	57·3	10 56·8	17 5	N. 22·0	−14·3
2 30·9	57·1	10 40·5	16 11	N. 23·1	−14·4
3 26·8	56·7	10 26	16 1·4	N. 22·1	−14·3
4 30	56·5	10 14·6	14 9·3	N. 20·6	−14·5
5 31·4	56·9	10 16·9	13 5·4	N. 16·1	−14·2
6 28·3	51·1	10 37	12 10	N. 12·5	−14·4
7 29·3	56·6	11 21·7	12 10·3	N. 7·3	−14·5
8 28·4	57	11 47·7	14 1·2	N. 0·9	−14·6
9 28·6	57·5	11 54·1	16 0·7	S. 4·6	−14·7
10 30	57	11 47·9	16 10·4	S. 11·4	−14·3
11 32·7	57·8	11 30·5	17 11·9	S. 16·4	−14·6

Lower (Interpolated) Transits, P.M.

30·2	57·4	11 15·9	18 2·2	N. 19·7	−14·4
1 32·1	56·8	10 57·4	17 3·1	N. 22·7	−14·2
2 33·5	57·1	10 39·1	16 10·9	N. 22·6	−14·4
3 34·2	56·9	10 24·4	15 7·3	N. 21·9	−14·1
4 35·1	56·8	10 14·1	14 0·5	N. 19·8	−14
5 28·1	56·8	10 17·5	12 11·7	N. 16·9	−14·2
6 31·1	56·6	10 43·4	11 9·4	N. 12·6	−14·4
7 33	57·2	11 24·7	12 7·6	N. 5·6	−14·1
8 31·5	56·9	11 50·3	13 10·8	N. 0·3	−14·5
9 31·7	57·4	11 53	15 8	S. 6·0	−14·4
10 27·6	57·5	11 46·9	16 9·1	S. 11·0	−14·4
11 29·5	57·2	11 33·6	17 9	S. 15·9	−14·5

December.

Upper Transits, P.M.

34·1	57·7	11 7·8	17 3·2	S. 23	− 3·3
1 31·1	57·4	10 53·7	17 2	S. 22·2	− 3·6
2 29·8	57·1	10 40·2	16 7·8	S. 19·2	− 3·1
3 30·8	57·1	10 29·5	15 6·4	S. 15·4	− 3·2
4 33·1	57·1	10 22·8	14 5	S. 11·2	− 3·5
5 32·9	56·8	10 31·5	13 1·2	S. 4·4	− 3
6 32·7	56·9	10 55·2	13 1·1	N. 1·0	− 2·3
7 30·7	57·3	11 23·3	13 8·6	N. 6·7	− 3·4
8 29·2	57	11 46·9	13 9·5	N. 12·3	− 3·9
9 29·2	57·4	11 48·7	15 4·4	N. 16·8	− 3·4
10 25·5	57·5	11 43·5	16 0·5	N. 19·7	− 3·2
11 28·9	57·3	11 28·3	17 0·6	N. 21·7	− 3·4

Lower (Interpolated) Transits, A.M.

26·8	57·3	11 13·5	17 8	S. 23·2	− 4
1 31·3	57·3	10 53·8	17 9	S. 21·8	− 3·3
2 32·6	57·4	10 38·8	17 5·8	S. 19·2	− 3·7
3 27·5	57·6	10 27·9	16 11·6	S. 16	− 3·1
4 27·9	56·7	10 23·1	15 9·6	S. 11·2	− 3·3
5 30·8	57·1	10 29·5	14 11·3	S. 5·6	− 3·6
6 31·8	57	10 52·1	13 8·8	N. 1·1	− 3·8
7 30	57	11 24	13 11·1	N. 6·6	− 3·4
8 29·7	57·2	11 44·5	14 6·6	N. 12·3	− 4
9 27·9	57·7	11 46·5	15 10·2	N. 15·9	− 4
10 32·6	57	11 41·7	16 4·4	N. 20·9	− 3·6
11 35·2	57·2	11 28·5	16 11·5	N. 22	− 3·5

Upper Transits, A.M.

32	57	11 11·3	17 2·2	N. 23·6	− 4·1
1 33·9	57·1	10 53·8	17 4·6	N. 21·9	− 3·2
2 31	56·9	10 39·8	16 9·6	N. 19·1	− 3·4
3 29	56·9	10 27·7	16 1·7	N. 16	− 3·4
4 31·6	56·6	10 23·8	14 11·6	N. 11	− 2·9
5 31·2	56·7	10 30	14 2·3	N. 6	− 4·2
6 32·4	57	10 53·8	12 11	S. 1·5	− 2·8
7 32·4	56·9	11 26·5	13 1·1	S. 6·8	− 4·1
8 29·8	57·5	11 45·4	14 0·2	S. 12·3	− 4
9 24·6	57·5	11 49·1	15 2·7	S. 16·1	− 4
10 27·2	57·6	11 43·2	16 5	S. 19·8	− 3·9
11 34·2	57·7	11 26·9	17 0	S. 22·7	− 3·4

Lower (Interpolated) Transits, P.M.

33·2	57	11 13·3	17 1·6	N. 22·4	− 3·5
1 29·2	57·2	10 57·5	16 11·4	N. 21·9	− 3·4
2 25·3	57	10 42·2	16 3·2	N. 19·6	− 3·5
3 29·2	56·4	10 29·5	15 0	N. 16·2	− 3·4
4 32·2	57	10 26·3	14 4·3	N. 10·2	− 2·8
5 28·6	56·6	10 31·9	12 9·2	N. 5·1	− 2·5
6 29·3	56·8	10 52·8	12 6·7	S. 0·3	− 3·9
7 31·6	57·3	11 24·2	13 0·5	S. 6·7	− 3·5
8 29·7	57·2	11 44·9	13 11·1	S. 12·5	− 3·2
9 31·9	57·4	11 48·9	15 4·4	S. 17·5	− 3·2
10 33·8	58·1	11 41·4	16 2·9	S. 20·2	− 3·8
11 26·6	57	11 29·7	17 4·8	S. 21·5	− 3·5

TABLE XXVIII.

Upper Transits, P.M.

Moon's Transit. P.M.	January.		February.		March.		April.		May.		June.	
	Diurnal inequality.	Moon's Declination.	Diurnal inequality.	Moon's Declination.	Diurnal inequality.	Moon's Declination.	Diurnal inequality.	Moon's Declination.	Diurnal inequality.	Moon's Declination.	Diurnal inequality.	Moon's Declination.
h m	feet.	°	feet.	°	feet.	°	feet.	°	feet.	°	feet.	°
30	−·41	S. 18·6	−·52	S. 10·1	−·75	N. 1·5	−·18	N. 13·1	−·33	N. 20·9	+·20	N. 22·8
1 30	−·14	S. 15	−·10	S. 2·8	−·82	N. 8·5	−·50	N. 16·8	−·11	N. 22·2	+·16	N. 22·6
2 30	−·06	S. 10	−·73	N. 2·7	−·38	N. 12·6	−·20	N. 20·6	+·05	N. 22·7	−·04	N. 19·4
3 30	−·22	S. 3·6	−·79	N. 8·6	−·41	N. 17·4	−·21	N. 22·7	+·09	N. 21·9	+·35	N. 15·4
4 30	−·56	N. 2·8	−·32	N. 13·5	−·39	N. 21·2	−·07	N. 22·7	+·19	N. 20·4	+·38	N. 11·1
5 30	−·60	N. 7·6	−·63	N. 18·2	−·43	N. 22·6	−·17	N. 22·1	+·21	N. 16	+·16	N. 5·1
6 30	−·51	N. 14	−·12	N. 21·3	−·11	N. 22·5	+·01	N. 20·2	+·32	N. 11·6	+·26	S. 0·7
7 30	−·62	N. 17·7	+·06	N. 22·5	−·15	N. 22·6	+·08	N. 15·6	+·28	N. 5·5	+·29	S. 7·5
8 30	−·17	N. 21·1	+·33	N. 22·8	0	N. 19·9	+·02	N. 10·8	+·20	S. 0·5	+·02	S. 11·3
9 30	−·07	N. 22·8	+·41	N. 21	+·07	N. 14·8	+·17	N. 5·7	+·10	S. 6·9	−·10	S. 18·0
10 30	+·03	N. 22·5	+·23	N. 19·1	+·27	N. 10·3	+·06	S. 0·7	+·16	S. 11·8	−·12	S. 20·1
11 30	+·30	N. 21·6	+·22	N. 14·5	+·07	N. 5	−·19	S. 7·0	−·06	S. 17·5	−·14	S. 22·4

	July.		August.		September.		October.		November.		December.	
30	+·25	N. 18·9	+·37	N. 11·2	+·40	0	+·02	S. 11·7	−·06	S. 19·9	0	S. 23
1 30	+·55	N. 15·3	+·17	N. 4·2	+·25	S. 7·3	+·34	S. 15·8	+·03	S. 21·9	−·12	S. 22·2
2 30	+·60	N. 10·8	+·54	S. 1·0	+·18	S. 11·5	+·19	S. 20·5	−·27	S. 23	−·14	S. 19·2
3 30	+·35	N. 4·8	+·17	S. 7·1	+·14	S. 17·2	+·05	S. 22·4	−·35	S. 22·1	−·35	S. 15·4
4 30	+·42	S. 1·1	+·29	S. 13	+·01	S. 20·4	+·01	S. 22·7	+·05	S. 20·3	−·48	S. 11·2
5 30	+·11	S. 7·6	+·17	S. 17	+·05	S. 22·4	−·48	S. 21·9	−·54	S. 16·6	−·69	S. 4·4
6 30	+·37	S. 13·3	+·08	S. 20·4	−·03	S. 22·8	−·19	S. 20·7	−·53	S. 12·1	+·02	N. 1·0
7 30	+·06	S. 17·7	+·16	S. 22·5	−·37	S. 22·3	−·44	S. 16·1	−·64	S. 6·4	−·49	N. 6·7
8 30	−·07	S. 20·3	−·24	S. 23	−·16	S. 20·4	−·42	S. 12·8	−·38	S. 1·2	−·26	N. 12·3
9 30	−·14	S. 22·4	−·22	S. 21·7	−·22	S. 15·5	−·55	S. 5·9	−·61	N. 5·6	+·03	N. 16·8
10 30	−·65	S. 22·7	−·31	S. 19·8	−·38	S. 11·1	−·45	S. 0·5	−·39	N. 10·7	−·28	N. 19·7
11 30	−·41	S. 21·8	−·41	S. 15·7	−·36	S. 6·2	+·43	N. 5·2	−·60	N. 17·2	−·08	N. 21·7

Upper Transits, A.M.

A.M.	January.		February.		March.		April.		May.		June.	
30	+·03	N. 18·3	+·28	N. 9·8	+·23	S. 1·9	−·08	S. 11·5	+·09	S. 19·8	−·67	S. 23·8
1 30	−·09	N. 14·9	+·71	N. 3·6	+·44	S. 6·8	+·22	S. 17·4	+·06	S. 22	−·42	S. 22
2 30	+·25	N. 10·4	+·62	S. 2·4	+·28	S. 13·1	+·34	S. 20·7	−·16	S. 22·7	−·59	S. 18·9
3 30	+·39	N. 4·7	+·07	S. 7·4	+·44	S. 18·2	+·10	S. 22	−·28	S. 22·1	−·65	S. 17·1
4 30	+·36	S. 4	+·13	S. 13·8	+·15	S. 20·8	−·12	S. 23·4	−·19	S. 19·7	−·39	S. 10·6
5 30	+·48	S. 8·1	+·21	S. 18·7	−·06	S. 22·3	−·18	S. 22·1	−·51	S. 16·6	−·63	S. 6·0
6 30	+·26	S. 13·7	−·11	S. 20·8	+·03	S. 23·2	−·14	S. 19·9	−·78	S. 12·3	−·44	N. 0·9
7 30	+·16	S. 18·3	−·35	S. 22·9	−·17	S. 21·6	−·38	S. 16	−·50	S. 6·0	−·47	N. 6·9
8 30	+·06	S. 21	−·06	S. 23	−·27	S. 19·4	−·50	S. 12	−·48	N. 0·5	−·27	N. 12·4
9 30	−·11	S. 22·3	−·49	S. 21·7	−·20	S. 15·6	−·46	S. 6	−·52	N. 7	−·16	N. 16·8
10 30	−·46	S. 22·5	−·15	S. 18·2	−·64	S. 11·2	−·20	N. 0·6	−·13	N. 11·8	−·08	N. 20·7
11 30	−·47	S. 22	−·48	S. 15	−·31	S. 5·4	−·53	N. 6·3	−·16	N. 16·5	+·03	N. 22·0

	July.		August.		September.		October.		November.		December.	
30	−·49	S. 19·7	−·60	S. 10·7	−·51	N. 1·6	−·07	N. 11·2	−·20	N. 19·8	− ·08	N. 23·6
1 30	−·75	S. 16·5	−·60	S. 5·2	−·54	N. 6·9	−·54	N. 16·5	−·29	N. 22	+ ·08	N. 21·9
2 30	−·63	S. 10·9	−·75	N. 0·6	−·44	N. 12·9	−·26	N. 20·6	−·07	N. 23·1	+ ·01	N. 19·1
3 30	−·61	S. 5·0	−·59	N. 7·2	−·39	N. 16·7	−·44	N. 21·9	+·06	N. 22·1	+ ·23	N. 16
4 30	−·53	N. 1·7	−·48	N. 12·4	−·24	N. 20·2	+·07	N. 23·4	+·27	N. 20·6	+ ·04	N. 11
5 30	−·57	N. 6·7	−·50	N. 17·9	−·32	N. 22·1	+·47	N. 21·6	+·12	N. 16·1	+ ·38	N. 6
6 30	−·51	N. 12·4	−·31	N. 20·5	−·03	N. 23·3	+·23	N. 20·1	+·55	N. 12·5	− ·13	S. 1·5
7 30	−·39	N. 17·3	−·17	N. 21·6	−·12	N. 22·1	−·06	N. 17·1	+·24	N. 7·3	−1·14	S. 6·8
8 30	−·05	N. 19·5	+·16	N. 23·1	+·04	N. 19·8	+·57	N. 12·6	+·19	N. 0·9	− ·05	S. 12·3
9 30	+·04	N. 22·1	+·22	N. 21·9	+·14	N. 16·8	+·18	N. 6·3	+·48	S. 4·6	− ·04	S. 16·1
10 30	+·33	N. 22·6	+·14	N. 19·5	+·41	N. 11·1	+·33	S. 0	+·13	S. 11·4	+ ·05	S. 19·8
11 30	+·21	N. 22·1	+·31	N. 15·3	+·07	N. 5·0	+·27	S. 6·4	+·44	S. 16·4	− ·17	S. 22·7

Table XXVIII. (Continued.)

Lower (Interpolated) Transits, A.M.

Moon's Transit. A.M.	January.		February.		March.		April.		May.		June.	
	Diurnal inequality.	Moon's Decli-nation.	Diurnal inequality.	Moon's Decli-nation.	Diurnal inequality.	Moon's Decli-nation.	Diurnal inequality.	Moon's Decli-nation.	Diurnal inequality.	Moon's Decli-nation.	Diurnal inequality.	Moon's Decli-nation.
	feet.	°	feet.	°	feet.	°	feet.	°	feet.	°	feet.	°
30	+·61	S. 19·1	+·86	S. 10·1	−·03	N. 1·3	+·28	N. 12·1	+·01	N. 20·4	−·33	N. 23
1 30	+·50	S. 14·6	+·83	S. 3·5	+·59	N. 7·1	+·24	N. 17·1	−·30	N. 21·9	−·45	N. 21·7
2 30	+·93	S. 9·9	+·56	N. 2·1	−·32	N. 13·0	−·08	N. 20·3	−·37	N. 23·0	−·68	N. 19·4
3 30	+·78	S. 3·6	+·52	N. 8·5	+·24	N. 18·2	−·05	N. 22·0	−·33	N. 22·1	−·54	N. 16·3
4 30	+·65	N. 2·0	+·16	N 13·6	+·12	N. 20·5	−·29	N. 22·9	−·33	N. 19·6	−·59	N. 11·8
5 30	+·82	N. 7·1	+·43	N. 18·8	+·08	N. 22·2	−·27	N. 22·1	−·45	N. 16·0	−·34	N. 4·7
6 30	+·28	N. 13·7	+·42	N. 21·1	−·32	N. 22·4	−·24	N. 18·9	−·26	N. 12·6	−·49	N. 0·7
7 30	+·48	N. 18·2	+·07	N. 21·9	−·15	N. 21·8	−·11	N. 16·2	+·08	N. 5·5	−·14	S. 7·1
8 30	+·17	N. 20·3	−·24	N. 23·4	−·13	N. 19·5	+·03	N. 12·0	+·06	N. 0·5	+·06	S. 12·0
9 30	−·03	N. 22·5	−·17	N. 21·6	−·28	N. 15·0	+·05	N. 5·1	+·15	S. 7·3	+·20	S. 16·6
10 30	−·08	N. 22·3	−·06	N. 18·7	−·54	N. 10·0	−·07	S. 0·4	+·17	S. 11·9	+·24	S. 20·0
11 30	−·07	N. 22·0	−·02	N. 15·0	−·09	N. 4·7	+·18	S. 7·5	+·35	S. 16·4	+·33	S. 22·3
	July.		August.		September.		October.		November.		December.	
30	−·31	N. 19·9	−·47	N. 10·4	−·44	S. 0·8	−·30	S. 11·5	+·07	S. 20·4	+·43	S. 23·2
1 30	−·36	N. 16·2	−·52	N. 6·4	−·31	S. 6·2	−·02	S. 16·5	+·28	S. 22·2	+·47	S. 21·8
2 30	+·47	N. 10·3	−·43	S. 1·2	−·48	S. 12·5	−·08	S. 19·8	−·10	S. 23	+·70	S. 19·2
3 30	−·59	N. 4·6	−·26	S. 6·8	−·12	S. 17	+·31	S. 22	+·64	S. 22·6	+1·04	S. 16
4 30	−·30	S. 1·5	−·34	S. 13	−·09	S. 19·6	+·40	S. 23	+·72	S. 20·4	+·82	S. 11·2
5 30	−·22	S. 7·6	−·08	S. 18·1	+·16	S. 22·3	+·57	S. 22·2	+·68	S. 16·2	+1·11	S. 5·6
6 30	−·14	S. 12·6	+·12	S. 21·3	−·04	S. 23·2	+·35	S. 20·3	+1·07	S. 12·5	+·69	N. 1·1
7 30	−·37	S. 17·6	+·36	S. 22·6	+·72	S. 22·1	+·46	S. 16·9	+·38	S. 6·5	+·71	N. 6·6
8 30	+·30	S. 21·3	+·41	S. 22·6	+·32	S. 19·9	+·40	S. 12·8	+·30	S. 0·7	+·48	N. 12·3
9 30	+·40	S. 22·0	+·33	S. 22·2	+·60	S. 16·1	+·46	S. 5·9	+·24	N. 4·6	+·53	N. 15·9
10 30	+·43	S. 23·3	+·62	S. 18·4	+·78	S. 11	+·46	S. 0·5	+·09	N. 10·6	−·04	N. 20·9
11 30	+·41	S. 22·4	+·81	S. 15	+·58	S. 5·1	+·67	N. 5	+·04	N. 16·5	−·23	N. 22

Lower (Interpolated) Transits, P.M.

P.M.	January.		February.		March.		April.		May.		June.	
30	−·33	N. 18·1	−·31	N. 9·3	−·18	S. 1·3	+·21	S. 13·0	+·34	S. 20·3	+·25	S. 22·9
1 30	−·41	N. 15·1	+·19	N. 3·9	+·01	S. 8·0	+·20	S. 16·5	+·46	S. 22·3	+·45	S. 22·5
2 30	−·40	N. 9·4	−·28	S. 2·5	−·03	S. 12·7	+·21	S. 20·7	+·46	S. 22·6	+·60	S. 19·7
3 30	−·18	N. 4·8	−·67	S. 7·2	0	S. 17·6	+·25	S. 22·6	+·59	S. 22·0	+·36	S. 15·6
4 30	−·52	S. 3·3	−·24	S. 13·9	+·09	S. 21·1	+·49	S. 22·6	+·46	S. 20·3	+·90	S. 11·0
5 30	−·50	S. 8·7	−·02	S. 18·2	+·09	S. 22·2	+·44	S. 22·2	+·54	S. 15·5	+·62	S. 5·3
6 30	+·31	S. 14·0	−·32	S. 21·2	+·45	S. 22·9	+·44	S. 20·5	+·68	S. 11·6	+·76	N. 0·9
7 30	+·37	S. 18·0	+·02	S. 22·3	+·57	S. 21·7	+·27	S. 16·3	+·49	S. 5·5	+·56	N. 7·4
8 30	−·03	S. 20·5	+·33	S. 22·8	+·22	S. 20·1	+·27	S. 10·8	+·18	N. 0·9	+·14	N. 12·1
9 30	+·13	S. 22·8	+·40	S. 22·0	+·17	S. 14·8	+·26	S. 5·1	+·23	N. 6·3	+·03	N. 17·6
10 30	+·38	S. 22·8	+·15	S. 19·5	+·08	S. 9·8	+·15	N. 0·8	−·01	N. 11·9	−·09	N. 20·2
11 30	+·69	S. 21·6	−·09	S. 14·1	+·23	S. 5·0	+·15	N. 7·0	−·13	N. 17·0	−·31	N. 22·2
	July.		August.		September.		October.		November.		December.	
30	+·37	S. 19·2	+·98	S. 10·3	+·84	N. 0·8	+·27	N. 11·1	+·23	N. 19·7	−·11	N. 22·4
1 30	+·74	S. 15·7	+·56	S. 5·4	+·69	N. 6·3	+·32	N. 16·8	−·46	N. 22·7	−·35	N. 21·9
2 30	+·61	S. 11·2	+1·04	N. 1·5	+·68	N. 12	+·19	N. 20·5	−·05	N. 22·6	−·60	N. 19·6
3 30	+·65	S. 4·2	+·47	N. 6·7	+·38	N. 16·9	+·08	N. 21·5	−·31	N. 21·9	−·90	N. 16·2
4 30	+·81	N. 1·8	+·50	N. 13·4	+·37	N. 20·1	−·47	N. 23·2	−·36	N. 19·8	−·57	N. 10·2
5 30	+·58	N. 7·0	+·35	N. 16·8	+·13	N. 21·5	−·24	N. 22·2	−·40	N. 16·9	−1·07	N. 5·1
6 30	+·31	N. 13·3	+·17	N. 20·8	−·18	N. 22·2	−·36	N. 20·6	−·50	N. 12·6	−·50	S. 0·3
7 30	+·42	N. 17	+·02	N. 22·3	−·37	N. 22	−·46	N. 16·9	−·04	N. 5·6	−·18	S. 6·7
8 30	−·08	N. 21·4	−·28	N. 23·1	−·10	N. 20·5	−·33	N. 12·6	−·09	N. 0·3	−·15	S. 12·5
9 30	−·22	N. 22·2	−·34	N. 21	−·58	N. 15·5	−·05	N. 7·1	+·01	S. 6·0	−·02	S. 17·5
10 30	−·35	N. 22·9	−·47	N. 19·2	−·54	N. 11·3	−·29	0	−·95	S. 11·0	−·18	S. 20·2
11 30	−·43	N. 21·4	−·54	N. 16	−·56	N. 5·9	−·29	S. 5·5	+·26	S. 15·9	+·23	S. 21·5

Table XXIX.

Conjectural, formed by interpolation and by arbitrary alterations from Table XXVIII.

Moon's Transit.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
h	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
P.M. 0	−·35	−·50	−·45	−·20	·00	+·22	+·35	+·50	+·45	+·20	·00	−·22
1	−·40	−·53	−·42	−·22	+·03	+·28	+·40	+·53	+·42	+·22	−·03	−·28
2	−·47	−·50	−·35	−·17	+·10	+·36	+·47	+·50	+·35	+·17	−·10	−·36
3	−·50	−·45	−·30	−·02	+·20	+·46	+·50	+·45	+·30	+·02	−·20	−·46
4	−·51	−·38	−·20	+·09	+·30	+·51	+·51	+·38	+·20	−·09	−·30	−·51
5	−·50	−·30	−·12	+·16	+·40	+·51	+·50	+·30	+·12	−·16	−·40	−·51
6	−·40	−·18	−·05	+·23	+·46	+·50	+·40	+·18	+·05	−·23	−·46	−·50
7	−·28	−·07	+·05	+·15	+·30	+·32	+·28	+·07	−·05	−·15	−·30	−·32
8	−·10	+·07	+·16	+·12	+·12	+·13	+·10	−·07	−·16	−·12	−·12	−·13
9	+·10	+·12	+·22	+·15	+·08	·00	−·10	−·12	−·22	−·15	−·08	·00
10	+·24	+·19	+·28	+·18	+·04	−·09	−·24	−·19	−·28	−·18	−·04	+·09
11	+·33	+·21	+·36	+·19	+·02	−·15	−·33	−·21	−·36	−·19	−·02	+·15
A.M. 0	+·35	+·50	+·45	+·20	·00	−·22	−·35	−·50	−·45	−·20	·00	+·22
1	+·40	+·53	+·42	+·22	−·03	−·28	−·40	−·53	−·42	−·22	+·03	+·28
2	+·47	+·50	+·35	+·17	−·10	−·36	−·47	−·50	−·35	−·17	+·10	+·36
3	+·50	+·45	+·30	+·02	−·20	−·46	−·50	−·45	−·30	−·02	+·20	+·46
4	+·51	+·38	+·20	−·09	−·30	−·51	−·51	−·38	−·20	+·09	+·30	+·51
5	+·50	+·30	+·12	−·16	−·40	−·51	−·50	−·30	−·12	+·16	+·40	+·51
6	+·40	+·18	+·05	−·23	−·46	−·50	−·40	−·18	−·05	+·23	+·46	+·50
7	+·28	+·07	−·05	−·15	−·30	−·32	−·28	−·07	+·05	+·15	+·30	+·32
8	+·10	−·07	−·16	−·12	−·12	−·13	−·10	+·07	+·16	+·12	+·12	+·13
9	−·10	−·12	−·22	−·15	−·08	·00	+·10	+·12	+·22	+·15	+·08	·00
10	−·24	−·19	−·28	−·18	−·04	+·09	+·24	+·19	+·28	+·18	+·04	−·09
11	−·33	−·21	−·36	−·19	−·02	+·15	+·33	+·21	+·36	+·19	+·02	−·15

IX. *Geometrical Investigations concerning the Phenomena of Terrestrial Magnetism. Second Series:—On the Number of Points at which a magnetic needle can take a position vertical to the Earth's surface. By* THOMAS STEPHENS DAVIES, *Esq., F.R.S. L. & E. F.R.A.S. Royal Military Academy, Woolwich.*

Received January 28,—Read February 4, 1836.

THE plan and objects of this series of papers have been so far explained already as to render it superfluous to enumerate them here.

At the close of my former paper* I have given the rectangular equation of the curve of verticity,—or that in any point of which a magnetic needle being placed, its line of direction would pass through the centre of the earth, and consequently be vertical to the horizon at the point where it cut the surface: but as the form and character of the curve could not be directly obtained from that equation, nor from any other into which it could be transformed; and as, moreover, the process by which they could be obtained required considerable preliminary investigations, I preferred to leave it in that state rather than give the partial and incomplete solution, which, in the midst of the deep domestic affliction that I was involved in, I must then have done. It is here, however, by pursuing a different course fully determined, as far, at least, as it is subservient to our physical problem: and I did not feel myself at liberty to insert in a paper on that subject any collateral inquiries, which, however interesting in a geometrical point of view, would be irrelevant to the immediate discussion professedly before me.

By transforming the rectangular equation (76.) of the curve of verticity into a polar one, I have shown that there are two values of the radius-vector, and only two, for every value of the polar angle; and a few of the consequences which seemed likely to facilitate our inquiry are deduced from it. The genesis of the curve, however, pointed out the necessity of a more complete examination of the magnetic curve itself: and it will appear that even in a geometrical view, and independently of any of its physical applications, this latter curve (the magnetical) is amongst the most elegant and interesting we possess. The method of investigation is, as far as I know, a new one: but it is one that in many cases, besides the present, may be very effective, and therefore valuable. Still as I could not lay down the principles of the method in this paper, so as to justify my processes, I have so modified it by a combination with the method of rectangular coordinates as fully to answer my present purposes. The method consists in taking as the variables in the equation of the curve, the *angles*

* Philosophical Transactions, 1835, p. 246, equation 76.

made by radiants drawn from two given points (in the present case the magnetic poles) with the line joining those two points: and it was suggested to my mind several years ago when considering a question proposed by Professor WALLACE in Professor LEYBOURN'S Mathematical Repository, viz., to "rectify the magnetic curve." The modification I have here used consists in the expression of the differential coefficients of a rectangular equation in terms of the polar angles, θ_i and θ_{ii} and the constants of the given equation: but I hope soon to complete a dissertation on coordination generally, and to give the necessary differential expressions that are requisite in the investigation of loci, plane, spherical, and solid; in which case several of the following processes may be considerably abbreviated.

I was compelled to employ the method here specified in consequence of the complicated form under which the rectangular and polar equations of the magnetic curve present themselves, being such as not to encourage the least hope of effecting my object by means of either of them, or by both of them conjointly.

From these investigations it appears, That both systems of branches, the convergent and the divergent, are comprised in the same angular equation of the magnetic curve already referred to, and deduced at page 238 of the Philosophical Transactions of last year: that the divergent branches on one side of the magnetic axis are continuous (algebraically and geometrically) of the convergent branches on the other, to the same parameter β: that the divergent branches are asymptotic, and the geometrical construction of the asymptote is very easy: that the continuous branches have the poles for points of inflexion, and that these are the only points of inflexion within finite limits, of the whole system: that the geometrical construction of a tangent at any point, that is the direction of a small needle whose centre is at that point, is always possible, and the process very simple: together with other properties not less interesting, though less easy to express in brief phraseology. An elegant curve is thus brought within the domain of geometry, which, when its properties are fully developed, will, I think, be second only to the conic sections themselves in point of mathematical interest: whilst its adaptation to at least one important physical inquiry will tend to enhance its value in the estimation of those who take an interest in such applications as are now, or may hereafter be, made of it, and even in this respect render it not inferior in point of value to any other loci except the conic sections, and perhaps the logarithmic curve*.

As both systems of branches of the magnetic curve are found to be involved in the same angular equation, and in the same rectangular one also by means of the double

* It is certainly a remarkable circumstance, that so few of our most elegant curves (geometrically considered) are capable of being rendered subservient to physical inquiries: for with the exceptions above mentioned, there is, besides the cycloid and the harmonic curve, with perhaps one or two of the spirals, scarcely one which could not be expunged from our geometry without any serious injury to physical science. This, together with the fact that in the dynamical problems which occur in physics, it is found to be generally most convenient to assume the time as the independent variable, has led some writers, too hastily as it appears to me, to conclude

sign of the radicals,—so also, as we should expect from the same principles being employed in both cases, in the equation of the curve of verticity the branches adapted to like poles and those adapted to unlike poles, are expressed in the same equation given in my former paper. It required, then, the previous separation of the two systems in the magnetic curve as a preliminary step to the separation of those in the curve of verticity. Such is the course I have pursued, but I have carefully abstained from the insertion of any properties of either curve except those which were essential in the determination of the number of points of *terrestrial verticity*. However, to avoid the long and complicated reductions into which I found the algebraical investigation was leading me, I have in one instance recurred to a geometrical method of investigation, founded on an elegant property of the magnetic curve, (first given by Professor LESLIE in his Geometrical Analysis, p. 400,) by means of which, and the genesis of the curve of verticity which it suggests, enables us to establish the required conclusion with ease and simplicity.

No apology is necessary, I conceive, for the introduction of new methods of investigating a problem in pure science, where those already existing are either insufficient or inconveniently operose in their application to that problem: nor yet for the employment of several different methods which in treatises on pure science are usually, and in good taste, kept distinct, when we are investigating a physical problem to which any one of them, taken separately, is inadequate. I readily and fully admit the desirableness and superior elegance of unity of mathematical method, even in physical investigations; and, doubtless, repeated efforts made by different geometers tends to the gradual formation of such united and systematic methods of development: nevertheless it is rarely the case that a unity in the full sense of the word can be brought into the mathematics of physical inquiries,—the unity that exists in the most perfect of them being more apparent than real. Mere symbolical notation does not constitute sameness of method. I have made no attempt of the kind in these inquiries, but have employed the language, methods, and notations, that seemed to me to be best adapted to obtaining the results after which the conditions and objects of the problem led me to search; or otherwise more apparent symmetry might have been easily given to the several investigations into which I have entered.

I have prefixed a few geometrical lemmas which were necessary to substantiate and facilitate the mode of reasoning here employed. Most of them are required in the present paper, the earlier one subservient to some of the others, and these for direct quotation. There is one indeed, (the sixth) which is not essential, but it is

that all consideration of curves may be advantageously banished from such inquiries. Such a plan may, indeed, furnish an elegant abstraction fitted for the higher order of minds and the highest degree of mathematical skill: but it would at the same time effectually preclude the possibility of elementary acquirement, and in very many cases that of original inquiry in *new* departments of physical science. The consequence would therefore in reality be, to retrograde the science instead of facilitating its progress. Curves, though but so few of them, enter into so many branches of philosophical investigation, that science would, save to one in a thousand, be rendered unintelligible by their abolition.

added on account of its connexion as to form of enunciation with the third, and from its admitting of a very neat analysis and construction.

The conclusion, I may, finally, add, at which I have arrived, is:—*That when two centres of magnetic force of equal intensity and opposite direction are situated anywhere within the earth, there are always two, and never more than two, points on its surface at which the needle can take a direction perpendicular to the horizon.*

GEOMETRICAL LEMMAS.

LEMMA I. LOCAL THEOREM.—*If from two given points lines be inflected to meet, and have a given ratio, the locus of their intersection is a given circle.*

This proposition was known to the Greek geometers, and is employed by EUTOCIUS in his preface to the Conics of Apollonius. The analysis and synthesis of it are given by SIMSON, in two different ways, in his Restoration of the Plane Loci, lib. ii. prop. ii. The latter of these is also given by Professor LESLIE in his Geometrical Analysis, book iii. prop. 13, and is that generally employed by geometrical writers. The following one is, so far as I know, different from any that has been given: still, but for its better answering the purposes I have in view, I should not have inserted it here, as on no other account can it, perhaps, be entitled to such an appropriation.

Let T and U (Plate X. fig. 1.) be the given points, and TN, NU, a pair of corresponding lines in the given ratio. Bisect the interior and exterior angles TNU and UNK by the lines NC, ND meeting the line TU at C and D respectively. Then

$$TC : CU :: TN : NU,$$

and

$$TD : DU :: TN : NU.$$

Hence the sum, or the difference, of two lines and their ratio being given, the lines themselves are given, and hence the points C and D are given, and the line CD between them is given in magnitude and position.

Again, since the interior and exterior angles at N, which are together equal to two right angles, are bisected by NC and ND, the angle CND is a right angle, and therefore also given in magnitude. And since CD is given, and the angle CND is a right angle, the locus of N is a circle on CD as a diameter. Hence the following

Construction.—Divide the given line internally and externally in the given ratio in CD, and on CD describe a circle. This will, as is evident from the analysis, be the locus sought.

Corollary.—The line TU is divided harmonically in C and D; for from the two analogies above, we have

$$TC : CU :: TD : DU.$$

Scholium.—A similar division may be made, the points C and D lying on the other side of M, the middle of TU. This implies, however, an inversion of the antecedent and consequent of the terms of the ratio.

Fig. 1.

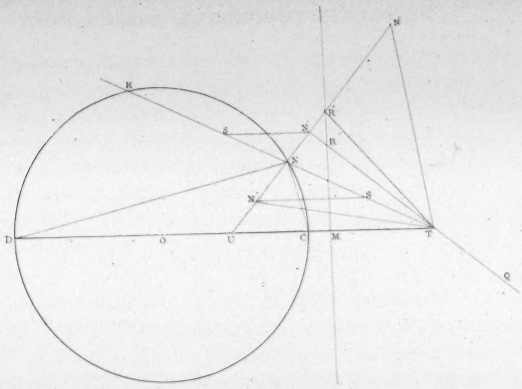


Fig. 2.

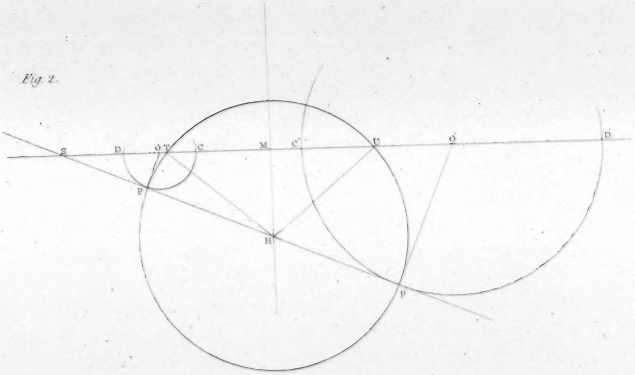
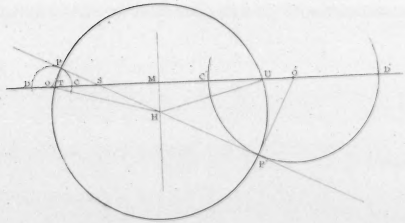


Fig. 3.



LEMMA II. THEOREM.—*If lines be inflected to a point* without *the circle* T N U *from* T *and* U, *the ratio* T N′ : N′ U *will be* less *than the ratio* T N : N U; *but if to a point within the circle, the ratio will be* greater. (Plate X. fig. 1.)

First. Let N′ be *without* the circle. This divides itself into two cases.

1. Where N′ is on the same side of M R (drawn from M at right angles to T U) as the points C and D are.

Join T N′, N′ U, and let N′ U cut the circle in N. Join also T N and draw N′ S parallel to T U, meeting T N in S, and produce T N to Q.

Then, by parallels, the angles Q T U, T N′ S are equal. And since N′ T U is less than a right angle, (for it is also an angle of the triangle T M R, of which T M R is a right angle, the point N′ being by hypothesis on the other side of R M from T,) the angle Q T U is greater than a right angle; and hence T N′ S is greater than a right angle, and consequently N′ S T is less than a right angle, and, *à fortiori*, less than T N′ S. The line T S is therefore greater than T N′.

But by similar triangles T N : N U :: T S : N′ U. Hence since the line T N′ is less than T S, the ratio T N′ : N′ U is less than the ratio T S : N′ U, and hence less than the ratio T N : N U.

2. Let N′ and C D be on opposite sides of M R. Join N′ U cutting M R in R′ and join T R′. Then the angles R′ T U, R′ U T are equal. But N′ T U is greater than R′ T U, and hence greater than N′ U T. The side N′ U of the triangle N′ T U is therefore also greater than N′ T. Hence the ratio N′ T : N′ U is a ratio of less inequality, whilst the ratio T N : N U is by hypothesis a ratio of greater inequality. The ratio T N′ : N′ U is therefore, in this case also, less than the ratio T N : N U.

Secondly. Let the point N′ lie *within* the circle. Produce U N′ to meet the circle at N and join N T. Draw N′ S parallel to T U, to meet T N at S.

Then it may be proved as before that T N′ is greater than T S. And by similar triangles T N : N U : : T S : N U′. But since T N′ is greater than T S, the ratio T S : N′ U, that is the ratio T N : N U, is less than the ratio T N′ : N U.

Scholium.—The antecedent of the lines in the expression of the ratio are considered to be drawn from the more distant point T: but these conditions will be reversed when the order of the terms is reversed.

LEMMA III. PROBLEM.—*From two given points* T *and* U (Plate X. figg. 2. and 3.) *to inflect lines to meet in a right line given by position, so that their ratio shall be the least or greatest possible.*

This problem is divisible into two cases according as the intersection S of the given line H S with the line T U drawn through the given points T and U, is on the same side of M with the antecedent or with the consequent of the lines which are in the required ratio. As, however, both cases are constructed by the same operation, it will be more convenient to give the analysis of them in juxtaposition by means of parallel vertical columns. Also for convenience of comparison, I shall employ the same letters in both cases, merely accentuating one set for the sake of distinction.

Let T U be produced to meet S H in S. Then it is obvious from the preceding lemmas that the problem is reducible to the description of two circles which shall touch the given line H S; and each divide the line T U internally and externally in the same ratio, or divide it harmonically; that is, in the one case U C : C T :: U D : D T, and in the other T C′ : C′ U :: T D′ : D′ U.

Suppose the points of contact P and P′ to be found. Draw M H from M the middle of T U perpendicular to T U, and let it meet H S in H. Draw the lines P O and P′ O′ from the points of contact perpendicular to H S; then O and O′ are the centres of the circles.

First case. Where the line T P is the antecedent, and the ratio the least possible.

By Leslie's Geom., vi. 7. $MT^2 = MC.MD$, that is,

$$MT^2 = (MS - SC)(MS - SD)$$
$$= MS^2 - MS(SC + SD) + SD.SC$$
$$= MS^2 - 2MS.SO + SD.SC.$$

But by the similar triangles SPO, SHM,

$$MS.SO = PS.SH;$$

and by the circle $SC.SD = SP^2$.

Hence

$$MT^2 = MS^2 - 2HS.SP + SP^2$$
$$= HS^2 - 2HS.SP + SP^2 - HM^2.$$

Hence

$$(HS - SP)^2 = MT^2 + MH^2 = HT^2.$$

That is $SP = HS - HT$.

Second case. Where the line T P′ is the antecedent, and the ratio the least possible.

By Leslie's Geom., ib. $MU^2 = MC'.MD'$, that is,

$$MU^2 = (SC' - SM)(SD' - SM)$$
$$= SC'.SD' - SM(SC' + SD') + SM^2$$
$$= SC'.SD' - 2SM.SO' + SM^2.$$

But by the similar triangles SP′O′, SHM,

$$MS.SO' = P'S.SH;$$

and by the circle $SC'.SD' = SP'^2$.

Hence

$$MU^2 = SP'^2 - 2HS.SP' + SM^2$$
$$= SP'^2 - 2HS.SP' + HS^2 - HM^2.$$

Hence

$$(SP' - HS)^2 = MU^2 + MH^2 = HU^2 = HT^2.$$

That is $SP' = HS + HT$.

Hence this

Construction.—Draw the perpendicular M H (from M) to T U, meeting H S in H. With centre H and distance H T or H U describe a circle cutting H S in P and P′. These are the points at which the ratios are those sought, as is too evident from the analysis to need a formal demonstration.

The ratio T P : P U is hence the *least*, and T P′ : P′ U the *greatest* that lines drawn from T and U to meet in the line H S, can possibly have; or the greatest and least, if the order of the terms of the ratio be changed. At H, midway between P and P′, they have a ratio of equality.

Lemma IV. Theorem.—*Not more than two pairs of lines can be inflected from the same two points* T *and* U *to meet in the same straight line* H S, *and have to one another a given ratio, the order of the terms of the ratio being given.*

For since the locus of all the intersections of all lines which can be so drawn is a circle, (Lemma 1.) and a circle can cut a straight line in only two points, the truth of the proposition follows.

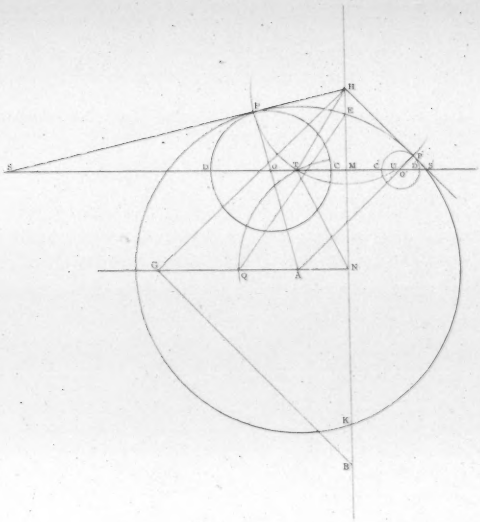


Fig. 5.

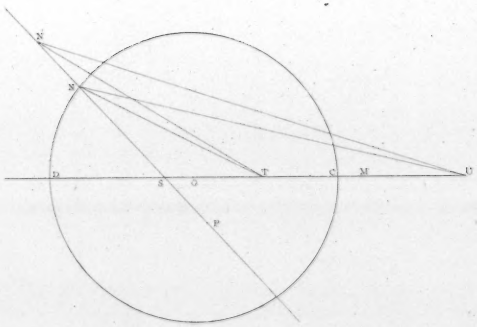


Fig. 4.

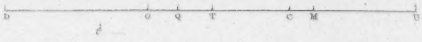


Fig. 6.

Fig. 7.

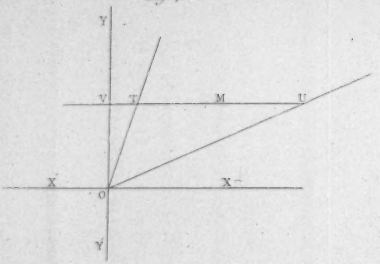


Fig. 8.

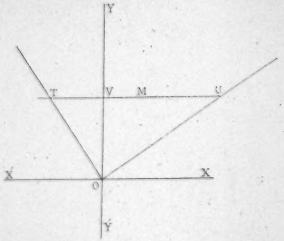


Fig. 9.

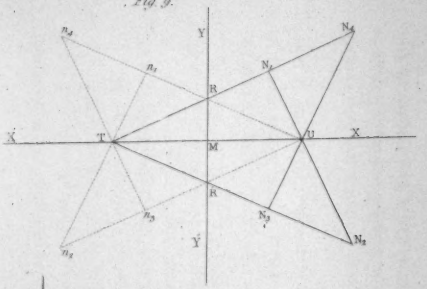


Fig. 13.

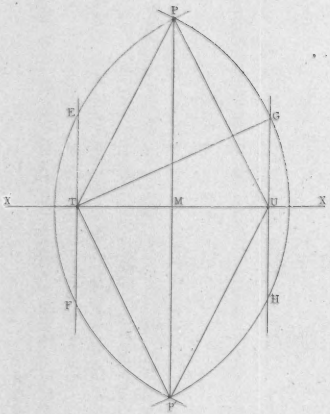
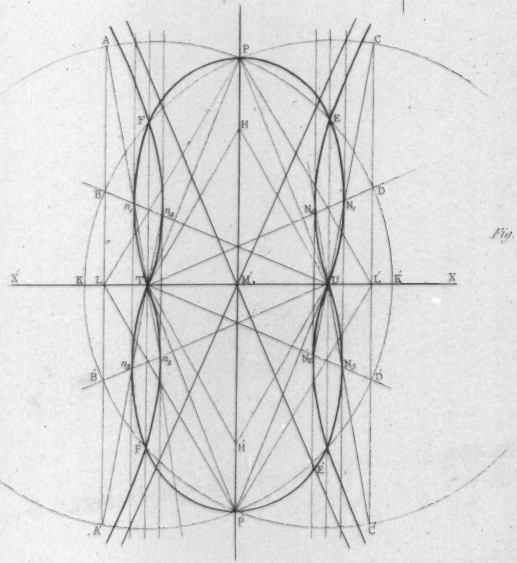


Fig. 10.



T. Bayne, Sc.

LEMMA V. THEOREM.—*If lines be inflected from two points in a given straight line to two given points, the pair which is more remote from the point at which the ratio is a minimum will have a greater ratio than those inflected from the point which is nearer. And when their order is changed, the ratio will be less.*

Let T, U be the points, and suppose T the point nearer to the given line N S to be the point from which the antecedent line is drawn. Let P be the point found in the last lemma, and N nearer to P than N′ is; then the ratio N′ T : N′ U is greater than the ratio N T : N U. (Plate XI. fig. 4.)

For divide the line T U harmonically in the points C and D in the ratio N T : N U, and describe the circle on C D, passing through N (Lemma 1.).

Then since N is nearer to P than N′ is, the point N′ falls without the circle D N C; and hence (Lemma II.) the ratio N T : N U is less than N′ T : N′ U.—*Q. E. D.*

LEMMA VI. PROBLEM.—From two given points T, U *within* a circle to inflect lines to the circumference, so that they shall have the *greatest* or the *least* ratio possible. (Plate XI. fig. 5.)

Suppose the points to be found at P and P′; and draw the tangent P H meeting the perpendicular M H from M, the middle of the line joining the given points.

Then from the reasoning in Lemma II., the circle of ratios will touch the given circle in P and divide T U harmonically in C and D; and from Lemma IV.* we learn that H P = H T. Hence the problem is reduced to finding a point H in the line M H, from which tangents being drawn to the given circle R P E they will be equal to H T or H U.

Though A the centre of the given circle draw A N parallel to T U; then it is perpendicular to H M or E K, and E K is bisected in N. Then

$$\begin{aligned} \mathrm{H\,P^2 = K\,H\,.\,H\,E = H\,T^2} &= \mathrm{T\,M^2 + M\,H^2} \\ &= \mathrm{T\,M^2 + (N\,H - N\,M)^2} \\ &= \mathrm{T\,M^2 + N\,H^2\ 2\,H\,N\,.\,N\,M + N\,M^2.} \end{aligned}$$

$$\begin{aligned} \text{Hence}\quad \mathrm{2\,H\,N\,.\,N\,M} &= \mathrm{T\,M^2 + N\,E^2 + K\,H\,.\,H\,E - K\,H\,.\,H\,E + N\,M^2} \\ &= \mathrm{T\,M^2 + M\,N^2 + N\,E^2} \\ &= \mathrm{T\,N^2 + N\,E^2.} \end{aligned}$$

Whence we have to form a rectangle whose area is $\mathrm{T\,N^2 + N\,E^2}$, and one of whose sides is 2 N M; and the other side of the rectangle is the distance of H from N.

Construction.—With centre N and distance N T describe a circle cutting A N in Q. Join E Q, and make N G = E Q, and N B = 2 N M. Join B G, and draw G H perpendicular to it, cutting M H in H. Then with centre H and distance H T or H U, describe a circle cutting the given circle in P and P′, and these will be the points required.

* Since D P T touches the line H P and divides T U harmonically.

The demonstration is obvious from the analysis.

The problem admits of several other cases, but the same analysis and construction, *mutatis mutandis*, serves for them all.

LEMMA VII. THEOREM.—*If a ratio be one of greater inequality, the triplicate of that ratio is greater than the ratio itself; but if it be a ratio of less inequality, the triplicate ratio is less than the ratio itself. Also conversely, the subtriplicate of a ratio of greater inequality is less than the ratio itself; but of a ratio of less inequality, the subtriplicate is greater than the ratio itself.*

This is too obvious to need a formal proof here.

LEMMA VIII. PROBLEM.—*If a line* T U *be divided in any undetermined ratio, viz.* $r_I : r_{II}$ *in* C *and* D, *and* O *be the middle of* C D; *and if the same line be divided in* Q *in the triplicate ratio of* $r_I : r_{II}$; *it is required to find whether* Q *and* O *can ever coincide for any value of the ratio* $r_I : r_{II}$. (Plate XI. fig. 6.)

Suppose they can; and let us first investigate the values of T O and T Q generally. Then if T U $= 2a$, we have

$$\left.\begin{aligned} \mathrm{TC} &= \frac{2ar_I}{r_{II}+r_I} \\ \mathrm{DT} &= \frac{2ar_I}{r_{II}-r_I} \end{aligned}\right\}$$

Hence

$$\mathrm{DT}+\mathrm{TC}=\mathrm{DC}=\frac{4ar_Ir_{II}}{r_{II}^2-r_I^2}$$

and

$$\mathrm{OC}=\frac{1}{2}\mathrm{DC}=\frac{2ar_Ir_{II}}{r_{II}^2-r_I^2}$$

and therefore

$$\mathrm{TO}=\mathrm{CO}-\mathrm{CT}=\frac{2ar_I^2}{r_{II}^2-r_I^2}$$

Again,

$$\mathrm{TQ}=\frac{2ar_I^3}{r_{II}^3-r_I^3}$$

And since we have admitted the hypothesis of the equality of T O, T Q, we have

$$\frac{2ar_I^3}{r_{II}^3-r_I^3}=\frac{2ar_I^2}{r_{II}^2-r_I^2}$$

which reduces to

$$r_I^2r_{II}^2=0.$$

And this again to the two equations $r_I^2=0$ and $r_{II}^2=0$; which indicates that it takes place at T and U, or when the ratios are infinitely great and infinitely small.

LEMMA IX. PROBLEM.—*To ascertain which is the greater,* T O *or* T Q.

Put T O − T Q = c; that is, $c =$

$$2a\left\{\frac{r_{\prime}^{3}}{r_{\prime\prime}^{3}-r_{\prime}^{3}}-\frac{r_{\prime}^{3}}{r_{\prime\prime}^{3}-r_{\prime}^{3}}\right\}=\frac{2a r_{\prime}^{2} r_{\prime\prime}^{2}}{(r_{\prime\prime}^{2}-r_{\prime}^{2})(r_{\prime}^{2}+r_{\prime}r_{\prime\prime}+r_{\prime\prime}^{2})}=\frac{2a r_{\prime}^{2} r_{\prime\prime}^{2}}{(r_{\prime\prime}-r_{\prime})(r_{\prime\prime}+r_{\prime})(r_{\prime}^{2}+r_{\prime}r_{\prime\prime}+r_{\prime\prime}^{2})}.$$

Now whilst $r_{\prime\prime}$ is greater than $r_{\prime}$, that is whilst the distances T O, T Q are reckoned to the left of T, this is essentially positive, since all the factors except $r_{\prime\prime}^{2}-r_{\prime}^{2}$ are essentially positive, however the quantities be reckoned; and hence the point O lies more remote from T than Q does. In precisely the same way it may be shown to be true when the points C and D, &c. are taken respectively to the right of the middle. Hence we may infer that, under all circumstances, except those determined in the last lemma, the quantity c is *finite*: and it may be easily shown to increase, as $r_{\prime}$ and $r_{\prime\prime}$ increase, *ad infinitum*.

XX.—*On the Points at which the Magnetic Needle takes a Position vertical to the Surface of the Earth.**

At the close of my last paper, art. XIX., I stated that I had been unable to resolve equation (78.) into its simple or quadratic component factors, and was therefore unable by means of it to assign positively the number of points on the earth's surface at which the needle can take a vertical position. That difficulty may, however, be obviated by a different process from that which I then indicated; and as this new method fully meets all the objects, physical and geometrical, which led to the formation of that equation, any further discussion of it in that form may now be dispensed with. It will here be proved that on the hypothesis of two poles of equal intensity and of different kinds, *there never can be more than two points on the earth's surface at which the needle can take the position in question.*

For this purpose let us return to the equation (76.), and take the axis of x parallel to the axis of the terrestrial magnet T U (Plate XII. figg. 7, 8.), the centre of the earth O being still the origin of the coordinates. Draw O V perpendicular to T U, which will coincide in position with the axis of y. Denote the angles which O U and O T make with O V by $\alpha_{\prime\prime}$ and $\alpha_{\prime}$, the line O V itself in magnitude by b, and the current polar coordinates of the curve of contact (of the tangents from O to the magnetic curves whose common poles are U and T) by r and θ.

Then we have V T $= a_{\prime} = b \tan \alpha_{\prime}$, V U $= a_{\prime\prime} = b \tan \alpha_{\prime\prime}$, and $b_{\prime} = b_{\prime\prime} = b$; and likewise $x = r \sin \theta$, and $y = r \cos \theta$. Make these substitutions in (76.), viz. in

$$(b_{\prime}x - a_{\prime}y)^{2}\{(a_{\prime\prime}-x)^{2}+(b_{\prime\prime}-y)^{2}\}^{3} = (b_{\prime\prime}x - a_{\prime\prime}y)^{2}\{(a_{\prime}-x)^{2}+(b_{\prime}-y)^{2}\}^{3};$$

and there will result, after a few reductions too easy and obvious to need indication here, the following polar equation of the curve of contact:

* Continued from the Philosophical Transactions, 1835, p. 248.

$$\left.\begin{aligned} &r^2\cos^2\alpha_{ii}\sin^2\overline{\theta-\alpha_i}\{r^2-2br\sec\alpha_{ii}\cos\overline{\theta-\alpha_{ii}}+b^2\sec^2\alpha_{ii}\}^3\\ &=r^2\cos^2\alpha_i\sin^2\overline{\theta-\alpha_{ii}}\{r^2-2br\sec\alpha_i\cos\overline{\theta-\alpha_i}+b^2\sec^2\alpha_i\}^3\end{aligned}\right\}\quad(79.)$$

and this resolves at once into the two equations

$$\left.\begin{aligned} &\cos^2\alpha_{ii}\sin^2\overline{\theta-\alpha_i}\{r^2-2br\sec\alpha_{ii}\cos\overline{\theta-\alpha_{ii}}+b^2\sec^2\alpha_{ii}\}^3\\ &=\cos^2\alpha_i\sin^2\overline{\theta-\alpha_{ii}}\{r^2-2br\sec\alpha_i\cos\overline{\theta-\alpha_i}+b^2\sec^2\alpha_i\}^3\end{aligned}\right\}\quad(80.)$$

$$\text{and } r^2=0\quad(81.)$$

If now we extract the cube root of both sides of (80.), and resolve the quadratic for r, we shall obtain the following equation of the system of branches of the curve of contact:

$$r=b\cdot\frac{\sec\alpha_{ii}\cos\overline{\theta-\alpha_{ii}}(\cos\alpha_{ii}\sin\overline{\theta-\alpha_i})^{\frac{2}{3}}\pm\left\{\begin{matrix}(\sec^2\alpha_{ii}\cdot\ 2\sec\alpha_i\sec\alpha_{ii}\cos\overline{\theta-\alpha_i}\cos\overline{\theta-\alpha_{ii}}+\sec^2\alpha_i)(\cos\alpha_i\cos\alpha_{ii}\sin\overline{\theta-\alpha_i}\sin\overline{\theta-\alpha_{ii}})^{\frac{2}{3}}\\ -\sec^2\alpha_{ii}\sin^2\overline{\theta-\alpha_{ii}}(\cos\alpha_{ii}\sin\theta-\alpha_i)^{\frac{2}{3}}-\sec^2\alpha_i\sin^2\overline{\theta-\alpha_i}(\cos\alpha_i\sin\overline{\theta-\alpha_i})^{\frac{2}{3}}\end{matrix}\right\}^{\frac{1}{2}}}{(\cos\alpha_{ii}\sin\overline{\theta-\alpha_i})^{\frac{2}{3}}-(\cos\alpha_i\sin\overline{\theta-\alpha_{ii}})^{\frac{2}{3}}}\quad(82.)$$

From this equation we learn the important fact, that *no more than two points* of the curve of contact can exist for each value of θ. To render it subservient to the completion of our object in this inquiry, it will be necessary to establish two other properties, viz. that the quantity under the radical is essentially positive, so as to render the curve real for all values of θ, and that of these values one is greater and the other less than $b\sec\theta$. The slightest attention, however, to the form of the expression will convince us that this would be a work of great labour if performed in a perfectly satisfactory manner; and that probably it would exceed the means at present in our possession for conducting such a discussion to a successful termination. It is fortunately as unnecessary as it is difficult, since by recurring to the genesis of the curve itself both these conclusions may be readily established; and as these are all that are essential to the present investigation, I do not think myself under any necessity to examine those characters of the curve which are mere matters of mathematical curiosity, even though some of them may be very readily obtained from the equation itself. Except, therefore, for facilitating some few steps of the succeeding course of inquiry, and for the establishment of the above-named general principle (the duality of the values of r for each value of θ), I shall rarely have occasion to again employ this equation, the objects of its introduction being hereby fully answered.

XXI.—By referring to art. XIX. (pp. 245, 246.) it will be obvious that the geometrical expression of the hypothesis belongs equally to the case of the convergent and divergent magnetic curves*; and hence the equations just obtained (81, 82.) must express both those cases. Moreover, it embraces the cases where N, O are on the

* These appropriate terms were first used by Professor LESLIE in his "Geometry of Curve Lines," p. 400, to designate the curves when the poles were respectively of different kinds and of the same kind. Professors ROBISON and PLAYFAIR had only considered the convergent curve; and I am not aware that any other author except Dr. ROGET has taken up the subject. See Library of Useful Knowledge, art. MAGNETISM, and Journal of Royal Institution, February 1831. Mr. BARLOW has followed LESLIE, Encyclopædia Metropolitana, p. 794.

same, and on different sides of the magnetic axis T U, as the slightest consideration will render obvious. In order, therefore, to separate these cases and adapt them to our immediate subject, we must recur to the magnetic curves themselves, and examine their particular characters with more care than has hitherto been done. We shall thus be enabled to establish the duality of the vertical points where the centres of force are, as assumed in the hypothesis, only two, and of equal intensity. The same method, it will readily appear after a little consideration, will establish analogous conclusions, whatever be the relative nature and intensities of the forces $F_{,}$ and $F_{,,}$ resident in the two centres T and U, should, at any future time, such an investigation be considered necessary, and prove that in no case can there be more than four such points on the earth's surface. In the present paper it will be shown, that could we imagine such an hypothesis to have any foundation in nature, the existence of two poles of the same kind and of equal intensities would in certain cases produce four points on the earth's surface, at which the needle would be vertical, and in others only two.

XXII.—*To trace the Magnetic Curve, and determine the Nature of its Branches and Singular Points.*

The equation of the curve* is

$$\cos\theta_{,} + \cos\theta_{,,} = 2\cos\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83.)$$

(A.) Let T and U be the poles; then since the equation is to be fulfilled by the cosines of $\theta_{,}$ and $\theta_{,,}$, we have the four following systems of equations, each of which fulfils the condition of (83.) for the same numerical value of $\theta_{,}$ and $\theta_{,,}$. (Plate XII. fig. 9.)

1. $\cos\theta_{,} + \cos\theta_{,,} = 2\cos\beta.$
2. $\cos(-\theta_{,}) + \cos\theta_{,,} = 2\cos\beta.$
3. $\cos(-\theta_{,}) + \cos(-\theta_{,,}) = 2\cos\beta.$
4. $\cos\theta_{,} + \cos(-\theta_{,,}) = 2\cos\beta.$

(B.) Hence the equation of the cosines determines the four points N_1, N_2, N_3, N_4 corresponding to the four forms of the equation just given respectively; and each of these points will trace out a branch of the system of curves as $\theta_{,}$, and consequently $\theta_{,,}$ is made to vary its actual angular value, $\cos\beta$ retaining the same value throughout the whole. When $\theta_{,}$ and $\theta_{,,}$ are both on the same side of the axis, that is both + or both −, the branches traced out are the convergent ones; but when on different sides, or one + and the other −, the branches are the divergent ones.

(C.) The four branches thus traced form two pairs of symmetrical portions, viz. N_1 is symmetrical to N_3, and N_2 to N_4.

(D.) When $\theta_{,}$ and $\theta_{,,}$ have interchanged their values, the four points will have at-

* See Philosophical Transactions, 1835, p. 238.

tained symmetrical positions with respect to the line T U, but inverted in respect to the extremities T and U, viz. the positions n_1, n_2, n_3, n_4 of the figure.

(E.) The latter system of points is symmetrical with the former, taken with respect to Y M Y′ at right angles to the magnetic axis, and passing through its centre M, viz. N_1 with n_1, N_2 with n_2, &c.

(F.) The points N_1 and N_3 trace out branches which are continuous from U to T, respectively above and below the magnetic axis.

(G.) The points N_4 and N_2 trace branches which diverge more and more from Y M Y′ as the angles approach to equality, since the angles T N_4 U and T N_2 U become more and more acute (being the differences of N_4 T U and N_4 U X, and of N_2 T U and N_2 U X, respectively), and when these are equal the points N_4 and N_2 become infinitely remote; that is, these are infinite branches.

(H.) For the same reason the points n_4 and n_2 trace a pair of infinite branches turned from the perpendicular Y M Y′ to the middle of the magnet, in the opposite direction to the former.

(I.) The branches are symmetrical to the magnetic axis and to the perpendicular through its centre, in the same cases in which the tracing points were severally symmetrical.—The general figure of the several systems of branches is represented in Plate XI. fig. 10.

(K.) No other points than N_1, N_2, N_3, N_4 fulfil the equation for the same angular values of θ_i and θ_{ii}; and hence no other branches than these can exist.

(L.) Describe the parallelogram T P U P′, having T U for its diameter, and the angles at T and U above and below the line T U each equal to β. Through M (the centre of T U) draw the lines q_4 M Q_2 and q_2 M Q_4 parallel to the sides of the parallelogram. Then the infinite branches traced out by N_4, N_2, n_4 and n_2 have these lines for rectilinear asymptotes, whilst P and P′ are the vertices of the finite branches passing from T to U above and below the axis of the magnet. (Figg. 10, 11, 12.)

(M.) Moreover, if from centres T and U with radii T P, U P circles be described cutting the axis in G and H, and from these points lines be drawn parallel to P P′ uniting the circles in K K′ and L L′, then the parallelogram formed by drawing the radii through these points till they meet in Y and Y′, will have its sides tangents to all the branches of the curve that pass through T and U respectively in those points.

(N.) The points T and U are true points of inflexion of the branches, the convergent ones above being continuous of the divergent ones below the axis, and the convergent ones below the axis being continuous of the divergent ones above the axis, the first series n_2 T n_1 P N_1 T N_2 being marked in full line, and the second in dotted line. (Plate XIII. figg. 11, 12.)

(O.) If we conceive the plane of the curve to be a sphere of infinite radius*, then

* The idea of considering the plane as an infinite sphere was first, I think, employed by the veteran geometer Hachette, for any real mathematical purpose except that of tracing a mere analogy between plane and spherical trigonometry, in his solution of Vieta's "Problem of Spherical Tangencies." This use of it is now

Fig. 11.

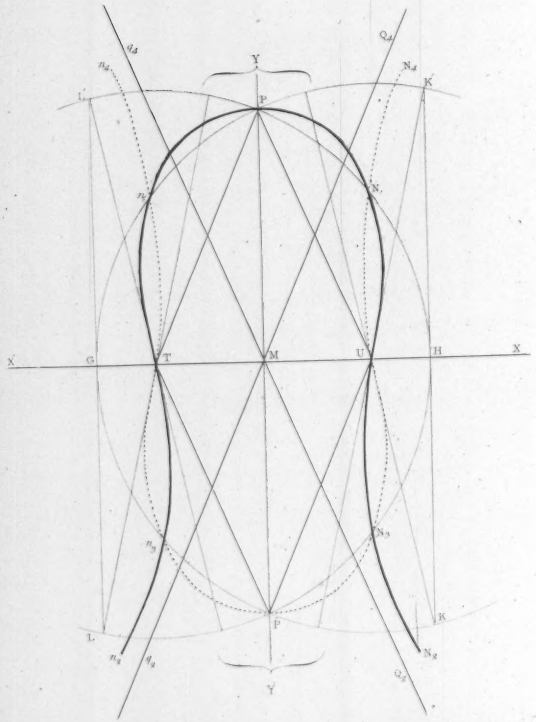
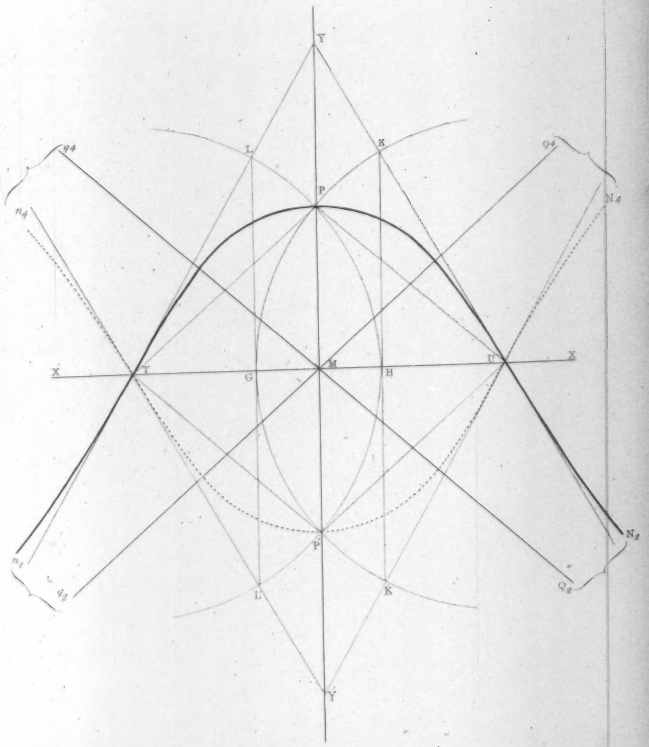


Fig. 12.



the dotted and full-line branches meet at the opposite side of the sphere from M; and if we call this point M′, the point M′ is also a double point of inflexion, and the whole system of branches may in this sense be said to be continuous.

(P.) The following construction gives the vertices of the curve with respect to the line P M P′. (Plate XII. fig. 10.)

Describe an equilateral triangle L P L′, whose perpendicular is P M, and from centres T and U two circles P K P′ and P K′ P′. Through L and L′ draw lines parallel to P P′, meeting the circles in A, B, B′, A′, and C, D, D′, C′. Draw radii to the several points of intersection of the lines A A′ and C C′ with the circles; these will intersect in the points n_1, n_2, n_3, n_4 and N_1, N_2, N_3, N_4, which are the several vertices sought. The outer points n_1, n_3, N_1 and N_3 are those at which the vertices of the convergent branches are situated, and the inner ones n_2, n_4, N_2, N_4 are those of the divergent branches.

When the points L and L′ fall without the double segment P K P′ K, the construction fails, and there are no such points in the curve when this takes place. This obviously will be the case when L and L′ fall between the poles T and U*.

(Q.) The points in which the convergent and divergent branches intersect are found as follows:—(Figg. 10, 13.)

Draw through T and U the indefinite perpendiculars to the axis, and with centres T and U describe the circles P G H P′, P E F P′ cutting the perpendiculars in G, H, and E, F, respectively; then E, F, G, H are the points sought.

When T P is less than T U the construction fails, and the existence of such points becomes impossible. When T P = T U, the points coalesce in the poles, and the branches all *touch* there.

We shall now proceed to establish the truth of such of the preceding properties of the curve as are not immediately evident.

XXIII.—*The Vertices; or the Points at which the Tangent is perpendicular or parallel to the Axis.*

That it may be more easily effected, resume the general equation (38.), the poles being endowed with equal absolute intensities of force.

$$\frac{dy}{dx} = \frac{\frac{y}{r_I^3} - \frac{y}{r_{II}^3}}{\frac{x+a}{r_I^3} - \frac{x-a}{r_{II}^3}} = \frac{\frac{\sin\theta_I}{r_I^2} - \frac{\sin\theta_{II}}{r_{II}^2}}{\frac{\cos\theta_I}{r_I^3} + \frac{\cos\theta_{II}}{r_{II}^3}} = \frac{r_{II}^2 \sin\theta_I - r_I^2 \sin\theta_{II}}{r_{II}^2 \cos\theta_I + r_I^2 \sin\theta_{II}} \quad . \quad . \quad . \quad (84.)$$

sufficiently familiar to the Continental geometers; but I am not aware that any further use of the principle has been made even by them. The application here made of the idea was suggested to my mind several years ago in considering the nature of infinite branches in a case analogous to the present one; and it seems to supply a desideratum, the want of which all writers on the higher geometry have felt when discussing the characters of certain particulars respecting curve lines.

* These considerations will be rendered subservient to an investigation of the state of the forces in what is commonly, though very improperly, called a "suturated" bar-magnet, which will hereafter be laid before the Royal Society.

But by the triangle T N U we have

$$r_{ii} \sin \theta_{ii} = r_i \sin \theta_i,$$

which substituted in (84.) gives

$$\frac{dy}{dx} = \frac{\sin^3 \theta_i - \sin^3 \theta_{ii}}{\sin^2 \theta_i \cos \theta_i + \sin^2 \theta_{ii} \cos \theta_i}. \quad \ldots\ldots\ldots \quad (85.)$$

First. That the tangent may be parallel to the magnetic axis, we must have

$$\sin^3 \theta_i - \sin^3 \theta_{ii} = 0,$$

which resolves itself into

$$\sin \theta_i - \sin \theta_{ii} = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (86.)$$

and

$$\sin^2 \theta_i + \sin \theta_i \sin \theta_{ii} + \sin^2 \theta_{ii} = 0. \quad \ldots\ldots\ldots \quad (87.)$$

The latter of these equations (87.) being imaginary, (for $\sin \theta_{ii} = \frac{-1 \pm \sqrt{-3}}{2} \sin \theta_i$) the only points that exist where the property holds good are to be determined from the former (86.).

This equation may be fulfilled by the four following systems of values:

1. $\pm \theta_i$ and $\pm \theta_{ii}$; 2. θ_i and $\pi \mp \theta_{ii}$; 3. $\pi \mp \theta_i$ and θ_{ii}; and 4. $\pi \mp \theta_i$ and $\pi \mp \theta_{ii}$. But whichever of these we employ, it must be consistent with the equation of the curve itself; viz. with

$$\cos \theta_i + \cos \theta_{ii} = 2 \cos \beta.$$

1. The first of these obviously gives the points P and P', and is consistent with the equation of the curve.

2. The fourth is virtually the same as the first, if we consider that in taking one supplement we should take all the supplements.

3. The second and third are incompatible with the equation of the curve itself.

There are hence only the two points P and P' at which the needle can be parallel to the axis.

Secondly. That the tangent may be perpendicular to the axis, we must have

$$\sin^2 \theta_i \cos \theta_i + \sin^2 \theta_{ii} \cos \theta_{ii} = 0. \quad \ldots\ldots\ldots \quad (88.)$$

Expressing $\sin^2 \theta_i$, $\sin^2 \theta_{ii}$, and $\cos \theta_{ii}$ in terms of β and θ_i from the equation of the curve, and inserting the results in (88.), we shall obtain, after slight reductions,

$$\left.\begin{aligned} \cos \theta_i &= \cos \beta \pm \frac{\sin \beta}{\sqrt{3}}, \\ \text{and} \quad \cos \theta_{ii} &= \cos \beta \mp \frac{\sin \beta}{\sqrt{3}}. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (89.)$$

From these equations it appears that if β be less than $\frac{\pi}{3}$, there can be no point on the system at which the tangent is perpendicular to the axis, as in that case either $\cos \theta_i$ or $\cos \theta_{ii}$ would be greater than unity.

Fig. 14.

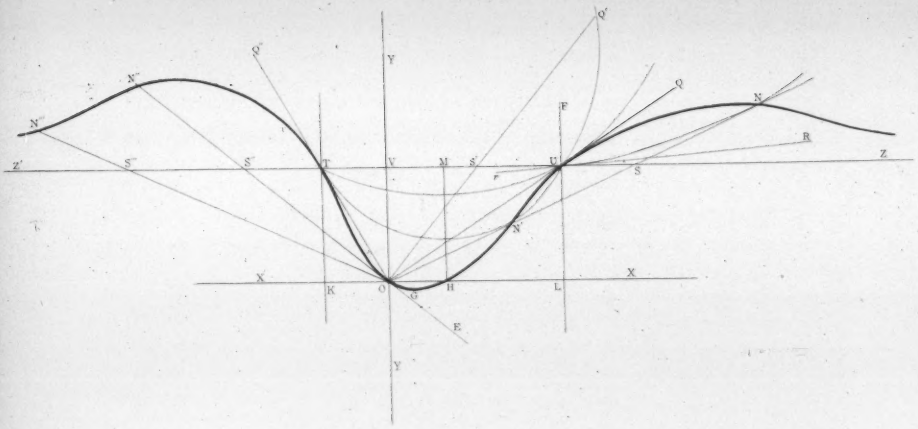
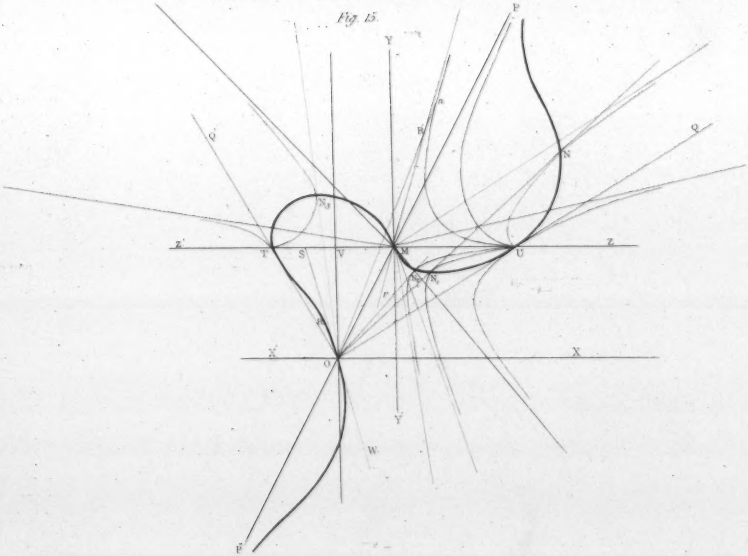


Fig. 15.



We may now establish the truth of the construction given at (P.) of art. XXII. for finding these points.

Let PT = radius = 1; then $PM = \sin\beta$, and $LN = \frac{\sin\beta}{\sqrt{3}}$. Hence $TL = \cos\beta - \frac{\sin\beta}{\sqrt{3}}$ and $LU = \cos\beta + \frac{\sin\beta}{\sqrt{3}}$ are the corresponding values of $\cos\theta_I$ and $\cos\theta_{II}$ in equations (89.): and the intersection, therefore, of the lines BU and DT give one of the points in question.

In the same manner, by taking the other combinations of + and − as signs of θ_I and θ_{II} in these equations, as signified in the opening of the last section (XXII. A.), the other points will be shown to be those given by the construction enunciated in (XXII. P.).

It is also clear that the limitation to which the construction is subjected is that expressed by the limitation of the equations themselves.

Moreover, the distribution of the points as to the particular branches of the curves to which they belong is properly made: for Tn_1, Un_1 are the radiants belonging to values of θ_I and θ_{II} on the *same side* of the axis. Hence n_1 is in the convergent curve. So, for the same reason, are n_3, N_1 and N_3. Again, since n_4 is found by the intersection of radiants n_2U and n_3T, whose corresponding values of $\cos\theta_I$ and $\cos\theta_{II}$ are estimated for θ_I and θ_{II} *on different sides* of the axis, n_4 is in the divergent curve. So also, for the same reason, are n_1, N_4 and N_1 in the divergent branches.

XXIV.—*The Points of Intersection of the Finite with the Infinite Branches: and on the Asymptotes.*

1. The construction of the points of intersection has been given in (XXII. Q.), and it is thus proved. (Plate XIV. fig. 13.)

When in the construction of the points, the two points N_1 and N_4 coincide, the radiants N_1U and N_4U coincide also. But $N_4UX = TUN_3 = TUN_1$. Hence when the radiants coalesce they form a line at right angles to TU.

Also in this case $\cos\theta_{II} = 0$, and the equation of the curve becomes at this point

$$\cos\theta_I = 2\cos\beta.$$

But taking $r_I = 1$, $TM = \cos\beta$ and $TU = 2\cos\beta$; and if GT be drawn it is = 1. Hence the construction is true.

2. The construction of the asymptotes has been given in (XXII. L.); and its truth may be thus established.

To prove that q_2MQ_4 and q_4MQ_2 are Asymptotes to the Infinite Branches. (Plate XIV. figg. 11, 12.)

Since the point of the curve is found by the intersection of two parallel lines PT and P′U, it is infinitely distant; and the two radii r_I and r_{II} themselves being infinite, are equal to one another. But the tangent to any point of the magnetic curve divides

the axis in the ratio of r_I^3 to r_{II}^3: and since these radii when infinite are equal, the tangent to the infinitely distant point of the curve bisects the magnet, or passes through M.

Now the line MQ_4 being parallel to TP and PU by construction passes through their common intersection; and dividing the axis TU in the ratio of r_I^3 to r_{II}^3 is a tangent to the curve at that infinitely distant point. That is, MQ_4 is an asymptote to the branch UN_4 of the curve.

In the same way the other branches are shown to have severally the lines drawn, as already described, through M for rectilinear asymptotes.

XXV.—*The Points of Inflexion.*

In this case the second differential coefficient is equal to zero.

By (85.) we have

$$\frac{dy}{dx} = \frac{\sin^3\theta_I - \sin^3\theta_{II}}{\sin^2\theta_I\cos\theta_I + \sin^2\theta_{II}\cos\theta_{II}}.$$

Also

$$\frac{x+a}{r_I} = \cos\theta_I, \text{ or } x = r_I\cos\theta_I - a,$$

and therefore

$$dx = \cos\theta_I\, dr_I - r_I\sin\theta_I\, d\theta_I,$$

which, since by the triangle

$$r_I = \frac{2a\sin\theta_I}{\sin\theta_I + \theta_{II}}$$

is convertible into

$$dx = \frac{\begin{matrix}-(\sin\theta_{II}\sin\theta_I\sin\overline{\theta_I+\theta_{II}} + \sin\theta_{II}\cos\theta_I\cos\overline{\theta_I+\theta_{II}})\, d\theta_I \\ +(\cos\theta_{II}\cos\theta_I\sin\overline{\theta_I+\theta_{II}} - \sin\theta_{II}\cos\theta_I\cos\overline{\theta_I+\theta_{II}})\, d\theta_{II}\end{matrix}}{\sin^2\theta_I + \theta_{II}}. \quad \ldots \quad (90.)$$

Now, by the differential equation of the curve * we also have

$$d\theta_{II} = -\frac{\sin\theta_I\, d\theta_I}{\sin\theta_{II}}\dagger,$$

which converts (90.) into

$$dx = -2a.\frac{\sin^2\theta_{II}(\sin\theta_I\sin\overline{\theta_I+\theta_{II}} + \cos\theta_I\cos\overline{\theta_I+\theta_{II}}) + \sin\theta_I\cos\theta_I(\cos\theta_{II}\sin\overline{\theta_I+\theta_{II}} - \sin\theta_{II}\cos\overline{\theta_I+\theta_{II}})}{\sin\theta_{II}\sin^2\theta_I + \theta_{II}}.d\theta_I,$$

or into

$$dx = -2a.\frac{\sin^2\theta_{II}\cos\theta_{II} + \sin^2\theta_I\cos\theta_I}{\sin\theta_{II}\sin^2\theta_I + \theta_{II}}.d\theta_I. \quad \ldots \quad (91.)$$

In a similar manner from the equation $y = r\sin\theta_I$ we obtain

$$dy = -2a.\frac{\sin^3\theta_I - \sin^3\theta_{II}}{\sin\theta_{II}\sin^2\theta_I + \theta_{II}}\, d\theta_I. \quad \ldots \quad (92.)$$

But this is more readily obtained at once from a comparison of (85.) with (91.).

* Philosophical Transactions, 1835, p. 238.

† In this case, as also in the formation of the angular equation (85.), θ_{II} is taken the supplement of the θ_{II} in the differential equations; and hence the change of sign from − to +.

To obtain the second differential coefficient, first differentiate the numerator and denominator separately.

$$\left.\begin{aligned} d\{\sin^3\theta_{,} - \sin^3\theta_{,,}\} &= 3(\sin^2\theta_{,}\cos\theta_{,}\,d\theta_{,} - \sin^2\theta_{,,}\cos\theta_{,,}\,d\theta_{,,}),\\ \text{or, by the equation } d\theta_{,,} &= \frac{\sin\theta_{,}\,d\theta_{,}}{\sin\theta_{,,}},\\ &= 3\sin\theta_{,}\{\sin\theta_{,}\cos\theta_{,} + \sin\theta_{,,}\cos\theta_{,,}\}\,d\theta_{,} \end{aligned}\right\} \quad . \quad . \quad (93.)$$

$$\left.\begin{aligned} d\{\sin^2\theta_{,}\cos\theta_{,} - \sin^2\theta_{,,}\cos\theta_{,,}\} &= (2\sin\theta_{,}\cos^2\theta_{,} - \sin^3\theta_{,})\,d\theta_{,} + (2\sin\theta_{,,}\cos^2\theta_{,,} - \sin^3\theta_{,,})\,d\theta_{,,}\\ &= \sin\theta_{,}\{2\cos^2\theta_{,} - \sin^2\theta_{,})\} - (2\cos^2\theta_{,,} - \sin^2\theta_{,,})\}\,d\theta_{,}\\ &= 3\sin\theta_{,}(\cos^2\theta_{,} - \cos^2\theta_{,,})\,d\theta_{,}. \end{aligned}\right\} (94.)$$

Hence from (91.), (93.), and (94.), omitting the constant factors, we have

$$\left.\begin{aligned} \frac{d^2y}{dx^2} &= \sin\theta_{,}\sin\theta_{,,}\sin^2\overline{\theta_{,}+\theta_{,,}}\cdot\frac{(\sin\theta_{,}\cos\theta_{,} + \sin\theta_{,,}\cos\theta_{,,})(\sin^2\theta_{,}\cos\theta_{,} - \sin^2\theta_{,,}\cos\theta_{,,}) - (\cos^2\theta_{,} - \cos^2\theta_{,,})(\sin^3\theta_{,} - \sin^3\theta_{,,})}{(\sin^2\theta_{,}\cos\theta_{,} + \sin^2\theta_{,,}\cos\theta_{,,})^3}\\ &= \frac{\sin\theta_{,}\sin\theta_{,,}\sin^2\overline{\theta_{,}+\theta_{,,}}\,(\sin^2\theta_{,,}\cos\theta_{,} + \sin^2\theta_{,}\cos\theta_{,,})}{(\sin^2\theta_{,}\cos\theta_{,} + \sin^2\theta_{,,}\cos\theta_{,,})^3}\\ &= \frac{\sin\theta_{,}\sin\theta_{,,}\sin^3\overline{\theta_{,}+\theta_{,,}}\,(\cos\theta_{,} + \cos\theta_{,,})(1 - \cos\theta_{,}\cos\theta_{,,})}{(\sin^2\theta_{,}\cos\theta_{,} + \sin^2\theta_{,,}\cos\theta_{,,})^3} = 0. \end{aligned}\right\} (95.)$$

Since the denominator of this cannot become infinite, the condition is only fulfilled by the numerator $= 0$: and this gives the five following equations:

$$\left.\begin{aligned} 1.\quad & \sin\theta_{,} = 0\\ 2.\quad & \sin\theta_{,,} = 0\\ 3.\quad & \sin^3\overline{\theta_{,}+\theta_{,,}} = 0\\ 4.\quad & \cos\theta_{,} + \cos\theta = 0\\ \text{and}\quad 5.\quad & \cos\theta_{,}\cos\theta_{,,} = 1. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (96.)$$

The first and second of these show that the poles themselves are true points of inflexion, and hence that the order of the branches as to continuity is, that the infinite are continuous of the finite branches on the opposite sides of the axis, as indicated by the full line and dotted line representations (Plate XIV. figg. 11, 12.), and as stated in (XXII.). It is evident that these conditions are consistent with the equation of the curve $\cos\theta_{,} + \cos\theta_{,,} = 2\cos\beta$; and, therefore, all the necessary conditions are thus fulfilled.

The third equation is fulfilled by the equation $\theta_{,} + \theta_{,,} = \pi$, which is also consistent with the equation of the curve. But this is the case when the radiants $r_{,}$ and $r_{,,}$ are parallel; and then the tangent, as has been already shown, is an asymptote. The view, then, which has been taken at (XXII. o.) of the infinite branches having points of inflection on the opposite side of the infinite sphere, is borne out by the analytical expression of the points of inflexion.

The third equation is, moreover, fulfilled also by $\theta_{,} + \theta_{,,} = 0$, $\theta_{,} = -\theta_{,,}$, which indi-

cates the opposite branches to those just described: and hence the same remark may be made respecting them.

The fourth and fifth equations are *generally* inconsistent with the equation of the curve, and hence in all those cases imaginary. When, however, $\cos\beta = 0$, or $\beta = \frac{\pi}{2}$, the equation of the curve is consistent with this equation. The divergent branches in this case respectively coalesce with the magnetic axis itself, and the convergent ones by their continual expansion outwards have then come to coalescence with the axis produced; each through its whole length. In these cases, *any* point in the magnetic axis may be considered as a point of inflexion, and the tangent in all cases so taken makes an angle 0 or π with the axis.

When the middle M, however, of the axis, and its opposite point on the infinite sphere are taken, *any line* through them may be considered a tangent, as the asymptote, properly speaking, has then ceased to exist, or to be expressed by the equation. In other words, the direction of the curve at these points is become properly indeterminate. That the expressions themselves indicate this, will be made to appear in the next section.

XXVI.—*To find the Multiple Points and the Directions of their Tangents of the Magnetic Curve.*

At a multiple point we shall have, in consequence of (83.) and (85.), the three equations

$$\left.\begin{aligned} &1.\ (\sin^3\theta_{\prime} - \sin^3\theta_{\prime\prime})\sin\theta_{\prime}\sin\theta_{\prime\prime}\sin^2\overline{\theta_{\prime}+\theta_{\prime\prime}} = 0,\\ &2.\ (\sin^2\theta_{\prime}\cos_{\prime} + \sin^2\theta_{\prime\prime}\cos\theta_{\prime\prime})\sin\theta_{\prime}\sin\theta_{\prime\prime}\sin^2\overline{\theta_{\prime}+\theta_{\prime\prime}} = 0,\\ \text{and}\ &3.\ \cos\theta_{\prime} + \cos\theta_{\prime\prime} = 2\cos\beta. \end{aligned}\right\} \quad . \ . \ . \quad (97.)^*$$

If these three equations be simultaneously fulfilled by the same values of $\theta_{\prime}$ and $\theta_{\prime\prime}$, the points indicated by those values will be multiple points. But this is the case with either of the values

$$\left.\begin{aligned} &1.\quad \sin\theta_{\prime} = 0,\\ &2.\quad \sin\theta_{\prime\prime} = 0,\\ &3.\ \sin^2\overline{\theta+\theta_{\prime\prime}} = 0, \end{aligned}\right\} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (98.)$$

that is, at the poles and at the point of symptotism; that is, at the point on the infi-

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* Since

$$d\left(\frac{\frac{x}{z}}{\frac{y}{z}}\right) = \frac{\frac{y}{z}\,d\left(\frac{x}{z}\right) - \frac{x}{z}\,d\left(\frac{y}{z}\right)}{\frac{y^2}{z^2}} = \frac{y\,\frac{z\,dx - x\,dz}{z^2} - x\,.\,\frac{z\,dy - y\,dz}{z^2}}{\frac{y^2}{z^2}}$$

$$= \frac{y\,z\,dx - y\,x\,dz - x\,z\,dy + x\,y\,dz}{z\,y^2} = \frac{y\,dx - x\,dy}{y^2} = d\left(\frac{z}{y}\right),$$

nite sphere diametrically opposite to M. These results accord with the statements made in article XXII., and justify them.

To find the values of $\frac{dy}{dx}$ corresponding to these multiple points, we may proceed thus:

When $\sin\theta_{I} = 0$, we have $\cos\theta_{I} = \pm 1$; and $\cos\theta_{II} = 2\cos\beta \mp 1$, the lower sign of which being impossible, so long as $\cos\beta$ is positive (which is our hypothesis) in these investigations; and though we might have so extended them as to include negative values of $\cos\beta$, yet nothing in point of generality would have been gained thereby, as only a reduplication of the branches of the curve would have resulted from it), and hence the only solution is $\cos\theta_{II} = 2\cos\beta - 1$. But this final direction of r_{II} is the direction of the tangent at U: and as the same holds for $\cos(-\theta_{II})$, there are two tangents at U equally inclined to the axis.

The value here given accords with the construction given (in XXII. M.). For (figg. 10, 12, 13.), taking P U as radius $= 1$, $U H = U T - T H = 2\cos\beta - 1$; and it is the cosine of the angle K U H by the construction. Hence it fulfils the condition of the equation and gives the tangents at the point U. In the same manner that construction gives the tangents at T.

The combination of the third equations of (97.) and (98.) may also be easily shown to coincide with the construction given for the asymptotes in (XXII. O.); but as the truth of that construction has already been proved at the close of XXIV., it is unnecessary to recur to it here.

But there occurs here a difficulty which is worthy of notice, but which is readily shown, however, to be only apparent. We have seen at (XXII. Q.) that there is another doubly symmetrical system of double points possible for values of β between

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it is quite clear that so long as we require only the differential fraction $\frac{\frac{x}{z}}{\frac{y}{z}}$, we may eleminate the denominators by the usual process *before* we commence the differentiation, even though they involve functions of the variable quantities that enter into an investigation. The same is true of factors not fractional: for

$$d\left(\frac{xz}{yz}\right) = \frac{yz(x\,dz + z\,dx) - xz(y\,dz + z\,dy)}{y^2z^2}$$

$$= \frac{yz^2\,dx - xz^2\,dy}{y^2z^2} = \frac{y\,dx - x\,dy}{y^2} = d\left(\frac{x}{y}\right).$$

It hence follows, that for seeking the second differential coefficient of the curve, we may eliminate by division or multiplication any common factor, integral, or fractional that enters into the numerator and denominator of the first differential coefficient, and hence that the process followed in (XXIII.) is legitimate. But when, on the contrary, each of the terms (numerator and denominator) of the first differential coefficient is to be equated to some other quantity, or to zero (as in finding the multiple points), then all the factors should, by the fundamental principles of algebraic equations, be retained in both. Hence the equations of condition (the first and second in (97.)) must retain *all* the factors which the process of first differentiation introduced into them. The want of due attention to this principle, simple and obvious as it is, has often led to very incomplete, and sometimes very erroneous, enumerations of the characters of certain curve lines.

certain limits; but our equations give no intelligence of their existence or character under the aspect we have yet viewed them.

The general expression of $\frac{dy}{dx}$ given in (85.) combined with (83.), viz.

$$\frac{dy}{dx} = \frac{\sin^3\theta_{,} - \sin^3\theta_{,,}}{\sin^2\theta_{,}\cos\theta_{,} + \sin^2\theta_{,,}\cos\theta_{,,}}$$

with $\cos\theta_{,} + \cos\theta_{,,} = 2\cos\beta$

is not in a *rational form*, and therefore does in itself for all corresponding values of $\cos\theta_{,}$ and $\cos\theta_{,,}$ involve multiple values of tangent of inclination, $\frac{dy}{dx}$. There are, indeed, for each value of $\cos\theta_{,}$ four points in the curve, all whose tangents *ought to be* included in the expression of the values of $\frac{dy}{dx}$. These four values are

$$\left.\begin{aligned} \frac{dy}{dx} &= \frac{\pm\left\{\sqrt{(1-\cos^2\theta_{,})^3} + \sqrt{\{1-(2\cos\beta-\cos\theta_{,})^2\}^3}\right\}}{(\cos\theta_{,}-\cos\beta)^2 - \tfrac{1}{3}\sin^2\beta}\sec\beta \\ \text{and} \\ \frac{dy}{dx} &= \frac{\pm\left\{\sqrt{(1-\cos^2\theta_{,})^3} - \sqrt{\{1-(2\cos\beta-\cos\theta_{,})^2\}^3}\right\}}{(\cos\theta_{,}-\cos\beta)^2 - \tfrac{1}{3}\sin^2\beta}\sec\beta \end{aligned}\right\} \quad . \quad (99.)$$

This method of proceeding, therefore, has the advantage (and in all cases where it is applied the same is true) of giving *directly* the several inclinations of the tangent to the axis of x or y corresponding to any assumed value of $\theta_{,}$: but for the *complete* and *certain* determination of the multiple points of the system, other and additional considerations, as stated in the prefatory remarks to this paper, are necessary. At present we may pass the subject over, as the only remaining multiple points have already been determined, as well as the conditions of their existence, in (XXII. Q.), and proved at the beginning of (XXIV.).

Finally. It was stated at the close of (XXV.), that when the curves themselves were the final ones, or coincided respectively with the magnetic axis, the tangent at M was indeterminate; and the statement is thus rendered evident.

When the curve coincides with the axis $\cos\beta = 0$, and the equation of the curve becomes $\cos\theta_{,} + \cos\theta_{,,} = 0$, or $\theta_{,} = \pi - \theta_{,,}$. Hence $\sin\theta_{,} = \sin\theta_{,,}$, and $\cos\theta_{,} = -\cos\theta_{,,}$. These values of $\theta_{,}$ and $\theta_{,,}$ inserted in (85.) give

$$\frac{dy}{dx} = \frac{\sin^3\theta_{,} - \sin^3\theta_{,}}{\sin^2\theta_{,}\cos\theta_{,} - \sin^2\theta_{,}\cos\theta_{,}} = \frac{0}{0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (100.)$$

Moreover, if we take the successive differential coefficients to infinity, (as the value of n,) we still evidently have

$$\frac{d^n y}{dx^n} = \frac{0}{0}.$$

Hence there is no factor in (100.) which is determinate; or the inclination of the tangent to the axis is at that point *essentially indeterminate*.

All the properties of the magnetic curve that are essential to our future investigations of the physical problem, and none else, have now been fully stated and established. We shall proceed now to their application.

XXVII.—*On the Curve of Magnetic Verticity: or that in any Point of which a Magnetic Needle being placed, it will be directed towards a given Point.*

At each point of the curve of verticity, the tangent to the magnetic curve through that point is directed to the given point, this being the defining character of the curve of verticity. Its polar equations are given in (81.) and (82.), and before we proceed to employ the properties of the magnetic curve to the determination of the properties of this, it is necessary to make one or two remarks on those equations, and deductions from them.

In the first place, the general equation (79.) involves the fourth power of r, and therefore, generally indicates that a line drawn through *any other point* in the plane of the curve will have four values, either all real, two real, or all imaginary, since a transformation of coordinates does not alter its dimension, and therefore the number of its roots; and it is easy to see that in its general form, the separation of its roots would be impracticable, in the literal state of the component data. But as by taking the coordinates in the particular way that is there done, a loss of two dimensions has occurred, or, more properly speaking, a separation of the general equation into two others, the utmost simplification that can possibly arise from the mode of assuming the system of reference, has been here effected. It is very probable that this is the only way in which that separation could have been made; and hence there is little hope of further improvement in the process by the transformation of coordinates, at least so far as origin of r is concerned*.

The point O (Plate XIV. fig. 14.) is a quadruple point, whilst any line drawn through O can cut the curve in only two points besides O. The values of r can therefore, except at this point, be only two, whilst in it they are four. Such is the obvious

* By transposing the origin of θ to the line bisecting the angle TOU, (that is, putting $\alpha_{II} = \varepsilon + \delta$, $\alpha_{I} = \varepsilon - \delta$, and hence $\overline{\theta - \alpha_{II}} = \overline{\theta - \varepsilon} - \delta = \chi - \delta$, and $\overline{\theta - \varepsilon} + \delta = \chi + \delta$,) we obtain a result in some respects better adapted to our final purpose; but still as the equation so transformed offers insuperable obstacles to a *complete* discussion of the curve, and we have been otherwise able to effect without that aid, it is unnecessary to do more than allude to it here. The same may be said of the rectangular equation (76.) itself, when the origin is transposed to T, V, M, or U, and referred either to oblique coordinates coincident with the asymptotes, or to rectangular coordinates bisecting the angles of the asymptotes, or having one coincident with the magnetic axis itself; or again, referring the system to rectangular coordinates through O, one of which is parallel to the magnetic axis; and so on. By one or other of these I have been able to obtain a few properties of the curve, but by no one, nor by all of them together, to deduce anything approaching to a complete development of its properties, the form of its branches, or its singular points. Could it have been so effected, there is no question that it would be the more elegant mode of proceeding, viewed in reference to mathematical symmetry; and for that reason I have spent a good deal of time in attempting it; but after repeated failures, I am compelled to admit that, in the present case, "*fallere et fugere est triumphus.*"

interpretation of equation (79.), or of its component ones (80.) and (81.). The system is, then, in this case, *coincident with* that of the curve, whose values of r are two, and a circle of infinitely small radius. Still we are not entitled to say that the whole system is actually so composed, since by taking the origin of polar coordinates in any other way, we should not find the equations expressive of *that* condition fulfilled. It is essential to keep this principle in view (and it is too often overlooked) in the discussion of the properties of curve lines.

In the second place, let us ascertain if (81.) admits of any infinite values of r. In this case, the denominator is equal to zero, (since the numerator cannot became infinite), or,

$$(\cos\alpha_{ii}\sin\overline{\theta-\alpha_i})^{\frac{1}{2}}-(\cos\alpha_i\sin\overline{\theta-\alpha_{ii}})^{\frac{1}{2}}=0. \quad . \quad . \quad . \quad . \quad . \quad (101.)$$

This is fulfilled by

$$\left.\begin{array}{ll} \text{1st,} & (\cos\alpha_i\sin\overline{\theta-\alpha_i})^{\frac{1}{2}}-(\cos\alpha_i\sin\overline{\theta-\alpha_{ii}})^{\frac{1}{2}}=0 \\ \text{and} & \\ \text{2nd,} & (\cos\alpha_{ii}\sin\overline{\theta-\alpha_i})^{\frac{1}{2}}+(\cos\alpha_i\sin\overline{\theta-\alpha_{ii}})^{\frac{1}{2}}=0. \end{array}\right\} \quad . \quad . \quad (102.)$$

Transpose and cube the first of these; then by expansion and aggregation, we find

$$\sin(\alpha_i-\alpha_{ii})\cos\theta=0,$$

or,

$$\theta=\pm\frac{\pi}{2}.$$

Hence the radius vector parallel to the magnetic axis is infinite, whilst θ is finite; and hence it is either an asymptote or parallel to an asymptote. To ascertain which, let us conceive the origin transferred to V (which, since it is only the *position* of a straight line we are seeking, we are entitled to do): then, since whether V be between the poles or beyond one of them, we have $\alpha_i-\alpha_{ii}=0$ or $\alpha_i-\alpha_{ii}=\pi$, the equation $\sin(\alpha_i-\alpha_{ii})\cos\theta=0$ is fulfilled by the coefficient of the variable, that line is the asymptote. This will, however, also appear from other considerations in the next section.

Again, transpose and cube the second of equations (102.), then we obtain

$$\tan\theta=\frac{1}{2}(\tan\alpha_i+\tan\alpha_{ii}),$$

which indicates the radius vector through the centre M of the magnet, and which, since θ is finite, whilst r is infinite, the line O M is either an asymptote or parallel to one. The determination which would be the case, would, from the equations themselves, be not difficult but rather laborious; and hence we shall employ another method in the next section to show that it is itself the asymptote to two infinite branches of the divergent curve.

Thirdly. To ascertain whether there be any equal values of r. In this case the quantity under the radical symbol becomes equal to zero, or, which is the same thing,

$$\left.\begin{array}{l} (\sec^2\alpha_{ii}-2\sec\alpha_{ii}\sec\alpha_i\cos\overline{\theta-\alpha_i}\cos\overline{\theta-\alpha_{ii}}+\sec^2\alpha_i)(\cos\alpha_i\cos\alpha_{ii}\sin\overline{\theta-\alpha_i}\sin\overline{\theta-\alpha_{ii}})^{\frac{1}{2}} \\ =\sec^2\alpha_{ii}\sin^2\overline{\theta-\alpha_{ii}}(\cos\alpha_{ii}\sin\overline{\theta-\alpha_i})^{\frac{1}{2}}+\sec^2\alpha_i\sin^2\overline{\theta-\alpha_i}(\cos\alpha_i\sin\overline{\theta-\alpha_{ii}})^{\frac{1}{2}} \end{array}\right\} \quad (103.)$$

which is fulfilled either by $\theta = \alpha_{,}$ or $\theta = \alpha_{,,}$. At the points T and U, then, the two values of r for such values of θ as lie to the right and left of these points coalesce, and the radius becomes a tangent to the branches of the curve that meet there. In the same way for values of θ between these points the branches of the curve coalesce, and the radii-vectores form at those points tangents also to these branches of the system. It also immediately appears that there are no other real values of θ which fulfil the condition, and hence there are no other such points besides those now determined.

By inserting these values of θ in (91.) we have in the two cases respectively, as we should anticipate, $r = b \sec \alpha_{,}$, and $r = b \sec \alpha_{,,}$ for the values of r.

XXVIII.—*The Branches of the Curve of Verticity corresponding to the Convergent System of Magnetic Branches.* (Plate XIV. fig. 14.)

Let T, M, U be, as before, the poles and the middle of the axis, and O the centre of the earth. Draw X′ X through O parallel to T U and O V, or Y′ Y perpendicular to it. Also draw M H parallel to O V, and the lines O T, O V, which produce to Q′ and Q respectively; and confining our attention to one side of the middle of the axis M, the corresponding branches may be thus investigated.

There is always one particular convergent curve to some parameter β which will touch the line O U at the point U, and which, since O and U are given points, is determinate and single (XXII. XXIII.). The same is true of O T; and the corresponding values are $\cos \beta = \sin^2 \frac{1}{2} \alpha_{,}$ and $\cos \beta = \sin^2 \frac{1}{2} \alpha_{,,}$.

Those curves only in which β enters as a larger angle, or in which $\cos \beta$ diminishes, can have tangents drawn to them from O. For if any line O S′ Q′ be drawn from O between O T and O U, or to cut the magnetic axis itself between the poles, the curve being wholly concave to O Y, it will *intersect* the curve at Q′; and since there are no points of inflexion (XXV.) between T and U in the branch U Q′, it cannot again meet the curve, and consequently cannot touch it. Hence only the branches of those convergent curves which depend on a parameter β greater than that already specified can be touched by lines from O.

No point of the convergent system of curves of verticity can therefore lie in the region Q′ T U Q.

Let any curve U N be taken to the right of the line O U and above the axis. Then since the curve is convex (its tangent U R at U making a less angle R U Z with the axis than O U Q) to the point O, a tangent can be drawn from O to a point N in it. And since there are no points of inflexion in the magnetic branch, there can be only one tangent so drawn to that branch.

The points of contact, or the points of this branch of the curve of verticity, are always at a finite distance from the line of the magnetic axis. For whilst the magnetic curve itself is finite, all its points are at a finite distance from the axis Z′ Z; and hence all the points of contact of lines from O to it, which constitute the curves of verticity, are also at finite distances from that line. Moreover, when the magnetic

curve becomes infinite, it coincides with the axis Z' Z (XXVI.), and hence the final tangent must also have a point of coincidence at an infinite distance from M with the line Z' Z. Nor can the curve meet the line Z' Z in any point to the right of U, and at a finite distance from it. Since, if it can, the convergent branch of the magnetic curve also meets the axis at that point. But the tangent to every finite branch of a convergent magnetic curve makes a finite angle with Z' Z at U, and having no points of inflexion, it cannot meet the tangent again. But if it meet Z' Z, it must have previously crossed its tangent at the point U. Hence the hypothesis of the curve of verticity meeting the axis Z' Z at a finite distance involves a contradiction. The magnetic axis Z' Z is therefore an asymptote to this branch of the curve of verticity.

Precisely the same circumstances take place in the branch lying above the axis and to the left of T. The convergent curve has therefore two asymptotic branches, the line of the magnetic axis being the rectilinear asymptote to them both; and no other points of the curve lie on the opposite side of that line from O.

In the next place, for the determination of the branch or branches of the curve lying on the same side of the magnetic axis with O:—

For the same reason as before, the line O Q' cannot be a tangent to any one of the curves lying below the axis; and the first curve that can have a tangent is that whose tangent at U coincides with T O; and as the same line is a tangent to the curve above and to the curve below the magnetic axes at their common point, the branches above and below are *continuous* ones.

For any other position, as O N, there is always one convergent magnetic curve which can touch it, as at N'; and the two distances O N and O N' are the two values of r, which correspond to any specified value of θ, as V O N, in the general polar equation of the curve (82.). The point N' will hence trace out another branch of the convergent curve U N' H (H being determined as already specified) corresponding to the asymptotic branch to the right of U, which branch will be finite, and comprised within the rectangle O V U L. Also, since by (82.) there are but two values of r corresponding to each value of θ, there are no other branches to the right of M H besides these two.

Divide T U produced in S', in the triplicate ratio of T O : U O, (or S' U^3 : S' T^3 :: O U : T U), then O S' is a tangent to the curve which passes through O. No curve which passes more remotely from the axis than O can have a tangent drawn to it from O, since O is on the concave side of all such curves, and they have no points of inflexion. Nor can curves passing through T have tangents drawn to them, for reasons before given, till β has become such as to render O T a tangent at T. From that state till the curve passes through O, the point O lies on the convex side of them, and hence from O two tangents can be drawn to each individual curve, one on each side of O, but which coalesce in the single tangent at that point. The curve is hence continuous from U to O, and from T to O; and since they have at that point a coalescent tangent, they form continuous branches not interrupted at O, their point of

union. The branch T O H is therefore also finite, and corresponds to the asymptotic branch to the left of T, as U N' H did to the asymptotic branch to the right of U.

In all these cases, and for all values of θ from α_1 to $-\frac{\pi}{2}$, and from α to $+\frac{\pi}{2}$, we have therefore found two values of r, and by equation (82.) these are all that can exist. Nor can any other tangents than those we have described be drawn to the *convergent* system of magnetic curves; *and hence it appears that the general equation* (82.) *applies to the convergent curves of verticity for no other than the values above assigned.*

XXIX.—*On the Branches of the Curve of Verticity corresponding to the divergent System of Magnetic Branches.*

Let O, T, M, U, and Z' Z denote the same things as in XXVIII.; and draw the line O T, O U, and O M, which continue indefinitely. (Plate XIV. fig. 15.)

Then since each individual divergent curve has an asymptote passing through M (XXIV.), the line O M is an asymptote to some one curve; and as the asymptote is itself a tangent from O to that curve at a point infinitely distant, the corresponding point in the divergent branch of the curve verticity is itself that same point. Whence the line O M is an asymptote to the branch of the curve of verticity; and since the asymptote itself lies in the angle Y M U, the branch to which that asymptote belongs emanates from U, or otherwise the magnetic curve must have crossed the vertical M Y.

There is also one curve which can touch O U at U, determined, as before explained, by the value of β; and there can be no one drawn between this and the produced axis U Z, which admits of a tangent from O; for in that case the curve is concave to O, and has no points of inflexion in that branch.

Take some intermediate position, as O N; then since the point O is on the convex side of the curve U N, a tangent can be drawn; and only one for the curve is convex to M Y, and has no points of inflexion in its finite branch. Nor again, can it be touched in the other part of the branch by a tangent from O, since O is on the opposite side of the tangent U T at the point of inflexion U. Also, as this is the case for all positions of the asymptote within the angle P M U, the series of contacts will trace out a curve, commencing at O, and having O M P for an asymptote; and there is only one branch situated within the angle U M P.

Again, let the curves be continued, whose asymptotes lie within the angle P M Y, as M P'. Then any line from O to the *left* of M will cut the asymptote before it meets the curve, and hence cannot be a tangent to that curve. None of the points of the curve of verticity corresponding to the divergent branches of the magnetic curve are therefore situated within the angle P' M Y.

Attending next to what takes place in the angle U M Y', we see that the first curve that can have a tangent drawn from O is that which has O U for its tangent: for any other would lie on the opposite side of its asymptote from O.

For every position of O N there is always one single magnetic divergent curve which can touch it in the angle U M Y′ as at N′, each of which will be successively found by giving to β successive values infinitesimally near to each other. Also as these curves approach more and more towards asymptotism with M Y′, they approach in their course more nearly to the point M: till when M Y′ becomes the actual position of the asymptote, the curve is reduced to the single *point* M; and it has been shown (XXVI.) that any line, as M O, is a tangent at this point to that individual case of the curve. The curve of verticity which corresponds to the divergent magnetic branches in the angle U M Y′ is, then, finite, and comprised between the lines U O and U M within that angle.

Moreover, as the same line O U Q is a tangent to the infinite branch in the angle P M U and to the finite branch in the angle U M Y′, these two branches are the one continuous of the other.

Proceeding to the angle Y M T, a precisely similar series of circumstances takes place as in its opposite angle U M Y′. The branch has O T for a tangent at T; it proceeds gradually round till it arrives at M and *meets* the branch U N′ M at M. We should be led to expect, from the principle of the continuity of the same law holding at all points in the course of a locus, that the two branches which meet at M are continuous: but as we have no other property of the point M before us except the *indeterminateness* of the tangent to the magnetic curve at that point, we might hesitate, did any conclusion of importance hinge upon it, to affirm the continuity of those branches positively. But by transposing the origin of rectangular coordinates in equation (76.) to M, and investigating the number and position of the tangents at the origin, the question is settled in the affirmative. The process is, however, long and rather intricate; and as we have no occasion to employ the property in our present inquiries, it is unnecessary to give its investigation here.

In the same manner as in the angle Y M U, we may divide its opposite and the only remaining region T M Y′ into two parts T M P″ and P″ M Y′, and consider them in order *.

The first curve which can have a tangent drawn to it from O is that which has O T for its tangent: and as before, the branch thus generated having a common tangent with the branch above the axis, they will form a continuous curve at that point.

To all the curves whose asymptotes lie in the angle T M P″ there can be one tangent drawn, and only one: for the point O is on the convex side of the curve viewed in reference to the tangent T U at the point of inflexion T of the magnetic curve. These will trace out a branch terminating at some point between O and T, as R. Whilst the magnetic curves vary through the interval of their passing from R to O, the point O will be not only on the convex side of the curve with respect to T U, but also between the curve and its asymptote. In this region, then, two tangents can be

* See also Plate XV. fig. 16, where this part of the work is drawn to a larger scale.

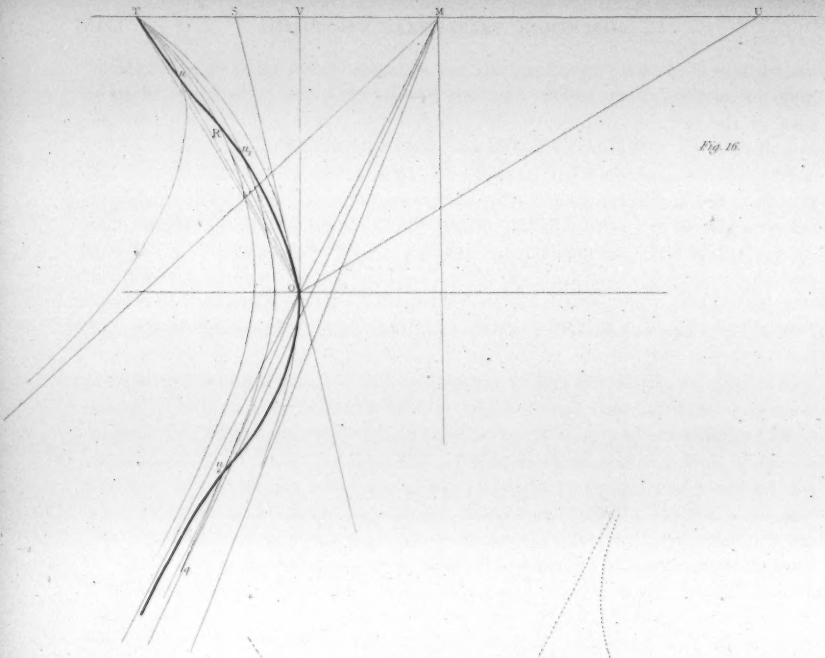
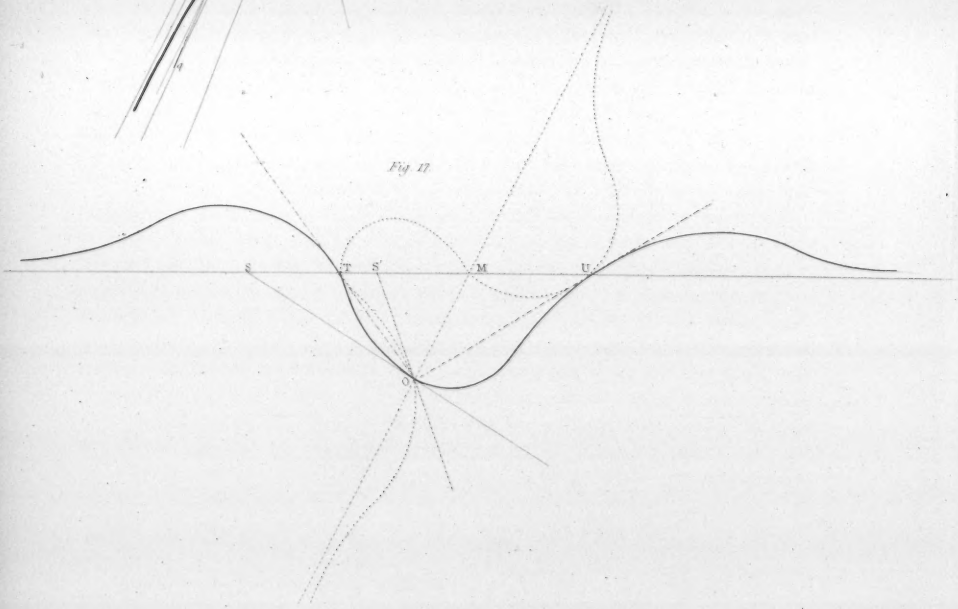


Fig. 16.

Fig. 17.



drawn from O to each curve, and trace out segments coalescing at O, where the two tangents coalesce in one, and the segments therefore also being continuous. Portions, then, of the curve of verticity will lie on both sides of M O in this region: one portion in the angle T M O between R and O, and the other in the angle O M Y'.

Now when the asymptote to the magnetic curve passes through O, we have seen that there is a tangent to it at R. But in the same case, M O P'' is itself a tangent to the same branch at a point infinitely distant. As the magnetic curves approach towards O, there will still be two tangents possible, the point of contact of one becoming continually nearer to O in the angle P'' M Y, and the other in the angle R M P'', till, as before shown, they coalesce continuously in O. The branch lying in the angle P'' M Y', therefore, has M O P'' for a rectilinear asymptote, as the branch in the opposite region had M P.

It would probably be difficult to establish, from any considerations furnished by the properties of the magnetic curves, the utmost angular extent to which the infinite branch lying in the angle O M Q extended from the asymptote M O P''. But recurring to the fact furnished by equation (82.), that for every value of θ which gives one real value of r there is also a second real value; and as for all values of θ comprised between α_{I} and α_{II}, or within the angular region T O U, we have shown that there is one real value of r; and, with the exception of the angular region M O W, (W O W' being the tangent to the magnetic curve at O,) we have established two values; it follows, that for completing the whole series, and fulfilling the conditions of (82.), there must be a second value of r for every direction which a line can take in the angle P'' O W'; or, which is the same thing, the infinite branch will touch the line O W', but can never pass to the right of it; that is, it lies wholly in the angle P'' O W', and never meets the line O W' again after it passes through O.

The course of each curve of verticity is thus fully made out: and it now appears that each is confined within specific and peculiar angular regions referred to lines drawn from O through the magnetic centres of force. Though in itself the divergent system is not required in the present physical problem, yet the separation of them as constituents of the equation (76.) was essential to enable us to ascertain the separate branches of the convergent one, which we have yet to discuss in its application to that problem. For the more complete and ready understanding of the whole system of branches, I refer to the figure of them as the representation of the complete equation (76.), the dotted branches representing the curve of verticity for the divergent branches of the magnetic curve, and the full-lined ones that for the convergent branches, to the consideration of which we shall hereafter return *. (Plate XV. fig. 17.)

* In the cases examined, the point O was without M H: but when it is in that line the divergent curve has its asymptotic branches both approaching the asymptotic on the same side of O, viz. on that on the opposite side of T U from O. The figure (Plate XVI. fig. 18.) will render further verbal detail unnecessary here.

XXX.—*Professor* LESLIE'S *Property of the Magnetic Curve, and a Genesis of the Curve of Verticity founded on it.*

If tangents be drawn from a given point in the magnetic axis to a series of magnetic curves, either convergent or divergent, the locus of the points of contact is a given circle.

For since the point S is given (Plate XVI. fig. 19.), the ratio S T : S U is given, and hence its subtriplicate N T : N U is also given; and the points T and U being also given, the conclusion follows from Lemma I.

The same conclusion also follows from our equation (76.), as in this case $b_i = b_{ii} = 0$ (taking the axis of a parallel to T U), and the equation is converted at once into

$$\left.\begin{array}{l} y^{\frac{1}{3}}\, a_i^{\frac{1}{3}}\{(a_{ii}-x)^2+y^2\} - y^{\frac{1}{3}}\, a_{ii}^{\frac{1}{3}}\{(a_i-x)^2+y^2\} = 0, \\ \text{or } y^{\frac{1}{3}}=0, \text{ and } (a_i^{\frac{1}{3}}-a_{ii}^{\frac{1}{3}})(x^2+y^2) - a_i^{\frac{1}{3}}\, a_{ii}^{\frac{1}{3}}\{a_{ii}^{\frac{1}{3}}-a_i^{\frac{1}{3}}\}x + a_i^{\frac{2}{3}}\, a_{ii}^{\frac{2}{3}}\{a_i^{\frac{2}{3}}-a_{ii}^{\frac{2}{3}}\} = 0. \end{array}\right\} (104.)$$

The former of these is a foreign factor introduced, so far as the locus of a lower order than 76 is concerned, by the eliminations through which that equation was obtained: the latter is the equation of a circle whose centre is in the axis, but not in the form best adapted for use; which would be to refer it to M, and thereby make $-a_{ii} = a_i = a$. As, however, we only require it for constructive purposes and geometrical reasoning, it is unnecessary to examine the equation further; and, except as a verification by a particular case of our general equation (76.), would not have been noticed here. Where it is possible, such verifications, it is admitted on all hands, should be made.

Genesis of the curve of verticity.—Take any point S in the magnetic axis, and find two lines in the triplicate ratio of S T : T U*. Describe the circle D N C, which is the locus of lines inflected from T and U in this triplicate ratio, (Lemma I.,) and let it cut the line O S in N. N is a point in the curve.

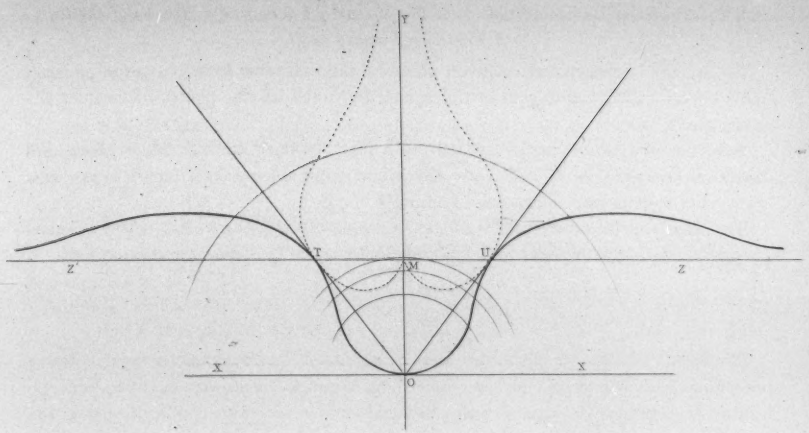
This does not enable us, however, to discriminate the branches themselves of the two classes of curves; nor, therefore, supersede the necessity of the preceding investigations.

XXXI.—*The Circle whose centre is* O *cannot cut the asymptotic branches of the convergent system of the Curve of Verticity in more than two points, one in each branch.*

Let N be a point in the curve of verticity where it cuts the earth's surface such that O N is the direction of the needle tending the centre of the earth. Draw the radiants N T and N U; and let N O intersect the magnetic axis in S. Then S T : S U is the triplicate of the ratio N T : N U, or of $r_i : r_{ii}$. Describe the circle D N C (Lemma I.), which is the locus of the point N corresponding to S; and with centre O and

* Dr. ROGET, Secretary of the Royal Society, has given a very elegant construction of this problem in the Journal of the Royal Institution for February 1831.

Fig. 18.



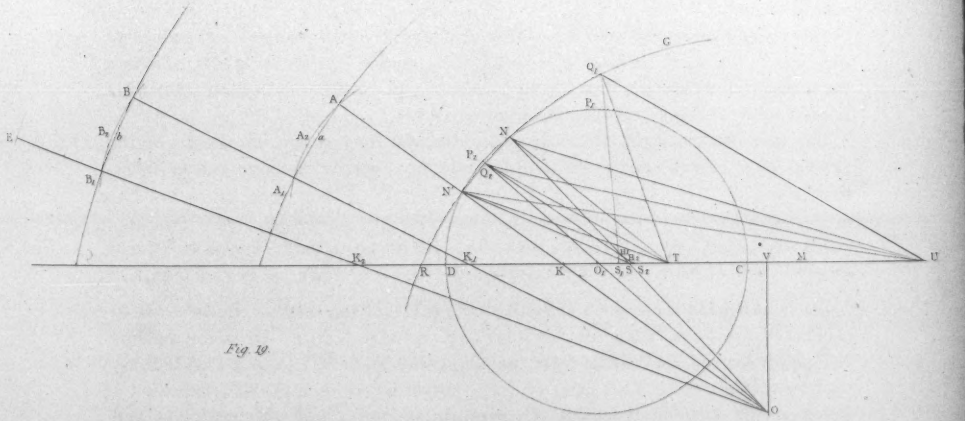


Fig. 19.

distance O N describe the circle R N G, which will be the magnetic meridian. Through the centres O and O_1 (O_1 being the centre of the circle D N C) draw $O O_1$, cutting the circles in Q_2 and P_2: for it will cut them both since it passes through both their centres.

Then (Lemma 8.) the point O_1 always lies to the left of S, and at a finite distance from it. Hence $O P_2$ is greater than O N, and hence the circle whose centre is O cuts that whose centre is O_1 in another point N. Moreover, the line $O O_1$ passing through both the centres, is perpendicular to their common chord N N', and bisects both it and the arcs N' Q_2 N and N' P_2 N in Q_2 and P_2 respectively.

But since the line $S O_1$ is finite in all cases (Lemma 8.), the angles N' O P_2 and P_2 O N are also finite, and hence O_1 K (K being the intersection of N' O with the magnetic axis) is also finite, and, obviously, greater than $S O_1$.

There are, hence, three finite segments of the magnetic meridian in which the needle may be placed, distinct from one another, and each requiring a distinct consideration: and we proceed to prove that in whichever of them placed, except at N, the line of its natural direction will not pass through O, the centre of the earth.

1. *When* Q_1 *is taken in the arc of the circle* R N G *between* N *and* G, *as the position of a magnetic needle.* Then drawing $Q_1 T$, $Q_1 U$, we have, by Lemma 2., the ratio $T Q_1 : Q_1 U$ less than the ratio T N : N U; and hence, drawing the tangent at Q_1 to cut the magnetic axis at S_1, the ratio of $S_1 T : S_1 U$, which is the triplicate of this, is also less than the ratio S T : S U. The point S_1, therefore, lies to the left of S, or more remote from T than S is. Consequently $Q_1 S_1$ cuts N S on the side of the magnetic axis at H_1, opposite to the centre O of the magnetic meridian; and as these lines have once intersected they cannot intersect again, and hence the line $Q_1 S_1$ cannot pass through O; or, in other words, the needle at Q_1 in the arc N G cannot be vertical to the earth's surface.

2. *When* Q_2 *is taken in the arc* N' Q_2 N *of the magnetic meridian.* Draw $Q_2 T$, $Q_2 U$, and the tangent at Q_2 cutting the magnetic axis in S_2. Then (Lemma 2.) the ratio $Q_2 T : Q_2 U$ is greater than the ratio N T : T U; and hence the ratio $S_2 T : S_2 U$ is also greater than S T : T U, these being the triplicates of those. The point S_2 falls, therefore, nearer to T than S does. The line of the needle's direction $Q_2 S_2$ at Q_2, therefore, cuts that at N, at a point H_2 on the side of the magnetic axis opposite to O, and hence, as in the last case, cannot pass through O, the centre of the earth.

3. *Neither can a needle placed at any point in the arc* R N *pass through* O. For, draw $N_1 S$. This is the direction of a needle at N', and hence this does not pass through O. Join N' O, cutting the magnetic axis in K. Then S K is a finite quantity, and hence the ratio K T : K U is less than S T : S U, and the point A of the curve of verticity corresponding to it has its radiants in a less ratio than N' T : N' N. That point, therefore, (Lemma 5.) must be more remote from the point of least ratio (which, obviously, from Lemma 3., lies on the opposite side of the axis T U, the angle T K O being acute), or beyond the point N.

With centre O, and distance O A, describe the circle A *a* A′, and with the ratio A T : A U, the circle A A_2 A_1; and, as in the former cases, these arcs will be finite. Join A_1 O, cutting the magnetic axis in K_1. Then also none of the needles either in A_1 A_2 A or in A G, except that at A, will pass through O, and that at A_1 passes through K_1.

In the same way, by producing O A_1 to B till B T : B U is the triplicate of the ratio K_1 T : K_1 U, we shall have another point B in the curve of verticity; through which, with centre O and distance O B, describe a circle cutting the circle of ratios of B in the points B and B_1; and join B_1 O, cutting the magnetic axis in K_2. And repeat this process as far as may be necessary both as to construction and reasoning.

Then, since the distances T S, S K, K K_1, K_1 K_2, ... are all finite, and the distance T R also finite, a continued repetition of these processes will at length conduct us to a point K_n, either coincident with R, or more remote from T than R is. Let E be a corresponding point in the curve of verticity. Then the segment of the curve joining N and A must lie in the mixtilineal angle formed by the line A N′ and the arc N′ Q_2 N; the segment A B is in the mixtilineal angle B A_1 A, and so on to E. But the arc of the magnetic meridian lies *wholly without* this series of angles, and hence cannot in any one point coincide with the segments of the curve which lies *within* them. The magnetic meridian, therefore, can only cut the asymptotic branch of the curve which lies to the left of T and above the axis, in one single point N.

In the same way, exactly, may it be shown that the magnetic meridian can only cut the other asymptotic branch to the right of U in one single point.

By processes of the same nature it may be proved that the finite branches of the convergent system can never be cut in more than two points by a circle whose centre is O; and that the same is true to the divergent system. But neither of these cases falls within the objects of the *physical problem* under consideration, it would be superfluous to enter upon them here; although for giving completeness to the *geometrical problem* such a discussion would be indispensable. However, after what has been done in the foregoing pages, this portion of it can present no difficulty to the geometer who may be disposed to follow it out, as the reasonings which I have employed in its solution, and which completely apply to all the cases, is essentially the same as that detailed in the case here discussed at length. It is only necessary to observe, that the positions of the finite and infinite branches in the two systems are so situated that, in the divergent system, all four branches, the two finite and the two infinite ones, *may be* cut by the same circle, or only the two infinite ones, depending upon the radius of that circle: whilst in the convergent system, which we have had occasion here to consider, only two of the branches, either the two infinite ones, or the two finite ones, or one of each, can be cut by the same circle.

XXXII.—The consequence of all this investigation, then, is, *that if we admit the hypothesis of two centres of magnetic force situated within the earth, there will be two,*

and only two, points on the earth's surface, at which the needle can take a position vertical to the horizon.

Whether this be the number actually existing on the surface of our earth, we are not at present in a condition to determine. One such undoubtedly there is, and a second is probable, but its position has not been assigned; neither from any observations yet published, can it be even approximately determined, nor, therefore, its existence positively affirmed. I am not aware that any observations give reason to suspect the existence of more than these two; and hence, so far as we can judge from the data before us, the conclusion now obtained as a consequence of two magnetic centres of force, is consistent with the phenomena for which the hypothesis is required to account. *It is therefore a strong argument, in the present state of our actual knowledge of the phenomena of terrestrial magnetism, for the truth of that hypothesis.*

XXXIII.—No particular specification of the cases which can arise from the relative positions of the centre of the earth, and the coordinates of the centres of force, is here necessary; as, except in the case of the magnetic axis passing through the centre of the earth, no considerable simplification of the equation, nor, therefore, any essential variation in the form of the curve of verticity, so far as I have remarked, can arise. When O is in the line $y\,\mathrm{M}\,y_{,}$ (fig. 18.) the branches are symmetrical, it is true, and the form rather more simple than when it is in any other position; and as we remove O to points further on either side from that line, the curve becomes more and more *bizarre*, but still it retains the same *general features* as in its more simple case; and its branches have in all cases the same character, whatever be the coordinates of O with respect to the magnetic poles, and not situated in the same line with them.

If, moreover, we have determined two points on the surface of the earth at which the needle can become vertical, and describe the great circle passing through them, we know that the poles themselves are in this plane, and situated somewhere in the concave angle formed by drawing the radii from those points to the centre. As, however, four quantities are necessary to express the coordinates of those poles, and we have only two conditions given, the actual position of the points themselves cannot be determined from these data. The problem is hence, even when both points of verticity are known, still left *indeterminate*. Nevertheless, by combining these with other observations upon the dip and variation made at different, and still better at distant, places, the problem becomes capable of solution.

Again, if we could determine the points on the earth's surface at which the *intensity* is a maximum (in respect to its contiguous points, in all directions from it), we should obtain other conditions, which united with those of the two points of verticity, from which the positions of the magnetic poles might be determined. To the solution of this problem, the determination of maximum-intensity points, I shall next direct the attention of the Royal Society. The investigations are already completed, and I hope shortly to find leisure to put them into order; and shall only premise here, that as

the results of the hypotheses of like and unlike poles are comprised in the same equation, it will, as was the case in the present discussion, require a peculiar mode of treatment to separate and define the curves of equal intensity belonging to each of them.

XXXIV.—It will be remarked, that in the equations of the magnetic curve, the final position of the needle, or that which it takes when its centre is coincident with either centre of force, is different from that which is exhibited by a needle acted on by an artificial magnet in all our experiments. This might at first sight seem to throw some doubt on the validity of the principle employed in deriving the equation of the magnetic curve; but a little reflection will convince us that the conditions of such experiments are different from those which obtain in the case before us.

In all our experiments, the length of the needle itself bears a finite ratio to the distance of its centre from the poles of the magnet upon which we experiment, and hence the action becomes mutual the two magnets, and the system of actions being thus rendered compound, its results must be expressed by a more complicated formula than in the case we have supposed, and from which our equations of the magnetic curve have been derived. This complexity of the conditions implies a corresponding complexity of the equation by which they are expressed; but uniform experience has shown that as we diminish the length of the needle, and increase its distance from the poles, the observed results are more nearly approximative to the results of the hypothesis from which we have started. In the case of the curves exhibited by iron filings strewed on a paper above a bar magnet, the approach of the observed curve to the calculated one is very close*. But in this case the length of the needle (or magnetized particle of the iron) is very small in comparison with the magnet and with its distance from the magnetic poles; and, moreover, has in itself so little magnetic intensity as to exert an insensible reciprocal influence on the state of the bar itself. This is precisely a miniature representation of the case of a small needle acted on by the terrestrial magnet, though the ratio of the particle of iron to the magnetic bar is many times greater than that of a needle to the terrestrial magnet; and hence the discrepancy between the observed and calculated result in the former case is many times greater than in the latter. No ground of exception to our plan of inquiry can therefore be found in this circumstance, but rather a confirmation of its validity in reference to the use we have made of it.

* Leslie's Geometrical Analysis, p. 405.

Royal Military Academy, Woolwich,
January 27th, 1836.

X. *On Voltaic Combinations. In a Letter addressed to* Michael Faraday, *D.C.L. F.R.S., Fullerian Prof. Chem. Royal Institution, Corr. Memb. Royal & Imp. Acadd. of Science, Paris, Petersburgh, &c. By* J. Frederic Daniell, *F.R.S., Prof. Chem. in King's College, London.*

Received January 26,—Read February 11, 1836.

My dear Faraday,

YOU know how deep an interest I have taken in your "Experimental Researches in Electricity," and how zealously I have availed myself of the opportunities, which you have ever kindly afforded me, of profiting by your oral explanation of such difficulties as occurred to me in the study of your last series of papers in the Philosophical Transactions. Having been early impressed with the conviction that the science of chemistry would date, from their publication, one of its great revolutions and eras of fresh impulse, I have been careful not only to store my own mind with the new facts and reasonings which they contain, but to impress them upon my pupils in my class room; and for this purpose I have been led to contrive some new apparatus and forms of experiments, by which the principles which you have promulgated have been verified, and, I think, in some instances demonstrated to advantage. In thus working in the mine which you have opened, you will be the last to be surprised if I should have moreover stumbled upon some threads of ore which you may have passed by, or temporarily abandoned, whilst following the main lode; and you will not be displeased that I venture to submit to your judgement whether there be enough of novelty or importance in the following observations to render them worthy of the attention of the Royal Society.

One result, I know, will gratify you; namely, that amongst the almost innumerable tests to which I have exposed your great discovery of the *definite chemical action of electricity*, I have found no fact to militate against it; and you will the more rejoice should I succeed in proving to you that, under the direction of this fundamental principle, I have been led to the construction of a voltaic arrangement, which furnishes a constant current of electricity for any length of time which may be required; and have thus been enabled to remove one of the greatest difficulties which have hitherto obstructed those who have endeavoured to measure and compare the different voltaic phenomena, viz. the variableness of the action of the common batteries.

You are aware of the vexatious accident by which I lost the original notes of my

experiments during the last year, which will prevent my giving you such full details of some of the results as I could have desired, although I have been careful to repeat all those in which measures are of fundamental importance.

I shall first beg to direct your attention to an arrangement which I purpose to distinguish by the name of the *Dissected Battery*; it has answered an analogous purpose to me of one of those optical instruments, by looking through which a multitude of confused lines assume a regular disposition; and many detached facts, well known before, have become clearer and of new importance, from their connexion and comparison with each other by its means. The battery consists of ten glass cells, a section of one of which is represented in the accompanying figures.

(a, b, c, d), Plate VIII. fig. 1. is a foot of solid glass, containing a cavity (e, f, g, h), the upper part of which is fitted with a stopper (g, h). Through this stopper the stems of the two plates (i, j, k, l, m, n) pass into the lower part of the cavity, which is divided into two cells by the partition (o, p), and each of which contains mercury, into which the wires respectively dip. This arrangement admits of the plates being changed at pleasure with little difficulty. The plates may be connected together, or with the plates of other cells, by means of wires (p, q,) passing through the lateral holes (t, u) and dipping also into the cups of mercury. To the glass foot thus arranged a glass shade (v, w, x, y, z, z) is fitted by grinding, and constitutes a cell for the reception of the liquid. A graduated glass jar (A, B) may be suspended over either plate by means of a brass clip, proceeding from a rod placed by the side of the cell in the manner represented by fig. 2, which is a perspective drawing of a circular arrangement of ten such cells.

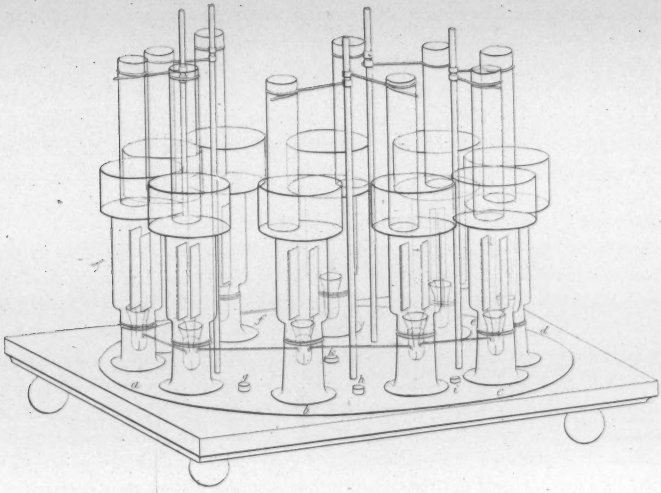
Fig. 3. represents the section of a cell which is adapted to the same purposes, but is less expensive in construction. It is supported in a perforated table (C, D) by its projecting rim (v, w, y, z), and the stems of the plates pass through the glass stopper (a, b, c, d) into the exterior mercury cups (o, p), by means of which all the necessary connexions may be made.

The active elements of the circuit which I adopted as standards of comparison were, for the metals, platinum and amalgamated zinc plates of the dimensions 3 inches by 1 inch; and for the electrolyte, water acidulated with sulphuric acid in the ratio of 100 parts by volume of the former to $2\frac{1}{4}$ of the latter (sp. gr. 1027·5), proportions which I adopted for the purpose of connecting some of my experiments with yours*.

This dilute acid was almost without local action upon the amalgamated zinc. When the plate was wholly immersed, its surface became, after some time, covered with bubbles of hydrogen gas, which strongly adhered to it; but after twenty-four hours it was found to have lost but a very small fraction of a grain of its weight. The force of heterogeneous adhesion, by which hydrogen and other gases are thus retained upon the surface of metals, I am disposed to regard as exerting a very important influence

* Experimental Researches, x. 1127. 1128.

Fig. 2.



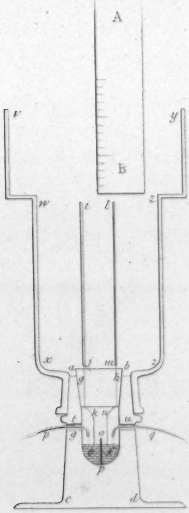


Fig. 1.

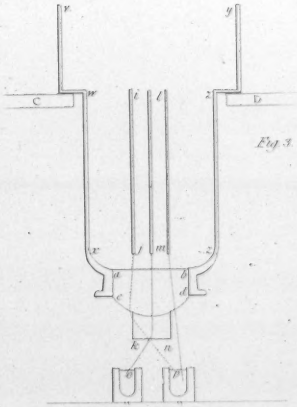


Fig. 3.

Dissected Battery.

both upon *local* and *current* affinity; and in the case before us I have little doubt that it is the affinity of the hydrogen thus tied down, as it were, to the surface of the plate which puts a stop to the decomposition of the water by the zinc. When a small quantity of nitric acid was added to the acidulated water, the same plate was entirely dissolved in a few hours without the extrication of any gaseous matter. It is probable that the elements of the nitric acid in this case acted by entering into combination with the hydrogen as it was evolved, and thus removing its opposing attraction. The decomposition also uninterruptedly proceeds when the hydrogen is evolved at a comparatively remote point in the voltaic circuit. The well-known energy of gases in what is called their *nascent state* may be referred to the same cause, namely, their adhesion to the surfaces against which they are evolved, by which their elasticity is counteracted; much in the same way, and by the same species of force, that the soluble gases are kept down by solution in water, and are enabled to exert their affinities for other forms of matter to advantage*.

Of the strength of the affinity of hydrogen in this state we have abundant evidence in the facility with which it deoxidates the oxide of copper, when sulphate of copper is exposed to it, and precipitates the metal upon the negative plate of a voltaic circuit. I shall also have occasion to prove to you in the course of this paper, that it is sufficient to deoxidate the oxide of zinc under similar circumstances; and we see at once the impossibility of the oxidation of the metal proceeding in the presence of an agent capable of abstracting oxygen from it when already combined.

When a metallic communication was made between the two plates of a cell in this its normal state, the evolution of hydrogen from the platinum was abundant, and at about the rate of three cubic inches in a quarter of an hour.

Before I proceed to describe the principal series of experiments which I instituted with this apparatus, it may perhaps be worth while to point out the facility with which, by its means, some of the principles of the voltaic battery may be demonstrated, which are the most elementary, but at the same time the most difficult to illustrate.

Taking one of the cells charged with water, acidulated with sulphuric acid, to which a small quantity of nitric acid has been added, its total inaction previous to the completion of the circuit is strongly contrasted with the torrents of gas which rise into the graduated jar placed over the platinum plate after the wires (p, q) are

* There is one well-known experiment which has given rise to much ingenious hypothesis, which may also receive its simplest explanation from the action of this force of heterogeneous adhesion, namely, the so-called metallization of ammonia. When mercury is made the negative electrode of a voltaic circuit in connexion with a strong solution of muriate of ammonia, hydrogen and ammoniacal gases are both evolved in contact with it: and not only does the adhesive force restrain the elasticity of the gases, but the latter also reacts upon the cohesion of the fluid metal, and causes it to expand and increase its volume in the manner which has been so often described. The results of the most careful examination of this amalgam by the first chemists are in entire conformity with this opinion.

brought into contact. If we take a second cell charged in the same way, and connect together the two zinc plates and the two platina, all is quiescent. No current can circulate, because the tendency of the generating plate in one cell to originate such a current in one direction of the circuit, is opposed by an equal tendency of the generating plate of the other to form one in the opposite direction. Substituting now, for the second cell, a third, charged with a solution of iodide of potassium and starch instead of the acidulated water, although it is easy to demonstrate that there is a similar tendency to form an opposing current, yet the current originating from the higher affinities of the first cell is sufficient to overcome that from the weaker affinities of the last cell, and iodine is abundantly precipitated upon the platinum plate of the latter, and immediately detected by the deep blue colour. Again, detaching the last cell, and connecting together its two plates in single circuit, its own current is established in the opposite direction, and the blue colour of the iodine speedily disappears from the platinum plate, under the influence of the hydrogen, which now takes its place. Thus your fundamental proof of the non-necessity of the contact of dissimilar metals for the establishment of an electric current is rendered very apparent and striking.

Recurring again to the two cells similarly charged with acidulated water, if instead of connecting them by their two platinum and their two zinc plates, we connect each platinum with the zinc plate of the other, the direction of the currents from both generating plates will coincide, and each, instead of opposing, will assist the other; and the completion of the circuit will be manifested by the evolution of gas in both the collecting jars.

I began my series of experiments by charging each cell with water acidulated with sulphuric acid alone, in the proportion which I have just stated; and upon connecting them in single circles I found that the amount of action of each, as measured by the quantity of hydrogen collected in their respective voltameters, in equal times, differed very materially, notwithstanding the apparent similitude of their circumstances. The difference between the highest and the lowest was nearly a third; and the inequality may probably be ascribed, partly to slight differences in the distances of the generating and conducting plates which it was not easy to avoid, and partly to differences in the amalgamated surfaces. Upon connecting them altogether in a single circular series, the inequalities disappeared: the amount of gas from each was equal, but was found to have fallen to that from the weakest cell.

The circular arrangement of the cells of the battery, fig. 2., admits of their being combined together in various ways with the greatest facility by means of small cups of mercury (*g*, *h*, *i*) placed at proper intervals. My next disposition was to connect all the platinum plates together by wires radiating from them to a central cup (*k*) of mercury, and all the zinc plates by wires dipping into a ring of the same metal placed in a groove (*a b c d e f*) surrounding the whole arrangement. In this state of things no action was of course manifest, for there was no complete circuit; but upon

making a connexion by means of a wire between the central cup and the exterior circle of mercury, the current was enabled to circulate, and was manifested by the simultaneous evolution of gas from all the cells. The inequality of action became again apparent, and the differences between the cells was nearly the same, as when they were connected in separate single circuits.

The result of this arrangement was virtually the same as that of a single pair of platinum and zinc plates, exposing the same extent of surface as that of the ten pairs added together; and the whole quantity of force generated in each cell must have passed through the single wire which connected the centre with the circumference in the progress of its circulation.

By these two arrangements of the same elements of the battery, the relations of quantity and intensity in the circulating affinity are placed in a very striking point of view. Setting aside, as we may do in our present comparison, the inequalities to which I have just referred, the same quantity of force is generated and expended in the cells in both combinations: but when in series, no connecting wire conveys more than the quantity generated in one cell to the next; whilst, in the single circuit, the whole quantity generated in all the cells must pass through the central connecting wire. This difference of quantity may be manifested by the elevation of temperature occasioned in a fine platinum wire when made the connecting medium of the latter combination, whilst the same wire will remain cold when employed for the same purpose in the former. On the other hand, the intensity of the force, though small in quantity, derived from the repeated impulses of the disposition in series, is shown by its projecting itself in the form of a spark through a break in the conductor; whilst ten times the quantity of the same force is effectually arrested in the simple circle by the slightest disruption of the continuity of its metallic path for want of this accumulated energy.

I proceeded next to combine the cells together in pairs, two platinum plates being connected, and two zinc; and the five pairs were afterwards arranged in series by wires leading from each pair of zinc to the adjoining pair of platinum. The irregularity of action again disappeared; the amount of gas was equal in all the voltameters; slightly exceeded that from the single-series arrangement; but did not quite come up to the amount of the most energetic single cell. The arrangement was equivalent to a series of five plates of double the standard size, and the quantity which circulated was determined by the least efficient pair.

Leaving one pair of cells thus connected together, the others were again disunited, and recombined with it in single series; the effect being that of a plate of double size interposed in a compound circuit with eight single. The gas collected in each of the voltameters of the double cell was exactly half of that in the several voltameters of the single cells; proving that the double plate had been reduced in efficiency to the exact standard of the single plates by its combination with them. The regulating effect of the voltaic series, by which all irregularities of its elements are equalized, is

strikingly exemplified in these experiments; and it is clearly manifest from them, as a primary law, that the circulating current must be uniform throughout its course.

In these arrangements, every cell had been a *generating* cell, and added something either to the quantity or intensity of the circulating force: I now proceeded to ascertain the effects of various *retarding* cells upon the elements of the battery. For this purpose I connected the cells together in single series, substituting a platinum plate for one of the zinc; and the obstacle reacted upon the whole series: the action was reduced by more than one third, and the quantities of gas collected from each generating cell was exactly equal to the hydrogen collected from the retarding cell. Repeating the experiment with a similar change in the next cell, the quantity of hydrogen in all the voltameters was equal, but reduced to little more than one tenth, and the current was entirely stopped by three retarding cells to seven generating cells.

When, instead of changing the zinc plate of one of the cells, I reversed its position in the series by turning the latter round, the more active opposition of the tendency in that cell to establish a contrary current was indicated by the decline of the quantity of gas in the nine voltameters of the regular cells to about one fourth; while it is remarkable that the quantity of hydrogen collected from the reversed zinc plate was considerably less. In several repetitions of this experiment this deficiency always occurred; and I also ascertained that the quantity of oxygen evolved from the corresponding platinum was the equivalent of the lesser quantity, and not of that which was evolved from the regular cells.

When one of the zinc plates was removed from the regular series, and replaced with a platinum plate which had been previously coated, by voltaic influence, with metallic copper, the phenomena were striking and instructive. No gas at first was evolved from the coppered plate, but it became slowly oxidated, and the progress of the oxidation could be traced by the gradual blackening of its surface. The oxide again was gradually dissolved, and the bright white surface of the platinum made its appearance, and oxygen gas began to be disengaged. At that moment the current received a check, which was quite appreciable by the voltameters.

As the progress of my experiments would require the use of an independent voltameter, I thought it desirable previously to determine its retarding power by comparison with that of a single retarding cell; and for this purpose I substituted one of your upright construction for one of the cells of the battery in single series: and I found, that notwithstanding its plates were only three inches long by six eighths broad, their nearer approximation counterbalanced their deficiency of surface; and the quantity of hydrogen which the instrument indicated was exactly equal to that in the voltameters of the retarding cell. The measure of the effects of different degrees of approximation in the plates of voltaic combinations was one of the objects of my experiments; but this I have for the present postponed, for reasons which will soon be apparent.

The next variation of the standard elements of the battery was to place conducting plates in connexion with one another on each side of the generating plates. This was easily effected by making both the plates, *ij*, *lm*, (fig. 3.) of platinum communicate with one mercury cup (*p*); while the amalgamated zinc plate placed between them communicated with the other cup (*o*). This arrangement was perfectly analogous in principle to the double or WOLLASTON plates of the common battery. Upon combining the cells thus arranged in single series, nearly as much gas was collected from each plate, on the opposite sides of the generating plates, as had been collected in an equal time from the single plates; the double surface of the platinum having enabled the single zinc plate to decompose very nearly a double portion of water.

During this experiment, and its frequent repetitions, I remarked that the bubbles of hydrogen arose not only from the surfaces of the platinum which were immediately opposed to the zinc, but from the opposite surfaces likewise; and when a little sulphate of copper was added to the liquid in the cells, both faces of the plates became coated with the reduced metal.

Almost the only serious difficulty with regard to your chemical theory of voltaic action which ever occurred to me, has arisen from the well-known case of voltaic protection, in which a small piece of zinc or iron has been found to defend from corrosion a surface of copper so many hundred times its own superficial dimensions. I was unable for a long time to understand how the hydrogen, which could only be the equivalent of the oxygen taken up by the former metals, became spread over so disproportionate a surface; and this effect seemed to me more intelligible upon the hypothesis of some condition of the copper, analogous to those produced by electrical or magnetic induction, than upon any known chemical principles. The following means of determining the question by experiment now occurred to me.

I took a silver plate, fifteen inches square, and, placing it in a shallow trough, covered it with the dilute sulphuric acid, to which a portion of sulphate of copper had been added. I then supported an amalgamated zinc wire of about one eighth of an inch in diameter, so as to allow one of its ends just to rest upon the centre of the plate. The instant the two metals came in contact, a circular spot of metallic copper was thrown down upon the silver, and rapidly spread itself in such a way that in a few hours it formed a well-defined circle of six inches in diameter. Particles of copper could be traced even beyond this; but none appeared to have reached the edges of the plate.

I varied this experiment by soldering to the centre of a silver plate, 8 inches by $5\frac{1}{2}$ inches, a small piece of amalgamated zinc $\frac{1}{4}$ths of an inch long by $\frac{3}{16}$ths wide, and placed it perpendicularly in a jar, and covered it with the solution of copper in acidulated water. The copper immediately began to precipitate itself upon the silver, in the form of an oval surrounding the zinc, and gradually extended itself equally on all sides. There was no extension of the copper on the upper side, to indicate that the hydrogen, the reducing agent, had been carried upwards by its levity; but it spread

itself downwards and laterally at the same rate, till in a few hours it reached the edge of the plate. It then made its appearance upon the opposite surface, and ultimately both sides were completely coated; but the deposition decreased in thickness as it receded from the central zinc. This effect, in a voltaic combination, of a small generating surface upon a large conducting one, is strikingly contrasted with the result when we reverse the circumstances of the arrangement; for by causing a rod of platinum to rest by its end upon a large surface of amalgamated zinc, covered with the acidulated water, hydrogen of course escapes from the former metal; but the oxidation of the latter is so local, that a hole is eaten through it at the point of contact.

These experiments set the question at rest which they were meant to resolve; and I think, upon attentively considering the matter, that we may find the cause of this great diffusion of the hydrogen in that force of *heterogeneous adhesion* to which I have already referred the power of nascent gases, reacting upon, or acting in conjunction with, its elastic or self-repulsive force. Under the influence of these two forces the gaseous matter seems to extend itself upon the metal, much in the same way that a drop of volatile oil spreads itself over a large surface of water; and it seems not improbable that the efficiency of the conducting plates of voltaic arrangements may in part, if not wholly, depend upon the extent of surface which they afford for this diffusion of the gas, by which it is carried beyond the sphere of reaction upon the generating plates. To test the accuracy of this hypothesis, I reduced the width of the zinc plates in the battery; and beginning by halving, and then diminishing them to a quarter of their original dimensions, I ultimately found that amalgamated wires of one eighth of an inch diameter and three inches long, were as efficacious as the inch-wide plates; and that under their influence the conducting plates evolved as much gas in equal portions of time as when connected with the larger generating surfaces.

Amongst the numerous experiments which I have tried by the substitution of different metals, both as generating and conducting plates, for the normal plates of the battery, I must only detain you on the present occasion with the results of two. The first was the change of common zinc plates for the amalgamated, when strong local action took place, which, however, did not appear to interfere with the current affinity when the cells were connected in single series; for while the disengagement of hydrogen from the zinc was at the highest, as much gas was evolved from the platinum as when they were combined with the amalgamated metal.

The second was the substitution of wrought iron for the amalgamated zinc plates. I had expected, from theoretical speculations upon its low equivalent number, that iron would have proved a very efficient generating metal; and my surprise was great upon discovering that its action in the battery was almost null. A slight local action took place upon the plates themselves, but the evolution of gas from the platinum was scarcely apparent, and quite immeasurable. This being the case, I converted them into conducting plates by removing the platinum, and opposing to them amalgamated zinc plates; and I was no less surprised to find that in their new office they

greatly exceeded in efficiency the platinum plates themselves; the quantity of gas collected in the voltameters in series being nearly double in amount to that from the battery in its normal state.

I repeated this experiment with fresh iron plates, and ascertained that previous opposition to the platinum had greatly contributed to this energetic action of the first set: the disengagement of gas from the second but little exceeded that from the normal plates.

I am inclined to believe that we may again find the explanation of these phenomena in the attraction of heterogeneous adhesion; and I think it probable that the hydrogen gas adhering with less force to the surface of the iron than to that of platinum, is more readily carried out of the sphere of action when evolved in the process of circulation upon the former than upon the latter. The efficacy derived from the previous combination of the iron with platinum may be ascribed to the cleansing of its surface, or possibly to some difference of mechanical structure developed in this particular position.

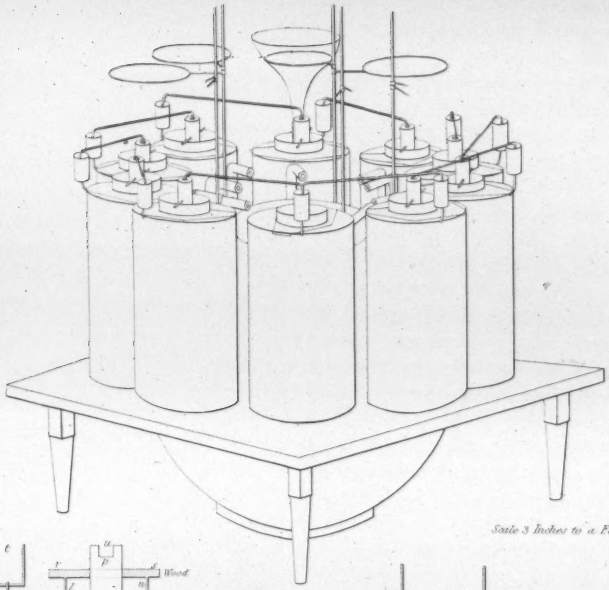
My attention was next turned to the effects of certain changes in the condition of the electrolyte employed in the excitation of the battery; and now it was that I began to experience those, apparently capricious, changes in the force of the current which soon obliged me to give up all hope of rendering the results comparable with one another in the present form of its construction. When I added to the acidulated water a volume of nitric acid equal to that of the sulphuric acid employed, the quantity of hydrogen evolved from the conducting plates of the cells was greatly diminished, and was irregular; but when the voltameter was substituted for one of the cells, the quantity of hydrogen which it indicated was at first nearly treble that from the battery in its normal state. This action, however, speedily declined by a quantity which was quite appreciable at intervals of five minutes. By breaking the connexion of the circuit this energy was partially recovered, and that even during the short cessation of action which arose from the mere turning of the voltameter. When the charge of nitric acid was increased, the evolution of hydrogen from some of the cells was wholly stopped, whilst in others it continued to a small but unequal amount; the quantity of the circulating current was at the same time increased, but rapidly declined as the action continued.

My efforts were now turned to discover the cause of this perplexing variation; but I shall not trouble you with the numberless experiments which I made upon the subject, any further than may be necessary to elucidate the steps by which I was ultimately led to its detection. I allowed the battery to exhaust itself by leaving it in connexion for thirty hours, at the expiration of which time scarcely any gas was evolved in the voltameter. I then removed the platinum plates and substituted fresh plates of the same metal; and when the connexions were made, the action was renewed with nearly the same activity as at the first; the falling off, however, was

rather more rapid than before. Even iron plates when substituted for the exhausted platinum renewed the action with great effect.

The decline thus seemed to depend upon some state taken up by the conducting plates, and this I endeavoured to remove in various ways. I polished them with rotten stone; I heated them red hot; and I boiled them in a strong solution of potassa, but without any decided effect. Boiling them, however, in nitric or muriatic acids completely restored them. Thus, frequently renewing the plates but retaining the old charge of acidulated water, the first action was always very brisk, but lasted for shorter and shorter periods, till the decline at last became almost immediate. Upon examining the plates after a long-protracted action of this kind, I found a roughness upon their edges and faces which conveyed the idea of a corrosion of the platinum. A more attentive investigation proved this to arise from a deposition upon them of metallic zinc; and upon leaving the battery in connexion for eighteen hours the coating increased to such a degree that it could be detached in large flakes. The precipitated metal was deposited chiefly upon the surface opposed to the zinc plate, but in some had extended considerably on the opposite side. It had mostly a beautiful mammillated appearance; but in one or two instances a crystalline structure was visible with a lens. I ascertained the weight of zinc thus deposited upon a plate by removing it with an acid, and found it 27·86 grains. The metal was pure, and when detached did not dissolve in the dilute acid till touched with a piece of platinum, when abundance of hydrogen gas was immediately given off from the latter. A similar deposition, in equal abundance, was formed upon iron when the conducting-plates were made of that metal. It originated doubtless from the oxide of zinc formed by the action of the battery at the generating-plate, and reduced by the *nascent* hydrogen on the conducting-plate; and its varying quantity and accumulation was amply sufficient to account for the variations and ultimate annihilation of the circulating force. In some instances, indeed, the casing of the platinum was so entire as to have the complete effect of one zinc surface opposed to another. This local deoxidating power of the hydrogen is exerted upon every oxide within its reach, and is more or less injurious to the action of the battery according as the resulting deposition upon the plates is more or less capable of generating a counter current by its reoxidation. The most injurious of all precipitations must, of course, be that of zinc upon the conducting-plate; and it must take place with a rapidity proportioned to the quantity of oxide held in solution by the electrolyte. In the common construction of the voltaic battery the solution of the metal is very rapid, on account of the local as well as the current action, and the deoxidating action must very soon be established; and the reason of the incrustation of zinc never having been remarked, is doubtless the extreme facility with which it is redissolved upon breaking the circuit. By its close contact with the copper it forms simple voltaic circles, and is under circumstances the most favourable for its removal. The momentary recovery of the strength of the

Fig. 2.



Scale 3 Inches to a Foot.

Fig. 1.

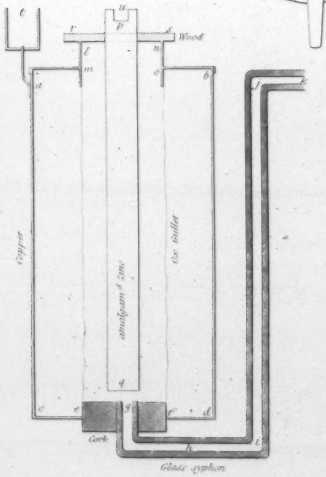
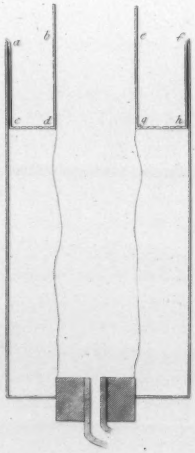


Fig. 3.



Scale 6 Inches to a Foot.

Constant Battery

J. Martin del.

C. Basire sc.

battery by breaking the metallic contact is now perfectly intelligible; and we can no longer be at a loss to understand all the phenomena of RITTER's secondary piles, the effects ascribed to polarity by M. BECQUEREL, and the experiments of MARIANINI and A. DE LA RIVE upon this interesting subject.

I have remarked in the dissected battery, that when any oxide of copper was present in the solution, it was thrown down in preference to the oxide of zinc; and that plates which had become covered with the former metal were, in some degree, protected from depositions of the latter. Upon adding, however, sulphate of copper in any considerable quantity to the liquid in the cells, notwithstanding the amalgamation of the zinc, there was local action enough upon that metal to disengage hydrogen; which, in however small a quantity, was sufficient to commence the precipitation of the copper upon it. Single circles were thus immediately formed by the two metals, and local action increased to such a degree as speedily to cover the zinc with reduced copper. The addition of nitrate of mercury is not liable to the same objection; and when I made use of this salt in the cells, the platinum plates soon became covered with metallic mercury in such quantities as ultimately to fall from them in drops. I did not carry this experiment far enough to ascertain whether the oxide of zinc would ultimately have become reduced with the oxide of mercury: but on one occasion, when some oxide of copper was present, the platinum plate became incrusted with a beautiful white soft amalgam; which upon analysis proved to be an amalgam of copper.

Notwithstanding the unfavourable effect upon the circulating force of this secondary deoxidating power of the *nascent* hydrogen, where the ultimate result is the deposition of a solid body having a strong affinity for the oxygen of the electrolyte, there can be no doubt that the removal of the hydrogen by combinations which do not give rise to such precipitation greatly promotes the efficiency of the current: and I have had abundant opportunities of confirming your explanation of the good effects of the addition of nitric acid to the battery-charge by its abstraction of the hydrogen*. It is, moreover, quite certain, that not only does the oxygen of the acid act in this way as a *dehydrogenating* agent, but the nitrogen also; for by neutralizing the exhausted charge of one of the cells of the battery with carbonate of soda, I not only obtained abundance of ammonia by distillation with lime, but a copious precipitate of ammonio-muriate of platinum with a solution of the chloride of that metal.

Having thus detailed, I hope sufficiently, the progressive steps by which I was led to the discovery of the cause of the variations and decline of the force of the voltaic battery, as well as of certain principles upon the application of which I have relied for the counteraction of its injurious effects, I will not detain you with any account of the less successful attempts which I made for that purpose, but proceed at once to describe a voltaic combination which I trust you will think worthy of the name of the CONSTANT BATTERY.

Fig. 1. of Plate IX. represents a section of one of the cells, ten of which are shown

* Experimental Researches (1021.).

in connexion at fig. 2.: (*a b c d*) is a cylinder of copper six inches high and three and a half inches wide; it is open at the top (*a b*) but closed at the bottom, except a collar (*e f*) one and a half inch wide, intended for the reception of a cork into which a glass siphon-tube (*g h i j k*) is fitted. On the top (*a b*) a copper collar corresponding with the one at the bottom rests by two horizontal arms. Previously to the fixing of the cork siphon-tube in its place, a membranous tube formed of a part of the gullet of an ox is drawn through the lower collar (*e f*) and fastened with twine to the upper (*l m n o*); and when tightly fixed by the cork below, forming an internal cavity to the cell communicating with the siphon-tube in such a way as that when filled with any liquid to the level (*m o*) any addition causes it to flow out at the aperture (*k*). In this state, for any number of drops allowed to fall into the top of the cavity, an equal number are discharged from the bottom. (*p q*) is a rod of cast zinc amalgamated with mercury, six inches long and half an inch diameter, supported on the rim of the upper collar by a stick of wood (*r s*) passing through a hole drilled in its upper extremity: (*t*) is a small cup for the reception of mercury, by which, and the cavity (*a*) at the top of the zinc rod, various connexions of the copper and zinc of the different cells may be made by means of wires proceeding from one to the other.

In fig. 2. the ten cells are represented as connected in single series, the zinc of one with the copper of the next. They stand upon a small table in a circle with the apertures of the siphon-tubes turned inwards, surrounding a large funnel communicating with the basin underneath for the reception of any liquid which may overflow. A smaller funnel is supported over the internal cavity of each cell by a ring sliding upon rods of brass placed between each pair of cells. One of these only is shown in the drawing to avoid the crowding of the sketch.

In the construction of this battery, I have availed myself of the power of reducing the surface of the generating plates to a *minimum*, the effective surface of one of the amalgamated zinc rods being less than ten square inches, whilst the internal surface of the copper cylinder to which it is opposed is nearly 72 square inches. My principal objects have been, to remove out of the circuit the oxide of zinc, which has been proved to be so injurious to the action of the common battery, as fast as its solution is formed, and to absorb the hydrogen evolved upon the copper without the precipitation of any substance which might deteriorate the latter.

The first is completely effected by the suspension of the rod in the interior membranous cell, into which fresh acidulated water is allowed slowly to drop from the funnel suspended over it, and the aperture of which is adjusted for the purpose; whilst the heavier solution of the oxide is withdrawn from the bottom at an equal rate by the siphon-tube. When both the exterior and interior cavities of the cell were charged with the same diluted acid, and connexion made between the zinc and the copper by means of a fine platinum wire $\frac{1}{100}$th of an inch in diameter, I found that the wire became red hot, and that the wet membrane presented no obstruction to the passage of the current.

The second object is attained by charging the exterior space surrounding the membrane with a saturated solution of sulphate of copper instead of diluted acid: upon completing the circuit the current passed freely through this solution; no hydrogen made its appearance upon the conducting plate, but a beautiful pink coating of pure copper was precipitated upon it, and thus perpetually renewed its surface.

When the whole battery was properly arranged and charged in this manner, no evolution of gas took place from the generating or conducting plates, either before or after the connexions were complete; but when a voltameter was included in the circuit, its action was found to be very energetic. It was also much more steady and permanent than that of the ordinary battery; but still there was a gradual, but very slow, decline, which I traced at length to the weakening of the saline solution by the precipitation of the copper, and the consequent decline of its conducting power.

To obviate this defect, I suspended some solid sulphate of copper in small muslin bags which just dipped below the surface of the solution in the cylinders; which gradually dissolving as the precipitation proceeded, kept it in a state of saturation. This expedient fully answered the purpose, and I found the current perfectly steady for six hours together. This arrangement I have since improved by placing the salt in a perforated colander of copper fixed to the upper collar.

Fig. 3. represents a section of this additional arrangement. ($a\,c\,f\,h$) is the colander with its central collar ($b\,d\,e\,g$), which rests by a small ledge upon the rim of the cylinder. The membrane is drawn through the collar, and turning over its edge is then fastened with twine.

After this alteration, the effective length of the zinc rods exposed to the action of the acid was found to be no more than four inches and a quarter. In ascertaining the powers of the battery in single series, the voltameter was the same that I have already described; the specific gravity of the solution of sulphate of copper was found to be 1198·5, and I commenced my experiments with the standard sulphuric acid, specific gravity 1027·3.

With this charge, after the circuit had been completed for ten minutes, the mean quantity of mixed gases taken at intervals of five minutes for two hours was 2·1 cubic inches, the results never varying more than 0·1 cubic inch from one another. The battery was then left in connexion, without the voltameter, for two hours, and again examined for three intervals of five minutes, when the mean quantity of gases was found as before. It was again left in connexion for two hours, and re-examined with the same result.

Upon adding nitric acid to the solution of sulphate of copper, I found that an injurious effect was produced; and that the mean quantity of gas in five minutes was lowered to 1·1 cubic inch: at this rate of action the battery, however, remained steady for six hours.

Returning to the original solution of sulphate of copper for the exterior cavity of the cells, I doubled the strength of the dilute acid for the interior; adding, for this

purpose, 4½ measures of oil of vitriol to 100 measures of water, sp. gr. 1056·2, and found the mean quantity of gas evolved in the voltameter increased to 3·8 cubic inches per five minutes. I then removed the meter, and connected the circuit, leaving the battery in action for four hours. Upon replacing the meter it was ascertained to be still working at the same rate. The addition of nitric acid to the solution of the sulphate again reduced the rate to 2·1 cubic inches.

I now made trial of an addition of nitric acid to the interior liquid of the cell, adding an equal volume to that of the sulphuric acid, and restoring the solution of the neutral sulphate to the exterior division. At the first impression, an increased effect seemed to be produced, and the action for the first quarter of an hour was as high as 4·2 cubic inches per five minutes; but it ultimately settled down, and remained at the former amount without the nitric acid, of 3·8 cubic inches.

Here I may remark, that at the first immersion of the rods the effect of the battery is always a little higher than it afterwards settles at: and this I found to depend upon the evolution of a small quantity of hydrogen from a residue of local action upon the zinc, which adhered to its surface, and slightly impeded the action of the acid. These bubbles scarcely ever rose to the surface, and after the first quarter of an hour the battery took up a steady rate of action. By gently agitating the rods in the acid, the action would momentarily return to its original amount, but always settled down to the lower rate.

How very small this local action is will be seen from the details of the following series of experiments, which I shall give at length. Upon this occasion I weighed the ten zinc rods, which were the same which I had used from the first, and were reduced to nearly half their original circumference, that I might judge how nearly the zinc oxidated would be equivalent to the volume of gases evolved: their weight was 2¼ lbs. and 13 grains (15,763 grains). I trebled the strength of the acid, adding 6¾ measures of oil of vitriol to 100 measures of water, sp. gr. 1079·4. I sometimes made use of a large voltameter, the plates of which were 3 inches by 1 inch, and which, having a bent tube, admitted of my receiving the gases into a large graduated jar, and sometimes of the same small meter which I had hitherto employed. The results are contained in the following Table.

Large Voltameter.			
Time.	Quantity of Gas.	Rate per quarter of an Hour.	Rate per five Minutes.
h m	cubic inches.	cubic inches.	cubic inches.
11 24			
11 39	16	16	5·3
11 45			
11 60	30	14	4·6
12 3			
12 18	44	14	4·6
12 19			
12 34	58	14	4·6
12 49	71	13	4·3
Small Voltameter.			
12 51			
12 56	.....	.....	4·6
12 58			
1 3	.....	.....	4·6
Hydrogen removed by agitating the Rods.			
1 6			
1 11	.....	.....	4·9
Large Voltameter.			
1 14			
1 29	85	14	4·6
1 31			
2 1	27	13·5	4·5
2 31	54	13·5	4·5
3 1	82	14	4·6
Small Voltameter.			
3 2			
3 7	.....	.....	4·8

Thus the total quantity of mixed gases collected amounted to 186 cubic inches, which, being corrected for pressure, were equal to 188·48. The zinc rods were removed from the cells, rinsed in water, and carefully dried, when they were found to have lost 933 grains.

Now adopting the data for computation, as laid down in your Tenth Series (1126.), of 100 cubic inches of the mixed gases being equivalent to 12·68 grains of water, and taking the equivalent number of water as 9, and that of zinc as 32·5, we have the following proportions:

Mixed Gases.		Water.				
cub. inch.		grs.		cub. inch.		grs.
100	:	12·68	::	188	:	23·84,

and

Equiv. Water.		Equiv. Zinc.				
				grs.		grs.
9	:	32·5	::	23·84	:	86·1;

showing the quantity of zinc equivalent to 188 cubic inches of the mixed gases to be 86·1 grains; differing from the quantity actually consumed in each cell only 7·2 grains, or 2·3 for each equivalent. The waste of metal, indeed, must have been even smaller than this; for at the bottom of the cells I found a small quantity of amalgam, which had fallen off the rods, and which, at the time, it was not convenient to collect, but which, if added to the weight, would have materially diminished the deficiency.

For this quantity of zinc, 166·5 grains of metallic copper must have been precipitated on each copper cylinder; for not a bubble of hydrogen made its appearance upon them, and 6623 grains of crystallized sulphate of copper must have been consumed in the whole battery, or only 377 grains short of a pound avoirdupois.

Upon examining the cells after the course of experiments, the fresh-precipitated copper had a most beautiful appearance, being of a bright pink colour. It was not only deposited upon the sides and bottom of the cylinders, but upon the under surface also of the colanders. In the angle formed by the junction of the bottoms with the sides and at the contact of the membrane with the collars of the cullenders, it was collected in the largest quantities, and had a very distinct mammillated structure. At these points it is probable that the diffusion of the hydrogen had been impeded, and it had consequently accumulated in greatest abundance.

The charge of sulphate of copper has been left for weeks together in the battery, only taking care to keep the bottom of the colanders covered with the solid salt; but the acidulated water was withdrawn from the interior tubes every morning by means of a siphon, and a fresh charge substituted. It was generally found very slightly tinged with the sulphate, which would have been injurious to the zinc rods.

The only disadvantage that I am aware of in this new construction of the battery, is the unavoidable distance of the generating from the conducting metal, which is five or six times greater than in the double Wollaston plates; and this I sought to obviate as much as possible by improving the conducting power of the interposed electrolyte. I made trial of sulphuric acid of four times the standard strength, or 9 volumes of oil of vitriol to 100 of water, sp. gr. 1105·4; and this I worked for six hours continuously, and found the result very steady; 5·0 cubic inches with the large voltameter, and 5·5 by the smaller, per five minutes of time.

Taking into consideration the great precipitation of the sulphate of copper, I am doubtful whether the solution could be kept in a state of saturation at much beyond this rate of work; and in cases where it may be desirable to maintain a constant action for a great length of time, it may not be desirable to carry it even so far; but this, with many other questions, I must leave for future examination.

Being now desirous to bring the *constant battery* into more immediate comparison with one of the usual construction, I took a trough with ten Wollaston plates four inches square, which had been but little used, and carefully cleaned and fitted a double case of copper to the last zinc, to make the series complete. I charged it with a mixture in the proportions of 100 water, $2\frac{1}{4}$ oil of vitriol, and 2 nitric acid, and

set it to work with the small voltameter. The following Table contains the results measured at intervals of five minutes.

Time. h m	Quantity of Gas. cubic inches.	Rate per 5 min. cubic inches.
10 22		
10 27	2·5	2·5
10 32	4·6	2·1
10 34	Voltameter refilled.	
10 39	2·6	2·6
10 44	4·4	1·8
10 46	Voltameter refilled.	
10 51	2·8	2·8
10 56	4·9	2·1
10 57	Voltameter refilled.	
11 2	2·4	2·4
11 7	4·2	1·8
11 8	Voltameter refilled.	
11 13	1·8	1·8
11 18	3·2	1·4
11 19	Voltameter refilled.	

Time. h m	Quantity of Gas. cubic inches.	Rate per 5 min. cubic inches.
11 24	1·3	1·3
11 29	2·2	0·9
11 34	3·0	0·8
11 39	3·55	0·55
11 44	4·1	0·55
11 49	4·5	0·4
11 54	4·9	0·4
11 59	5·25	0·35
12 0	Voltameter refilled.	
12 15	0·9	0·3
12 30	1·7	0·26
12 45	2·25	0·26
1 0	2·75	0·25
1 58	Battery left in connexion.	
2 13	0·1	0·03

This series of observations displays in a striking manner the peculiar irregularities of the common voltaic battery, and entirely agrees with those which I had previously made with the dissected battery; the energy of the first contact declining even in the first five minutes a fourth or a third; recovering itself again by one minute's rest, and again declining: even with five such intervals of interrupted action, falling off permanently one half in the first hour; in the next hour of unceasing action rapidly falling to one tenth; and after four hours' connexion almost entirely ceasing.

If we compare the surfaces of the generating and conducting plates in each cell of the two batteries, we shall find that of the zinc in the Wollaston battery 32 square inches, whilst in the constant battery it does not exceed $7\frac{1}{2}$ inches. If we calculate only the interior surface of the copper of the former, that is also 32 inches; but if both surfaces be efficient, (as I believe, though not to the same amount,) then it amounts to 64 inches. It can only, of course, be the interior surface of the copper cylinders of the *constant battery* which assists the action; and reckoning $5\frac{1}{2}$ inches in depth to be efficient, it amounts to $65\frac{1}{2}$ inches; to which if we add a small quantity for the bottom and the underside of the colander, the amount of surface but little exceeds the former; but then it is all disposed to the greatest advantage. Under these circumstances the power of the constant battery is double that of the common battery at its first impulse, and can be maintained for any length of time in an invariable condition.

I shall trouble you, on the present occasion, with but one comparison more; and that is, of the efficiency of the two batteries under a retarding force. The charge which I used for the *constant battery* was only the double acid (sp. gr. 1056); but nevertheless I found that, with the opposition of three voltameters, the amount of gases in each was 0·9 cubic inch per five minutes; whilst a fresh set of Wollaston plates, with a new charge in the same proportions as before, gave only 0·32 cubic

inch in the first five minutes: so that the superiority of the former, under these circumstances, was even greater than before.

I will only add, that I have kept six inches of platinum wire, $\frac{1}{100}$th of an inch diameter, permanently red hot for a considerable length of time when the battery was merely connected in single series, and that the spark between charcoal-points is remarkably beautiful; and shall reserve for a future communication, if you will permit it, some observations upon the different modes of combining the plates, for which it affords great facilities, and the different important applications to which, from the invariableness of its action, it is applicable.

My principal object in these researches has been the attainment of this constancy; but, in addition, this new combination will be found, I think, to possess advantages which will secure to it a more general application than I at first contemplated.

First, the abolition of all local action by the facility of applying amalgamated zinc:

Second, the trifling expense of replacing, the zinc rods when worn out (for they are easily cast and fitted in the laboratory); and the total absence of any wear of the copper:

Third, the non-necessity of employing nitric acid, and the substitution of the cheaper materials, sulphate of copper and oil of vitriol; to which I may add, the absence of any annoying fumes:

And, fourth, the facility and perfection with which all metallic communications may be made, and different combinations of the plates arranged.

Hoping that I may not have wearied you by these details,

I remain, dear FARADAY,

Your very faithful friend,

J. F. DANIELL.

King's College,
23*rd January,* 1836.

XI. *Additional Observations on Voltaic Combinations. In a Letter addressed to* Michael Faraday, *D.C.L. F.R.S., Fullerian Prof. Chem. Royal Institution, Corr. Memb. Royal & Imp. Acadd. of Science, Paris, Petersburgh, &c. By* J. Frederic Daniell, *F.R.S., Prof. Chem. in King's College, London.*

Received April 14,—Read April 21, 1836.

My dear Faraday,

THE Council of the Royal Society having done me the honour to order the publication of my observations upon "Voltaic Combinations" in the Philosophical Transactions, I should wish to add the results of some further researches, which may render the account of the *constant voltaic battery* more complete and practically useful.

My great object in this combination was to obtain an invariable current of force sufficient to effect chemical decompositions, after overcoming the resistance necessary to register its quantity by the voltameter; and having succeeded in this, it seemed to me almost a matter of indifference to the solution of the various important questions to which it might be applied, whether the quantity were large or small. I quickly, however, discovered that the battery might be rendered not only perfectly steady in action but very powerful; and that it would be extremely efficient and convenient for all the purposes to which the common voltaic battery is usually applied. I set myself therefore to perfect its construction with this view.

Before I state the results I wish to direct your attention to some observations which affect the construction and application of your voltameter, being convinced that anything which may tend to facilitate the use or establish the correctness of that most important instrument will prove of substantial benefit to electro-chemical science. In my previous communication (p. 112) I stated that I had found that the plates of a voltameter only six eighths of an inch wide evolved the same amount of gases as two platinum plates one inch wide, and of the same height, in one of the cells of the dissected battery, when the former was substituted in the circuit for the latter; and I imagined that "their nearer approximation had counterbalanced their deficiency of surface." The results of the experiments recorded in the Table (p. 121), also show that there was no difference in the indications of the same small meter and those of a larger, the plates of which were three inches by one inch when they were alternately used. The plates of the larger instrument were moveable, and admitted of adjustment to different distances from each other; and by these means I ascer-

tained that no alteration, within the distance of the generating and conducting plates of the battery, produced any difference in the results.

This rendered the non-influence of variation of surface still more remarkable; and wishing to push the observation to the utmost, I reduced the plates of a voltameter to the width of one eighth of an inch, retaining the same height of three inches, and still found its efficacy unimpaired. I next thrust two platinum wires one tenth of an inch diameter through a cork closing the mouth of a glass tube, exposing about two inches of each to the dilute acid without any diminution of effect, the battery generating in all these experiments at the rate of 2·7 cubic inches per five minutes.

When I even covered the wires with a resinous cement so as to have only one fourth of an inch of each exposed, the gases evolved in the same time amounted to 2·3 cubic inches. I ultimately coated the wires entirely with cement, and carefully bared their mere points with a file, when they still gave off the gases at the rate of 0·8 cubic inch per five minutes; and in this case the currents of the gases instead of rising at once from the points of the horizontal wires seemed to be projected forward into the liquid with some force. This independence of the results upon the metallic surfaces of the voltameter is curiously contrasted with the paramount influence of the surface of the conducting plates of the cells of the battery, but is probably owing to the absence of any active chemical affinity assisting or retarding the main current which circulates.

In the prosecution of my experiments I now began to perceive that it was by no means necessary to attend so closely to the supply of fresh acid to the battery as at first seemed advisable; and I ascertained that after an uninterrupted action of five hours, without its renewal, the voltameter only indicated a decline from 2·7 to 2·4. It being of great consequence to note to what extent the constancy of the action might be independent of the exact adjustment of the acid, I left the battery in connexion for twenty-four hours, at the expiration of which time 0·9 cubic inch of the mixed gases was collected in the voltameter in a quarter of an hour, or 0·3 cubic inch per five minutes. The acid was found to be almost perfectly saturated; a very small drop of dilute ammonia occasioning an instantaneous precipitation of oxide of zinc in the solution, which had a specific gravity of 1·276. In this state three quarters of a fluid ounce of fresh dilute acid was poured upon the top of the solution in each cell (the total charge of each cell being about 5½ fluid ounces), when the action not only rose to its original amount of 2·7 cubic inches, but to 4·2 cubic inches, at which rate of work it kept perfectly steady without any further renewal of the acid for four hours.

This increase of action I could only refer to the superior conducting power of the solution of sulphate of zinc; and two important points were thus indicated: 1st, that the conducting power of the electrolyte in the battery might be increased with great advantage; and 2ndly, that the quantity of the circulating force was still more independent of the surface of the generating metal than my experiments had yet shown

it to be. The length of the zinc rods exposed to the action of the fresh acid, which must have floated upon the saline solution in this experiment, could not much have exceeded one inch, and yet they proved perfectly efficient. I immediately confirmed this conclusion by shortening the zinc rods to one fourth of their original length, and found no diminution of the power of the battery when charged with fresh acid.

I next charged the battery with acid of the same strength as that which I employed in the voltameters, viz., eight parts of water to one part of oil of vitriol by measure (specific gravity 1136), and obtained at once a steady action of 11· cubic inches per five minutes; this was therefore the mixture which I employed in all my subsequent experiments.

This rate of action must obviously require some attention to the state of the solutions in the battery, when perfect steadiness is required to be maintained for a long period; as nearly 25 grains of oxide of zinc are formed, and 154 grains of sulphate of copper decomposed in each cell per hour. Nevertheless it will remain constant for an hour and a half even without any change of acid; and the addition every now and then of one fluid ounce of fresh acid will maintain the action for any desired time, provided the colanders be kept well supplied with the salt of copper.

Considering the great advantage which had been derived from the extension of the surface of the conducting metal of the battery, I now wished to ascertain whether this might be carried further; and for this purpose I caused some cells to be fitted up in the interior with ten small plates, one inch wide, extending from the bottom to the colander, and converging from the interior circumference towards the centre. In this way they just reached to the membranous tube, and the surface of the copper was more than trebled. I thought it possible that advantage might be thus derived, not only from the great extension of surface, but from the approximation of a part of the conducting plate to the generating rod.

My first experiment was made with five plain cells and five ribbed, working into separate voltameters, and I found the products equal, inclining, if anything, in favour of the former. The amount of gases from each was, however, only 5·5 cubic inches per five minutes, or one half of that from the whole series of ten cells. When the ribbed and the plain cells were connected together in one series, the amount in one meter was, as before, 11 cubic inches.

It now seemed evident that a series of five did not confer *intensity* enough upon the current to enable its whole quantity to overcome the resistance of the conducting fluid between the two metals at the existing distance; and this being the case with the plain cells, no increase of quantity could be expected to manifest itself from those with extended surfaces. When the latter were joined with the former, the smaller surface of course governed the whole series according to the established law. I therefore completed the number of the ribbed cells to ten, and setting them to work against an equal number of plain, the product of the latter was, as before, 11· cubic inches, but that of the former did not exceed 8·5. I have not yet been able to satisfy

myself, whether this unexpected difference, in the opposite direction to what I expected, is dependent upon the construction of the cells or upon some accidental circumstance which I have not been able to trace; but the result may certainly be taken to prove that no advantage arises from the extension or approximation of the conducting metal which I have described.

The increase of the number of the battery series requires for convenience a different arrangement from that which I described in my last communication; and I now place the cells in two parallel lines of ten each, upon a long table, the siphon-tubes arranged opposite to each other, and hanging over a small gutter placed between the rows, to carry off the refuse solution when it is necessary to change the acid; and as the uniformity of action may be completely maintained by the occasional addition of a small quantity of fresh liquid, I have been able to dispense with the cumbrous addition of the dripping funnels. This arrangement admits with facility of any combination of the plates which may be desired.

I proceeded now to connect the cells together in pairs; the zinc rod of each ribbed cell being in communication with that of a plain cell, and the copper with the copper. The ten pairs were then connected in a series of ten: the product of this combination was 17 cubic inches per five minutes, or exactly double that of the single-ribbed cells.

Considered in a theoretical point of view, these experiments seem to me to lead to the conclusion that the most perfect voltaic combination would consist of a solid sphere of a generating metal, surrounded by a hollow sphere of a conducting metal, with a stratum of intervening electrolyte perpetually renewed, and the metals communicating by a wire defended from the electrolyte by a glass tube covering that portion which it would be necessary should pass through it. In such a hypothetical arrangement, the resistance of the electrolyte would increase directly as the distances of the two spheres, or as the thickness of the stratum; while, supposing this resistance overcome, the quantity of force set in circulation would increase as the square of that distance from the centre, or as the surface of the exterior sphere. The number of a series required to give the necessary impulse would consequently only increase as the simple distance, while the advantage would increase as the square.

The rod of zinc within the cylinder of copper is probably the nearest practical approximation which can be made to such an arrangement; but the soundness of this deduction might doubtless be tested by varying the diameters of the cylinders.

The battery which I have now described, consisting of twenty cells, will, I think, be found amply sufficient for all the purposes of demonstration and investigation. It is competent to keep eight inches of platinum wire $\frac{1}{100}$th inch permanently red hot in the open air; and the amount of work which it is able to perform renders it even an economical source of the purest oxygen for laboratory purposes.

To facilitate this application, I have fitted up a cell by inclosing a platinum plate, instead of the zinc rod, within the membranous tube, which is closed at the upper end by a glass tube bent in a convenient form to deliver the disengaged gas under a

receiver. When this cell is included in the circuit of double cells, the hydrogen is absorbed as before by the oxide of copper; but the oxygen is evolved at the rate of 84 cubic inches per hour.

I shall conclude these observations with the result of an experiment which places the secondary action of the affinities of the disengaged gases in a striking point of view. The mixed gases collected in a voltameter in five minutes were found to amount to 17 cubic inches; the oxygen collected in an equal time, when the hydrogen was absorbed, was 7 cubic inches; which are equivalent to 21 cubic inches of the mixed gases: and the hydrogen collected when the oxygen was absorbed (as is readily effected by reversing the connexions of the cell which I have just described) amounted to 16 cubic inches, equivalent to about 24 cubic inches of the mixed gases. Thus the removal of the hydrogen from the sphere of attraction allowed of an increase of action equal to 4 cubic inches, while the increase from the like removal of the oxygen was very nearly double; a difference which is probably referable to the equivalent combining volumes of the two gases. This observation opens a new field of inquiry of great interest, upon which, with your permission, I hope to have the pleasure of addressing you at no very distant period.

I remain, my dear FARADAY,

Yours very faithfully,

J. F. DANIELL.

King's College,
6th April, 1836.

XII. *Researches on the Tides.—Fifth Series. On the Solar Inequality and on the Diurnal Inequality of the Tides at Liverpool.* *By the Rev.* WILLIAM WHEWELL, *F.R.S., Fellow of Trinity College, Cambridge.*

Received February 23,—Read March 3, 1836.

SECT. 1. *Present State of the subject.*

1. THE great success with which recent researches on the Tides have been attended, has encouraged me to attempt some further advances in this subject. The laws of the semimenstrual inequality of the times were shown by Mr. LUBBOCK, from the London observations, to agree very closely with the equilibrium-theory: and this result has been confirmed by the examination of observations made at many other places. I have shown, from the Liverpool observations, that the semimenstrual inequality of the heights presents a still more complete agreement with the equilibrium-theory*; and by the help of Mr. LUBBOCK's discussions of the London and of the Liverpool observations, I have shown, in the Second and Fourth Series of these Researches, that the inequalities depending on the changes of lunar parallax and declination may be very well represented by the equilibrium-theory, with certain modifications, which are far from inconsistent with the best mechanical views we can at present form of the laws of the motion of fluids.

The most obvious points which now remain requiring still to be made out and explained, are the diurnal inequality, and the solar inequalities of the time and height of high water.

2. The *Diurnal Inequality* of the tides is that which makes the tide of the morning and evening of the same day at the same place, differ both in height and time of high water, according to a law depending on the time of the year. This is called the diurnal inequality, because its cycle is a day.

The existence of such an inequality in the heights of high water has often been noticed by seamen and other observers, as I have stated in the First Series of these Researches†. But its reality has only recently been confirmed by regular and measured observations, and its laws have never been correctly laid down. Its existence appeared very palpably in the curves which I constructed in order to examine the results of the tide observations made by the coast-guard in June 1834, and also in the curves drawn by the self-registering tide-gauge erected near Bristol; although the Sheerness machine did not exhibit it, in consequence of the tide at that place

* Fourth Series: Philosophical Transactions, 1836, page 1.

† Philosophical Transactions, 1833, page 221.

being a compound of two others. But this inequality had never been obtained in numbers till the recent discussion of the Liverpool tides. Under Mr. Lubbock's direction, Mr. Dessiou has obtained it from the observations of Mr. Hutchinson. Mr. Bywater, who has calculated Tide-Tables for Liverpool for the present year, has also obtained this inequality from his own calculations, (suggested, as he states, by the remarks made in the First Series of these Researches,) and has introduced it, for the first time, into published tables; and I have just learnt from Mr. Bunt, who is at present employed in forming Tide-Tables for Bristol, that he has also obtained the general form of this inequality, agreeing, on the whole, with the other results, although with some discrepancies. These arise, I conceive, principally from the shortness of the series of observations which he had at his command (less than two years). The tide observations now going on at these ports, when hereafter discussed as the preceding ones have been, will, I have not any doubt, lead to the production of tide-tables possessing a degree of accuracy which, a little while ago, would have been considered unattainable.

3. It is natural for us to wish to refer the effects of the diurnal inequality, so far as they have yet been obtained, to the equilibrium-theory. I will say a few words on this subject.

The general relation of these results to the equilibrium-theory it is not difficult to see. When the moon is south of the equator, the equilibrium-tide corresponding to her upper transit at any place having southern latitude, would be greater than the tide corresponding to her lower transit: when she is north of the equator the contrary would be the case. When she is in the equator the two tides are equal. Now in one lunation she moves in an orbit inclined to the equator. Hence, while she moves from the sun to the sun again, (that is, while the time of her upper transit passes through the whole twenty-four hours from noon to noon again,) the tide which corresponds to her upper transit will, during one lunation, be greater during half the period, and less during the other half, than the tide of the next half-day; and at two particular times in the lunation the difference will vanish. The times of moon's transit for which the diurnal difference vanishes, correspond to the times when the moon is in the equator, and are therefore different at different seasons of the year. Now from the general correspondence of the phenomena of the tides with the equilibrium-theory, we may expect that the circumstances of the diurnal inequality will be the same as those which have been described; but that the time when the diurnal inequality vanishes will not be the time when the moon is in the equator, but some time afterwards.

4. By the statements above referred to on this subject it appears that the time at which the diurnal difference vanishes is when the moon's transit takes place about $9^h\ 30^m$ in January, and two hours earlier in each succeeding month, taking the general average of the facts. Now in the middle of January the sun is $4^h\ 30^m$ from the vernal equinox, and hence the moon is 5^h beyond the equinox when the diurnal inequality

vanishes. The same result would be obtained from the other months, since the sun's right ascension increases on an average two hours in each month*. Hence the evanescence of the diurnal inequality which, in the equilibrium-spheroid, would take place when the moon is in the equator, does not take place till she has described 5^h of right ascension after that time, and this requires six days and a quarter.

5. In the inequalities hitherto considered, which were the effects of the sun, and of changes of the moon's parallax and declination, the circumstances of the tide agree with those of the equilibrium-spheroid one day and a half or two days previous. It appears that such an interval suffices for the forces, when near their maximum, to accumulate and transmit their effects to Liverpool; and after this interval the diminution of the actual forces overbalances the increase arising from their continued action. But in this case of the diurnal inequality we find that above six days are required for this accumulation and transfer. The inequality vanishes six days after its cause vanishes, and in the same way it reaches its maximum six days after the producing force is greatest. On a little consideration we shall not be surprised at the great time required to bring this inequality to its full magnitude. The semidiurnal tides, alternately greater and less, which are transmitted from the Southern Ocean to Liverpool, may be compared to oscillations of the ocean, and these are augmented by the action of the forces occurring at intervals equal to those of the oscillations. Hence the oscillations go on increasing for a considerable period after the forces have gone on diminishing, and reach their maximum almost a week after the forces have passed theirs.

6. In January the tide at Liverpool, which follows the moon's upper transit by about eleven hours, is *less* than the following tide, when the moon is near the sun, and consequently when the moon is south of the equator. This agrees with the equilibrium-tide depending upon the position of the moon at the time of transit. But it is clear that the tide which reaches Liverpool does in fact come from the Southern Ocean; and hence the equilibrium-tide for the time of the moon's upper transit at Liverpool will be *greater* than the following one, when the moon is near the sun in January. Hence the tide at Liverpool is not that which corresponds to the equilibrium-tide *at* the time of transit, but to the equilibrium-tide about either twelve or thirty-six hours *earlier*. The latter is probably the right conclusion, and agrees with the inference already obtained from the effects of the solar forces, that the tide agrees with an equilibrium-tide thirty-seven hours and a half previous to the moon's transit.

8. The succeeding sections of this paper will be devoted to the investigation of the *Solar Inequalities* at Liverpool. By carefully eliminating the lunar effects, which the preceding researches enable us to do, I have determined, as I conceive beyond dis-

* In order to detect the diurnal inequality, the observations have been classed by calendar months; but as this inequality depends mainly on the moon's declination, it would probably have been obtained more distinctly if the observations had been classed according to the moon's declination, distinguishing north and south declination, and also increasing and decreasing declination.

pute, the approximate circumstances of the solar correction for the *height*. I have also obtained, though by no means with the same certainty, evidence of the existence and laws of the solar inequality of the *times*. These inequalities, as thus discovered, exhibit the same general agreement with the equilibrium-theory which has been disclosed in all the inequalities hitherto detected*.

9. But though the equilibrium-theory thus seems to suggest and express the laws of the various inequalities of the tides, I would by no means be understood to rate this theory above its true value. It is not the true theory, but a very inaccurate and insufficient substitute for it, which we are compelled to adopt in consequence of the extremely imperfect state of the mathematical science of hydrodynamics. The tides are a problem of the motion, not of the *equilibrium* of fluids; and we can never fully explain the circumstances of the phenomena till the problem has been solved in its genuine form. This solution is perhaps not beyond the powers of modern mathematics, but it has certainly never yet been given. Laplace's solution, besides being obtained by means of a precarious assumption, rests upon several arbitrary hypotheses, fatal to it even as a first approximation; and, I believe it will be found, leaves out of consideration an essential portion of the forces. To obtain any useful result, the question must be taken up afresh and treated in another manner.

10. I hope some mathematician will be found able and willing to execute this task. But in the mean time I may be permitted to observe, that what has been already done in the discussion of tide observations, and in bringing to light the empirical laws of the phenomena, has entirely altered the position of this branch of science with respect to the mathematical theory. A little while ago the theory was in advance of observation; at present observation is in advance of theory. A very few years since, the equilibrium-theory and the Laplacian theory were in a condition to assign laws regulating the changes of the times and heights under given astronomical circumstances, and it had not been shown from observation whether these laws were obeyed. We can now state what the agreement and disagreement is between such theoretical laws and the facts; and we call upon the mathematician to substitute for these two theories, both confessedly false, some other, which shall come nearer to the true state of the case, and, by that means, nearer to the laws of the phenomena. The performance of this task is requisite for the completion of the Newtonian theory of the universe.

§ 2. *On the Effect of the Moon's Declination on the Tides at Liverpool*†.

11. In order to obtain the Solar Inequality, it was necessary to eliminate the effect of the variations of the lunar tidal forces; the results of the two sets of forces being combined in the tides when tabulated according to months, as is done in Mr. Lub-

* While I write this, I am informed by Mr. Bunt that he has obtained the law of the solar inequality of the times at Bristol. The agreement with my results is very remarkable.

† In the calculations of this Memoir, I have been assisted by Mr. Nayler of Queen's College in this University.

BOCK's Tables I., II., III.* In these tables the parallax may be already considered as reduced to its mean value; for each number is the result of about ninety observations, distributed through nineteen years, in which time the moon's perigee moves twice round the ecliptic; and hence the deviations above and below the mean will balance each other. But it is otherwise with the declination; for since the mean moon may be considered as moving in the mean elliptic, a certain time of the year, with a certain hour of moon's transit, implies a certain declination; for the moon's time of transit added to the sun's right ascension is the moon's right ascension, which of course determines the declination. Hence the variations of the numbers in Tables II. and III. are those which arise from the varying forces of the mean moon and of the sun; and the greater part of the variation arises from the change of the declination of the moon. This part must be eliminated, in order to bring into view the solar effect; and we are able to perform this elimination by means of Table I., since that Table contains also the mean declination of the moon for each set of observations. In each set, the number of observations being large, and the limits of declination small, the mean correction for the lunar declination may be taken to be identical with the correction for the mean lunar declination, although the correction varies nearly as the square of the declination. But in order to apply this correction, we must have a table of the effect of every degree of declination; whereas Mr. LUBBOCK's Tables XII. and XVI. only contain the effect for every 3° of declination. It is obvious also, by inspection of this table, that it is affected by many casual irregularities which must be got rid of by interpolation. Among these irregularities, however, I do not include the apparent difference of the effect of north and south declination, which is shown in Tables XI. and XV.; since it will be seen by comparison with Table X. that the greater part of this difference arises from the effect of parallax; for the changes of parallax and declination so nearly recur in the same cycle, that their effects are not insulated in a period of nineteen years. The remainder of the difference arises from the solar inequality of which we are in search.

12. I have therefore laid down Mr. LUBBOCK's Tables XII. and XVI., and interpolated them by means of curves, and have thus obtained two new declination tables for the *Times* and the *Heights*. These tables I shall designate as Declination Tables (W. T.) and (W. H.) respectively; meaning by the former letter to distinguish them from Mr. LUBBOCK's Tables XII. and XVI. They may be used in calculating the time and height of high water at Liverpool (the table for the height being altered by a constant if the measures are not taken from Mr. HUTCHINSON's zero). When so applied, they take the place of Mr. LUBBOCK's Tables XXV. and XXVI., from which they differ—in including the semimenstrual inequality, in being interpolated independently, in being given for the middle of each hour instead of the beginning, and for every degree of declination instead of every three degrees. These Tables will be found at the end of this Memoir, namely,

* Philosophical Transactions, 1835, p. 282.

Declination Table (W. T.). *To be used in predicting the Time of high water at Liverpool, for the mean parallax.*

Declination Table (W. H.). *To be used in predicting the Height of high water at Liverpool, for the mean parallax.*

When these tables are used in predicting the tides, the times and heights found by these must be corrected for parallax by Mr. Lubbock's Tables XXIII. and XXIV., or by the formulæ given in my Memoir on the Empirical Laws of the Tides in the Port of Liverpool. I will insert, at the end of this paper, Tables of the correction for parallax given by these formulæ.

§ 3. *On the Solar Inequality of the Heights of High Water at Liverpool.*

13. By means of the Declination Tables (W. T.) and (W. H.) we can reject from Mr. Lubbock's Tables II. and III. the part which depends on the moon's declination, and we thus have the remainder, which is the solar correction as far as it can be obtained from the observations of Mr. Hutchinson. This is what I have done in the following calculations (Table A.). The third column of each table, marked (W. H.), contains the height due by the declination table; the fourth column, marked (L. III.), contains the observed height as given by Mr. Lubbock's Table III.; and the column (Diff. H.) is the excess of the latter, and is therefore the residual height due to the solar effect. In like manner the columns marked (W. T.), (L. II.) and (Diff. T.) contain the *Intervals* due to the moon's declination, the observed intervals, and the residual solar effect.

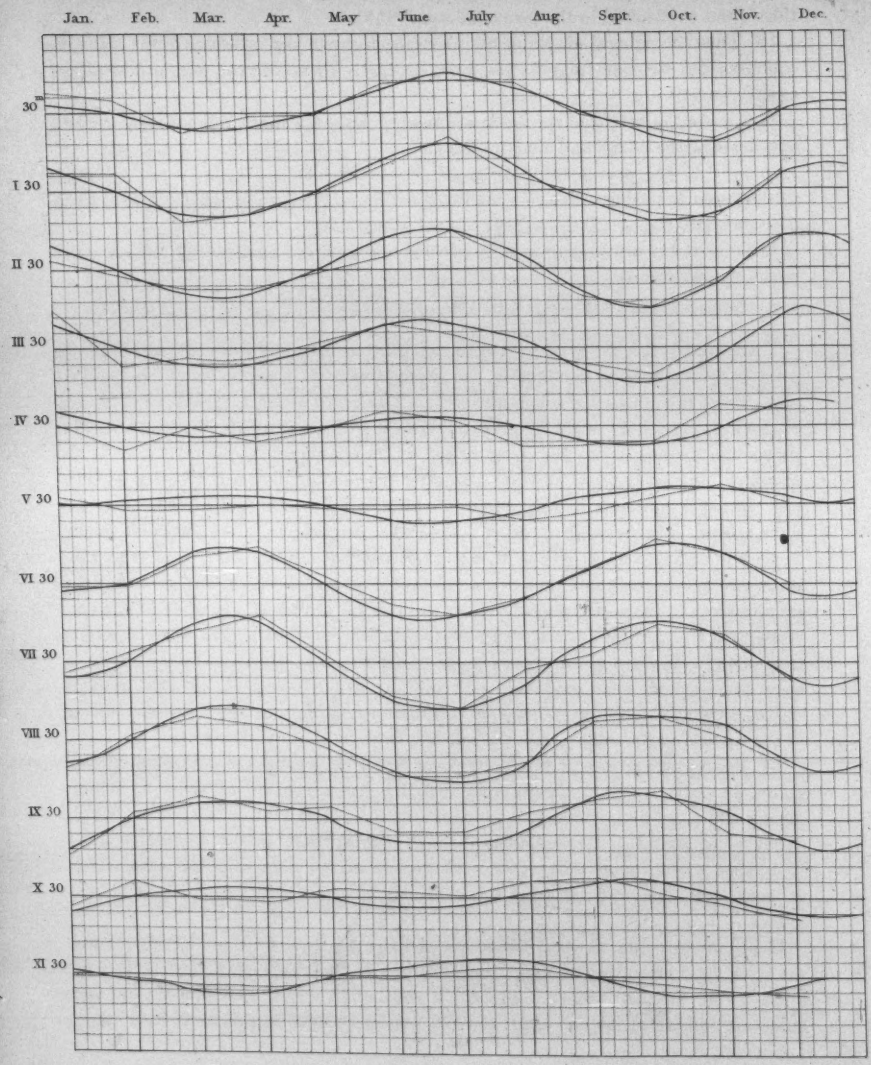
I shall consider the solar effect on the *Heights* in the first place, since its law is more manifest. It may be expected, like the other tidal inequalities, to be resolvable into a non-periodic and a periodic part. In order to effect this, I take the mean of each monthly column of differences, and subtract it from each number in the column. In this manner I obtain Table (B. H.).

In order to trace the law of the periodical part, which exists in the remainders, if at all, I lay down these remainders by coördinates, as in Plate XVII.

14. If we draw lines through the dots belonging to each hour of the moon's transit, it becomes manifest that these remainders really result from a solar inequality. For the curves (the dotted curves in Plate XVII.) have all the same general form, having a reference to an annual cycle. Thus there are, for all of them, a maximum or minimum in March, another in July, another in October, and another in December. Hence the effect of lunar declination has been nearly or altogether eliminated; for this effect is a maximum for different hours at different times of the year, as appears by the reason of the case, and as is shown in Mr. Lubbock's Tables IV. and V.

It appears also by the curves, that the effects of the changes of the solar force are the greatest for the hours of moon's transit, $1^h\ 30^m$ and $7^h\ 30^m$, and are in these two cases the opposite of one another, which agrees with the nature of the case; for the

SOLAR CORRECTION OF HEIGHTS. PERIODIC PARTS. LIVERPOOL.



The ordinates are here six times too large. The positive ordinates are measured downwards.

J. Basire sculp.

height of the solar tide is added to that of the lunar in the former case (at spring-tides), and subtracted in the latter (at neap-tides).

15. Thus the general course of the phenomena shows the existence of the solar inequality; but we shall trace its law more distinctly by taking the suggestions of the equilibrium-theory.

If h, h', be the height of the solar and lunar tides, φ the angular distance of the sun and moon, y the compound tide, we have

$$y = \surd\{h^2 + h'^2 + 2\,h\,h' \cos 2\,(\varphi - \alpha)\},$$

as shown on former occasions; α being a quantity which is to be determined so as to accommodate the equilibrium-theory to the actual case.

Let h undergo any change so as to become $h + \Delta h$, Δh being small. Then y becomes approximately

$$y + \Delta y = y + \frac{dy}{dh}\Delta h = y + \frac{h + h' \cos 2\,(\varphi - \alpha)}{\surd\{h^2 + h'^2 + 2\,h\,h' \cos 2\,(\varphi - \alpha)\}}\Delta h.$$

The quantity Δh varies according to the different season of the year, and depends principally on the sun's declination, the changes of solar parallax being too small to affect much the amount of solar tidal force. The curve which expresses the changes of Δh has, as appears in the figures, a maximum at the end of March, soon after the time when the sun is in the equator, and when, consequently, his tidal force is the greatest. It cuts the axis in May, soon after the sun has his mean effective declination: it has a minimum in July, soon after his greatest declination, at which time his force is least. The mean effect recurs in the end of August: there is another maximum in the end of September. About November, December, and January, there is another mean and another minimum, arising from the mean declination in November and the greatest declination in December. Thus the general course of the value of the correction for a given value of φ agrees with the equilibrium-theory. The want of perfect regularity in the form of the curves is due partly to the combination of the effects of solar parallax and solar declination. According to the theory, the greatest amount of the correction for solar declination is about one tenth, and the greatest amount of the correction for solar parallax about one twentieth, of the whole solar tide.

16. For a given season of the year, if we follow the changes of φ through twelve hours, we easily see from the formula that Δy has a continuous series of values, among which are a maximum and a minimum value. Hence the curves in Plate XVII. ought to be such that the ordinates for each month, taken in the successive hour-lines, form a continuous series. The subtraction of the non-periodical part, which we have performed, will not affect this continuity, since this part is constant for the month.

Hence in correcting the original curves of Plate XVII., so as to get rid of irregularities, we must endeavour to make them conform to these two conditions:—that the ordinates for the same month shall form a continuous series, with a maximum and minimum; and, that the curves for the different hours shall be similar to each other,

and have their maxima and minima at the same seasons. This latter condition may perhaps be slightly modified, so that α may be somewhat different for different values of Δh.

The curves drawn with full lines in Plate XVII., are drawn under these conditions, the ordinates being those contained in Table (C. H.). Their agreement with the original curves is such as to entitle us to consider them as correct interpolations, for the coincidence is almost complete in the cases where the corrections are the largest; as the hours $0^h\ 30^m$, $1^h\ 30^m$, $2^h\ 30^m$, $6^h\ 30^m$, $7^h\ 30^m$, $8^h\ 30^m$; and there are no material discrepancies except in the lines $4^h\ 30^m$ and $10^h\ 30^m$; and even in these the difference is only a displacement of a maximum of four inches by about a month. The observations, therefore, prove that there is a solar correction of the heights which follows, nearly, the law suggested by the equilibrium-theory. The greatest amount of the periodical part of this correction is about half a foot *plus* and *minus*; and to this must be added the non-periodical part, which at some seasons amounts to one fifth of a foot.

17. We may expand the formula above given for Δy into a non-periodical and a periodical part. We have

$$\Delta y = \frac{h + h' \cos 2(\varphi - \alpha)}{\sqrt{\{h^2 + h'^2 + 2hh' \cos 2(\varphi - \alpha)\}}} \cdot \Delta h,$$

$$= \frac{h'}{\sqrt{(h^2 + h'^2)}} \frac{c + \cos 2(\varphi - \alpha)}{\sqrt{\{1 + \frac{2c}{1 + c^2} \cos 2(\varphi - \alpha)\}}} \cdot \Delta h,$$

Making $c = \frac{h}{h'}$. Expanding and omitting c^2, &c.

$$\Delta y = \frac{h'}{\sqrt{(h^2 + h'^2)}} \Big(c + \cos 2(\varphi - \alpha)\Big) \Big(1 - c \cos 2(\varphi - \alpha)\Big) \Delta h$$

$$= \frac{h'}{\sqrt{(h^2 + h'^2)}} \Big\{ c + \cos 2(\varphi - \alpha) - c \cos^2 2(\varphi - \alpha) \Big\} \Delta h$$

$$= \frac{h'}{\sqrt{(h^2 + h'^2)}} \Big\{ \frac{c}{2} + \cos 2(\varphi - \alpha) - \frac{c}{2} \cos 4(\varphi - \alpha) \Big\} \Delta h$$

Hence the periodical part is

$$\frac{h' \Delta h}{\sqrt{(h^2 + h'^2)}} \Big\{ \cos 2(\varphi - \alpha) - \frac{c}{2} \cos 4(\varphi - \alpha) \Big\}.$$

This vanishes for two values of φ, which are a little less than $90°$ or 6^h from each other when c is small. In Table (B. H.) it appears that the periodical solar correction vanishes for two values of the hour-angle which are nearly 6^h from each other in each month. Hence we may suppose that the periodical part involving $\cos 2(\varphi - \alpha)$ is alone sensible. This vanishes when $\cos 2(\varphi - \alpha) = 6^h$ or 18^h, $\varphi = 3^h + \alpha$ or $9^h + \alpha$. Hence we find that in January and September α is $2^h\ 30^m$; in March, April, June, July, October α is about 2^h; in August and November it is $1^h\ 30^m$; in December it is 3^h.

These differences in the value of α may arise from the lunar declination not being completely eliminated in our previous calculation, the interpolation of the Table being slightly inexact. But it is by no means improbable that they are the discrepancies, arising from mechanical principles, which exist between the tides of water in motion and the results of the equilibrium-hypothesis. The general agreement of the equilibrium-theory with the facts (assuming $\alpha = 2^h$) is near enough to be very remarkable. It may be possible, by additional care and labour, to bring the solar interpolated curves nearer to the observations, preserving the requisite conditions: but I conceive that enough has been done to establish the general law.

§ 4. *On the Solar Inequality of the Time of High Water at Liverpool.*

18. The solar inequality of the time must be found in exactly the same manner as the solar inequality of the heights. By interpolation of Mr. LUBBOCK's Table XVI., I have obtained Declination Table (W. T.); and by comparing the time of high water due to lunar declination by this Table with the observed times as given in column (L. II.) of the calculations, I obtain the residual quantities in the columns (Diff. T.), which should exhibit the solar correction of the times.

I then find the mean of the column for each month, and subtract it from every number in the column, in order to separate the non-periodical from the periodical part of the residual quantities. The means and the remainders are exhibited in Table (B. T.). The remainders were laid down by coordinates, but I have not thought it necessary to give these curves.

The points thus found being joined by continuous lines, I had a series of curves which ought to exhibit the solar inequality. These lines were less obviously regular than those which we obtained by a similar treatment of the heights; but they were still apparently free from any lunar effect, which would have given a maximum passing successively from one month to another; and I could trace a solar cycle in them; namely, the curves for $1^h\ 30^m$, $2^h\ 30^m$, and $3^h\ 30^m$ had a maximum about April, and a minimum about August, while the curves for $5^h\ 30^m$, $6^h\ 30^m$, $7^h\ 30^m$ had these features inverted, and the rest of the curves had small ordinates only.

19. Let us compare the laws of the phenomena of which we thus catch a glimpse with the laws according to the equilibrium-theory. We have, on that hypothesis,

$$\tan(\theta' - \lambda') = -\frac{h \sin 2(\varphi - \alpha)}{h' + h \cos 2(\varphi - \alpha)} = t \text{ suppose,}$$

where θ' is the interval of the tide and moon's transit.

Now let h become $h + \Delta h$, and θ' become $\theta' + \Delta\theta'$, we have, approximately,

$$\tan(\theta' - \lambda') + \frac{d \cdot \tan(\theta' - \lambda')}{d\theta'} \Delta\theta' = t + \frac{dt}{dh} \Delta h$$

$$\sec^2(\theta' - \lambda') \cdot \Delta\theta' = \frac{dt}{dh} \Delta h$$

$$\Delta\theta = \frac{1}{1+t^2} \cdot \frac{dt}{dh} \Delta h$$

$$= -\frac{h' \sin 2(\varphi - \alpha)}{h^2 + h'^2 + 2hh' \cos 2(\varphi - \alpha)} \cdot \Delta h.$$

Hence for a given value of Δh, that is, for a given month, $\Delta\theta$ goes through a cycle which has a minimum and a maximum, when $\cos 2(\varphi - \alpha) = -\frac{2hh'}{h^2 + h'^2}$. If $\frac{h'}{h} = 3$, which has been found to be nearly the value by other phenomena, the two values of $2(\varphi - \alpha)$ are 144° and 216°, or 10^h and 17^h nearly; and if in this case, α be $-1^h\ 30^m$, the maxima and minima will agree with $\varphi = 3^h\ 30^m$, $\varphi = 7^m\ 0^m$, which appears best to represent the phenomena.

20. After drawing the lines described in Art. 18, I drew interpolated curves for the hours $1^h\ 30^m$, $2^h\ 30^m$, $3^h\ 30^m$, and $5^h\ 30^m$, $6^h\ 30^m$, $7^h\ 30^m$; the general agreement of the course of the interpolated and the original curves appeared to me to be such as to show that the errors arose from a solar inequality, following in its general changes the law given by the equilibrium-theory. I have given the corrections upon this supposition in Table (C. T.). The displacement of the zero points and maxima, and the want of proportionality in the maximum values, which appear in some of the lines, I have admitted, because such modifications may arise from my not having got rid of the whole of the non-periodical effect. Such irregularities may also arise from there being still some vestige of the lunar declination correction not got rid of by the processes which I have employed: but this, if it exists, must be very small, and I do not think there can be any doubt as to the general form of the solar correction of the time. I have, however, abstained from filling up the Table, as not thinking my present materials sufficient to enable me to do so with any confidence.

DECLINATION TABLE (W. T.). To be used in predicting the *Time* of high water at Liverpool for the mean parallax.

Moon's Transit.	0° Decl.	1° Decl.	2° Decl.	3° Decl.	4° Decl.	5° Decl.	6° Decl.	7° Decl.	8° Decl.	9° Decl.	10° Decl.	11° Decl.	12° Decl.	13° Decl.
h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
0 30	11 20	11 20	11 20	11 20	11 20	11 20	11 20·5	11 20	11 20	11 19·5	11 19	11 19	11 18	11 18·5
1 30	11 5	11 5	11 5·5	11 5	11 5	11 5	11 5	11 5	11 4	11 4	11 4	11 3·5	11 3	11 2·5
2 30	10 51	10 51	10 51	10 51	10 51	10 51	10 50	10 50	10 50	10 49·5	10 49	10 49	10 47·5	10 47·5
3 30	10 39	10 39·5	10 39·5	10 39·5	10 39·5	10 39·5	10 39	10 39	10 38·5	10 38	10 38	10 37	10 36·5	10 36
4 30	10 31	10 31·5	10 31·5	10 31·5	10 31	10 31·5	10 31·5	10 31	10 30	10 29	10 28·5	10 28	10 27·5	10 27
5 30	10 35	10 35	10 35	10 34·5	10 34·5	10 34·5	10 34	10 34	10 33·5	10 32·5	10 32	10 31	10 30	10 28·5
6 30	10 54	10 54	10 54	10 53·5	10 53	10 52·5	10 52	10 51·5	10 51	10 50	10 49	10 48	10 47	10 46
7 30	11 24	11 24	11 24	11 24	11 24	11 24	11 23·5	11 23	11 23	11 22	11 22	11 20·5	11 20	11 19
8 30	11 47	11 47	11 47	11 47	11 47	11 47	11 47	11 47	11 47	11 47	11 46	11 46	11 45	11 44·5
9 30	11 53·5	11 53·5	11 53·5	11 53·5	11 53·5	11 53·5	11 53	11 53	11 53	11 52·5	11 52	11 52	11 52	11 51·5
10 30	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46
11 30	11 35	11 35	11 35	11 35	11 35	11 35	11 35	11 35	11 35	11 34·5	11 34·5	11 34·5	11 34	11 34
	14° Decl.	15° Decl.	16° Decl.	17° Decl.	18° Decl.	19° Decl.	20° Decl.	21° Decl.	22° Decl.	23° Decl.	24° Decl.	25° Decl.	26° Decl.	27° Decl.
0 30	11 17·5	11 17	11 16·5	11 16	11 15	11 14	11 14	11 13	11 12·5	11 12	11 11·5	11 11	11 10	11 9
1 30	11 2	11 1·5	11 1·5	11 0·5	10 59·5	10 59	10 58·5	10 58	10 57·5	10 57	10 56	10 54	10 53	10 52
2 30	10 47	10 46·5	10 43·5	10 44	10 44	10 43	10 42	10 41	10 40	10 39	10 37·5	10 37	10 36	10 34
3 30	10 35	10 34	10 33	10 32·5	10 32	10 31	10 29·5	10 28	10 27	10 26	10 25	10 24	10 22	10 21
4 30	10 26·5	10 25	10 24	10 23	10 22	10 21·5	10 20	10 18·5	10 17·5	10 16	10 14	10 12·5	10 11	10 9
5 30	10 27	10 26	10 25	10 23·5	10 22·5	10 21·5	10 19	10 18	10 17	10 15	10 13·5	10 12·5	10 11·5	10 10
6 30	10 44·5	10 43·5	10 42·5	10 41	10 39	10 37·5	10 36	10 35	10 33	10 32	10 31	10 29	10 27	10 25
7 30	11 18	11 18	11 17	11 15	11 13·5	11 12·5	11 11	11 10	11 8·5	11 7	11 6	11 4	11 2·5	11 0·5
8 30	11 44·5	11 44	11 43	11 42	11 42	11 41	11 40·5	11 40	11 39	11 38	11 37	11 36	11 34	11 33·5
9 30	11 51	11 51	11 50	11 50	11 49	11 49	11 48·5	11 48	11 47·5	11 47	11 46·5	11 46	11 45	11 44
10 30	11 45·5	11 44·5	11 44·5	11 44	11 43	11 43	11 42·5	11 42	11 41·5	11 40	11 39·5	11 39	11 38	11 37·5
11 30	11 33·5	11 32·5	11 32·5	11 32	11 32	11 31·5	11 30	11 29	11 28·5	11 28	11 27·5	11 27	11 26	11 25

DECLINATION TABLE (W. H.). To be used in predicting the *Height* of high water at Liverpool for the mean parallax.

The heights in feet from Mr. HUTCHINSON's zero.

Moon's Transit.	0° Decl.	1° Decl.	2° Decl.	3° Decl.	4° Decl.	5° Decl.	6° Decl.	7° Decl.	8° Decl.	9° Decl.	10° Decl.	11° Decl.	12° Decl.	13° Decl.
h m	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
0 30	18·3	18·3	18·3	18·3	18·3	18·3	18·3	18·3	18·3	18·25	18·25	18·2	18·15	18·1
1 30	18·3	18·3	18·3	18·3	18·3	18·3	18·3	18·3	18·3	18·25	18·25	18·2	18·15	18·1
2 30	17·65	17·65	17·65	17·65	17·65	17·65	17·65	17·65	17·65	17·6	17·6	17·55	17·5	17·5
3 30	16·6	16·6	16·6	16·6	16·6	16·55	16·55	16·55	16·55	16·5	16·5	16·5	16·45	16·4
4 30	15·2	15·2	15·2	15·2	15·2	15·2	15·15	15·15	15·1	15·1	15·05	15·05	15·0	14·95
5 30	13·95	13·95	13·9	13·9	13·9	13·9	13·85	13·8	13·75	13·75	13·75	13·7	13·65	13·65
6 30	13·0	13·0	13·0	13·0	12·95	12·95	12·9	12·9	12·85	12·8	12·75	12·7	12·65	12·6
7 30	12·95	12·95	12·95	12·95	12·95	12·9	12·	12·9	12·85	12·8	12·75	12·7	12·65	12·6
8 30	13·95	13·95	13·9	13·9	13·85	13·85	13·85	13·8	13·75	13·75	13·7	13·65	13·65	13·55
9 30	15·35	15·35	15·35	15·35	15·35	15·35	15·35	15·35	15·3	15·3	15·25	15·25	15·2	15·15
10 30	16·8	16·8	16·8	16·8	16·75	16·75	16·75	16·75	16·7	16·7	16·65	16·6	16·55	16·5
11 30	17·7	17·7	17·7	17·7	17·65	17·65	17·65	17·65	17·65	17·65	17·6	17·55	17·55	17·45
	14° Decl.	15° Decl.	16° Decl.	17° Decl.	18° Decl.	19° Decl.	20° Decl.	21° Decl.	22° Decl.	23° Decl.	24° Decl.	25° Decl.	26° Decl.	27° Decl.
0 30	18·05	17·95	17·9	17·8	17·7	17·65	17·5	17·4	17·3	17·2	17·1	17·0	16·8	16·7
1 30	18·05	17·95	17·9	17·8	17·7	17·65	17·5	17·4	17·3	17·2	17·1	17·0	16·8	16·7
2 30	17·45	17·4	17·35	17·25	17·2	17·1	17·0	16·9	16·85	16·75	16·65	16·55	16·4	16·3
3 30	16·35	16·35	16·3	16·25	16·25	16·2	16·15	16·1	16·0	15·95	15·9	15·8	15·7	15·6
4 30	14·95	14·9	14·9	14·8	14·75	14·75	14·65	14·6	14·55	14·5	14·4	14·35	14·25	14·15
5 30	13·6	13·55	13·5	13·45	13·4	13·35	13·25	13·2	13·1	13·05	12·95	12·9	12·8	12·75
6 30	12·55	12·5	12·45	12·35	12·25	12·2	12·1	12·0	11·9	11·8	11·7	11·5	11·35	11·2
7 30	12·5	12·5	12·4	12·35	12·25	12·2	12·1	12·0	11·9	11·75	11·65	11·5	11·35	11·2
8 30	13·5	13·45	13·4	13·3	13·25	13·2	13·15	13·05	12·95	12·9	12·8	12·75	12·65	12·55
9 30	15·1	15·05	15·0	14·95	14·9	14·85	14·75	14·7	14·6	14·5	14·4	14·3	14·2	14·1
10 30	16·45	16·4	16·35	16·3	16·25	16·2	16·1	16·0	15·85	15·75	15·6	15·45	15·3	15·1
11 30	17·4	17·35	17·25	17·2	17·1	17·0	16·9	16·8	16·7	16·6	16·45	16·3	16·15	16·0

PARALLAX TABLE (W. T.). To be used in correcting the *Time* of high water at Liverpool, predicted for mean parallax 57′.

Moon's Transit.	54′.	55′.	56′.	57′.	58′.	59′.	60′.	61′.
h m	m	m	m	m	m	m	m	m
0 30	9	6	3	0	− 3	− 6	− 9	−12
1 30	7	5	2	0	− 2	− 5	− 7	− 9
2 30	5	3	2	0	− 2	− 3	− 5	− 6
3 30	3	2	1	0	− 1	− 2	− 3	− 3
4 30	1	1	0	0	0	− 1	− 1	− 1
5 30	1	1	0	0	0	− 1	− 1	− 2
6 30	4	3	1	0	− 1	− 3	− 4	− 5
7 30	9	6	3	0	− 3	− 6	− 9	−12
8 30	13	8	4	0	− 4	− 8	−13	−17
9 30	14	9	5	0	− 5	− 9	−14	−19
10 30	13	9	4	0	− 4	− 9	−13	−18
11 30	11	7	4	0	− 4	− 7	−11	−15

PARALLAX TABLE (W. H.). To be used in correcting the *Height* of high water at Liverpool, predicted for mean parallax 57′.

H. P. 54′.	H. P. 55′.	H. P. 56′.	H. P. 57′.	H. P. 58′.	H. P. 59′.	H. P. 60′.	H. P. 61′.
ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.
−1·2	−0·8	−0·4	0	+0·4	+0·8	+1·2	+1·6

TABLE (A).

Calculation of the Differences between the Time and Height of High Water at Liverpool, as due to the Moon's Declination, and as shown by Mr. HUTCHINSON's Observations, for every month of the year.

January.							
Moon's Transit.	Declination.	(W. H.) Height due to Declination.	(L. III.) Observed Height.	Diff. H.	(W. T.) Time due to Declination.	(L. II.) Observed Time.	Diff. T.
h m	°	feet.	feet.		h m	h m	m
0 30	18	17·70	17·47	−·23	11 15	11 13·7	−1·3
1 30	15	17·95	17·74	−·21	11 1	10 59·5	−1·5
2 30	10	17·60	17·52	−·08	10 48	10 47·0	−1·0
3 30	5	16·55	16·08	−·47	10 40	10 37·1	−2·9
4 30	5	15·15	15·14	−·01	10 31	10 31·1	+ ·1
5 30	8	13·75	13·19	−·06	10 33	10 33·8	+ ·8
6 30	14	12·55	12·61	+·06	10 44	10 49·3	+5·3
7 30	19	12·20	12·38	+·18	11 12	11 16·1	+4·1
8 30	19	13·20	13·56	+·36	11 41	11 37·8	−3·2
9 30	23	14·50	14·95	+·45	11 47	11 45·0	−2·0
10 30	22	15·85	15·98	+·13	11 41	11 38·5	−2·5
11 30	22	16·70	16·73	+·03	11 28	11 34·2	+6·2

TABLE (A). Continued.

Moon's Transit.	Declination.	(W. H.) Height due to Declination.	(L. III.) Observed Height.	Diff. H.	(W. T.) Time due to Declination.	(L. II.) Observed Time.	Diff. T.
February.							
h m	°	feet.	feet.		h m	h m	m
0 30	10	18·25	18·05	−·20	11 19	11 18·3	− ·7
1 30	5	18·30	18·04	−·26	11 5	11 8·2	+3·2
2 30	5	17·65	17·69	+·04	10 50	10 49·7	− ·3
3 30	8	16·55	16·73	+·18	10 39	10 37·3	−1·7
4 30	14	14·95	15·20	+·25	10 26	10 27·3	+1·3
5 30	18	13·40	13·39	−·01	10 22	10 26·3	+4·3
6 30	21	12·00	11·96	−·04	10 35	10 37·9	+2·9
7 30	21	11·95	11·78	−·17	11 10	11 8·0	−2·0
8 30	23	12·90	12·79	−·11	11 38	11 37·4	− ·6
9 30	22	14·60	14·47	−·13	11 47	11 51·0	+4·0
10 30	19	16·20	15·96	−·24	11 43	11 44·0	+1·0
11 30	15	17·35	17·39	+·04	11 32	11 31·5	− ·5
March.							
0 30	5	18·3	18·38	+·08	11 20	11 19·1	− ·9
1 30	8	18·25	18·49	+·24	11 4	11 3·9	− ·1
2 30	13	17·5	17·63	+·13	10 47·5	10 49·4	+1·9
3 30	16	16·3	16·31	+·01	10 33	10 33·3	+ ·3
4 30	21	14·6	14·47	−·13	10 19	10 20·3	+1·3
5 30	22	13·1	13·01	−·09	10 17	10 16·2	− ·8
6 30	22	11·9	11·42	−·48	10 33	10 30·2	−2·8
7 30	22	11·85	11·31	−·54	11 8·5	11 5·8	−2·7
8 30	20	13·1	12·68	−·42	11 40	11 40·7	+ ·7
9 30	15	15·05	14·62	−·43	11 50·5	11 50·3	− ·2
10 30	10	16·65	16·54	−·11	11 46	11 44·9	−1·1
11 30	6	17·65	17·67	+·02	11 35	11 33·4	−1·6
April.							
0 30	12	18·15	18·01	−·14	11 19	11 19·4	+ ·4
1 30	17	17·8	17·88	+·08	11	11 1·1	+1·1
2 30	20	17	17·01	+·01	10 42	10 43·4	+1·4
3 30	22	16·05	15·98	−·07	10 27	10 30·4	+3·4
4 30	23	14·45	14·44	−·01	10 15	10 14·2	− ·8
5 30	22	13·1	12·89	−·21	10 17	10 12·8	−4·2
6 30	20	12·1	11·44	−·66	10 36	10 31·8	−4·2
7 30	16	12·4	11·71	−·69	11 16·5	11 8·5	−8·0
8 30	11	13·65	13·28	−·37	11 45	11 44·0	−1·0
9 30	6	15·35	15·07	−·28	11 53	11 54·3	+1·3
10 30	5	16·75	16·61	−·14	11 46	11 48·2	+2·2
11 30	7	17·65	17·60	−·05	11 35	11 36·0	+1·0
May.							
0 30	20	17·5	17·33	−·17	11 14	11 16·3	+2·3
1 30	22	17·3	17·15	−·15	10 57	10 57·9	+ ·9
2 30	23	16·75	16·56	−·19	10 39	10 38·6	− ·4
3 30	22	16	15·75	−·25	10 27	10 25·1	−1·9
4 30	20	14·65	14·48	−·17	10 20	10 16·0	−4·0
5 30	16	13·5	13·33	−·17	10 25	10 18·8	−6·2
6 30	12	12·65	12·35	−·30	10 47	10 41·9	−5·1
7 30	7	12·85	12·58	−·27	11 23	11 20·6	−2·4
8 30	5	13·85	13·75	−·10	11 46·5	11 46·9	+ ·4
9 30	7	15·35	15·01	−·34	11 53	11 52·5	− ·5
10 30	12	16·55	16·28	−·27	11 46	11 46·9	+ ·9
11 30	17	17·2	17·01	−·19	11 32	11 31·2	− ·8

Table (A). Continued.

Moon's Transit.	Declination.	(W. H.) Height due to Declination.	(L. III.) Observed Height.	Diff. H.	(W. T.) Time due to Declination.	(L. II.) Observed Time.	Diff. T.
June.							
h m	°	feet.	feet.		h m	h m	m
0 30	23	17·2	16·73	−·47	11 12	11 13·4	+1·4
1 30	22	17·3	16·84	−·46	10 57	10 55·5	−1·5
2 30	20	17	16·74	−·26	10 42	10 39·8	−2·2
3 30	16	16·3	15·81	−·49	10 33	10 29·6	−3·4
4 30	11	15·05	14·69	−·36	10 28	10 24·9	−3·1
5 30	6	13·85	13·75	−·10	10 34	10 31·6	−2·4
6 30	5	12·95	13·07	+·12	10 52·5	10 52·9	+ ·4
7 30	8	12·8	13·09	+·29	11 22·5	11 24·6	+2·1
8 30	12	13·6	13·86	+·26	11 45	11 44·0	−1·0
9 30	17	14·95	14·93	−·02	11 50	11 49·2	− ·8
10 30	20	16·1	15·91	−·19	11 42	11 41·8	− ·2
11 30	22	16·7	16·57	−·13	11 28	11 28·9	+ ·9
July.							
0 30	19	17·6	17·08	−·52	11 14	11 13·5	− ·5
1 30	16	17·9	17·14	−·76	11 1	10 57·8	−3·2
2 30	11	17·55	16·84	−·61	10 49	10 45·2	−3·8
3 30	6	16·55	16·29	−·26	10 39	10 37·6	−1·4
4 30	5	15·15	14·96	−·19	10 31	10 33·1	+2·1
5 30	8	13·75	13·83	+·08	10 33	10 34·2	+1·2
6 30	13	12·6	12·87	+·27	10 45	10 51·6	+6·6
7 30	18	12·25	12·72	+·47	11 13	11 12·2	−0·8
8 30	21	13·05	13·39	+·34	11 40	11 38·9	−1·1
9 30	22	14·6	14·64	+·04	11 45·5	11 48·5	+3·0
10 30	23	15·75	15·61	−·14	11 40	11 39·6	− ·4
11 30	22	16·7	16·52	−·18	11 28	11 27·2	− ·8
August.							
0 30	11	18·2	17·68	−·52	11 19	11 17·5	−1·5
1 30	6	18·3	17·96	−·34	11 5	11 4·5	− ·5
2 30	5	17·65	17·37	−·28	10 51	10 51·6	+ ·6
3 30	7	16·55	16·46	−·09	10 39	10 35·4	−3·6
4 30	13	14·95	15·03	+·08	10 27	10 30·2	+3·2
5 30	17	13·45	13·46	+·01	10 23	10 28·7	+5·7
6 30	21	12	12·00	·00	10 35	10 38·4	+3·4
7 30	22	11·85	11·80	−·05	11 8	11 8·6	+ ·6
8 30	25	12·7	12·82	+·12	11 36	11 37·4	+1·4
9 30	22	14·6	14·34	−·26	11 47·5	11 46·9	− ·6
10 30	19	16·2	15·84	−·36	11 43	11 42·4	− ·6
11 30	16	17·25	16·99	−·26	11 32	11 30·1	−1·9
September.							
0 30	4	18·3	18·30	·00	11 20	11 20·4	+ ·4
1 30	7	18·3	18·39	+·09	11 10	11 3·0	−7·0
2 30	12	17·5	17·89	+·39	10 48	10 48·8	+ ·8
3 30	17	16·25	16·50	+·25	10 32·5	10 34·0	+1·5
4 30	20	14·65	14·94	+·29	10 20	10 21·3	+1·3
5 30	22	13·1	13·19	+·09	10 17	10 16·9	− ·1
6 30	23	11·8	11·63	−·17	10 32	10 32·1	+ ·1
7 30	22	11·85	11·78	−·07	11 8	11 4·8	−3·2
8 30	20	13·1	12·86	−·24	11 40	11 39·3	− ·7
9 30	16	15	14·78	−·22	11 50	11 50·3	+ ·3
10 30	11	16·6	16·38	−·22	11 46	11 46·4	+ ·4
11 30	6	17·65	17·69	+·04	11 35	11 35·2	+ ·2

TABLE (A). Continued.

Moon's Transit.	Declination.	(W. H.) Height due to Declination.	(L. III.) Observed Height.	Diff. H.	(W. T.) Time due to Declination.	(L. II.) Observed Time.	Diff. T.
		October.					
h m	°	feet.	feet.		h m	h m	h m
0 30	11	18·2	18·52	+·32	11 19	11 19·1	+ ·1
1 30	16	17·85	18·28	+·43	11 1	11 1·7	+ ·7
2 30	20	17	17·62	+·62	10 42	10 42·3	+ ·3
3 30	22	16	16·48	+·46	10 27	10 26·7	− ·3
4 30	23	14·45	14·76	+·31	10 15	10 14·7	− ·3
5 30	22	13·1	13·15	+·05	10 16	10 12·2	−3·8
6 30	20	12·1	11·70	−·40	10 36	10 29·1	−6·9
7 30	17	12·3	11·95	−·35	11 15	11 11·0	−4·0
8 30	13	13·55	13·37	−·18	11 45	11 45·1	+ ·1
9 30	7	15·35	15·13	−·22	11 53	11 54·0	+1·0
10 30	4	16·75	16·83	+·08	11 46	11 48·7	+2·7
11 30	7	17·65	17·92	+·27	11 35	11 35·3	+ ·3
		November.					
0 30	20	17·5	17·95	+·45	11 14	11 18·4	+4·4
1 30	22	17·3	17·73	+·43	10 57·5	10 58·7	+1·2
2 30	23	16·75	16·99	+·24	10 38	10 39·1	+1·1
3 30	22	16	16·01	+·01	10 27	10 24·1	−2·9
4 30	20	14·65	14·50	−·15	10 20	10 14·7	−5·3
5 30	16	13·5	13·33	−·17	10 24·5	10 17·1	−7·4
6 30	13	12·6	12·28	−·32	10 45	10 40·4	−4·6
7 30	7	12·85	12·62	−·23	11 23	11 21·3	−1·7
8 30	5	13·85	13·95	+·10	11 47	11 47·9	+ ·9
9 30	6	15·35	15·61	+·26	11 53	11 53·8	+ ·8
10 30	12	16·55	16·73	+·18	11 46	11 46·7	+ ·7
11 30	17	17·2	17·50	+·30	11 32	11 32·9	+ ·9
		December.					
0 30	23	17·2	17·25	+·05	11 12·5	11 12·9	+ ·4
1 30	21	17·4	17·28	−·12	10 58	10 54·8	−3·2
2 30	19	17·1	16·80	−·30	10 43	10 40·8	−2·2
3 30	16	16·3	15·89	−·41	10 33	10 28·7	−4·3
4 30	11	15·05	14·95	−·10	10 28	10 23·9	−4·1
5 30	8	13·75	13·82	+·07	10 33	10 30·6	−2·4
6 30	5	12·95	13·06	+·11	10 52·5	10 52·5	− ·0
7 30	7	12·85	13·21	+·36	11 23	11 23·4	+ ·4
8 30	12	13·6	14·07	+·47	11 45	11 45·7	+ ·7
9 30	17	14·95	15·35	+·40	11 50	11 48·2	−1·8
10 30	20	16·05	16·38	+·33	11 42·5	11 42·2	− ·3
11 30	21	16·8	17·14	+·34	11 29	11 27·7	−1·3

TABLE (B. H.). The Lunar Effect rejected.

Residual quantities. *Heights.*

☽'s Transit.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
Means	+·01	−·05	−·14	−·21	−·20	−·15	−·12	−·16	+·02	+·12	+·09	+·10
	Remainders.											
h m												
0 30	−·24	−·15	+·22	+·07	+·03	−·32	−·40	−·36	−·02	+·20	+·36	−·05
1 30	−·22	−·21	+·38	+·29	+·05	−·31	−·64	−·18	+·07	+·31	+·34	−·22
2 30	−·09	+·09	+·27	+·22	+·01	−·11	−·49	−·12	+·37	+·50	+·15	−·40
3 30	−·48	+·23	+·15	+·14	−·05	−·34	−·14	+·07	+·23	+·36	−·08	−·51
4 30	−·02	+·30	+·01	+·20	+·03	−·21	−·07	+·24	+·27	+·19	−·24	−·20
5 30	−·07	+·04	+·05	−·00	+·03	+·05	+·20	+·17	+·07	−·07	−·26	−·03
6 30	+·05	+·01	−·34	−·45	−·10	+·27	+·39	+·16	−·19	−·52	−·41	+·01
7 30	+·17	−·12	−·40	−·48	−·07	+·44	+·59	+·11	−·09	−·47	−·32	+·26
8 30	+·35	−·06	−·28	−·16	+·10	+·41	+·46	+·28	−·26	−·30	+·01	+·37
9 30	+·44	−·08	−·29	−·07	−·14	+·13	+·16	−·10	−·24	−·34	+·17	+·30
10 30	+·12	−·19	+·03	+·07	−·07	−·04	−·02	−·20	−·24	−·04	+·09	+·23
11 30	+·02	+·09	+·16	+·16	+·01	+·02	−·06	−·10	+·02	+·15	+·21	+·24

TABLE (C. H.). The Remainders in Table (B. H.) corrected by Interpolation.

Periodical and non-periodical part of the Solar Effect, in *tenths of feet.*

☽'s Transit.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
h m												
0 30	− 1	0	+ 2	+ 2	0	− 3	− 5	− 3	0	+ 3	+ 4	0
1 30	− 3	0	+ 3	+ 3	0	− 4	− 6	− 3	+ 1	+ 4	+ 3	− 2
2 30	− 3	0	+ 3	+ 3	0	− 4	− 5	− 2	+ 3	+ 5	+ 2	− 4
3 30	− 3	0	+ 2	+ 2	0	− 3	− 3	− 1	+ 3	+ 4	+ 1	− 5
4 30	− 2	0	+ 1	+ 1	0	− 1	− 1	0	+ 2	+ 2	0	− 3
5 30	0	0	− 1	− 1	0	+ 2	+ 2	+ 1	− 1	− 2	− 2	− 1
6 30	+ 1	0	− 4	− 4	0	+ 4	+ 4	+ 2	− 2	− 5	− 4	+ 1
7 30	+ 2	0	− 5	− 5	0	+ 5	+ 6	+ 3	− 3	− 5	− 3	+ 2
8 30	+ 3	0	− 4	− 4	0	+ 4	+ 5	+ 3	− 3	− 3	− 2	+ 3
9 30	+ 4	0	− 2	− 2	0	+ 3	+ 3	+ 2	− 3	− 3	− 1	+ 3
10 30	+ 2	0	− 1	− 1	0	+ 1	0	0	− 2	− 2	+ 0	+ 2
11 30	0	0	+ 2	+ 2	0	− 1	− 2	− 2	0	+ 2	+ 2	+ 1
Non-periodical	0	−1	− 2	− 2	−2	− 2	− 1	− 1	0	+ 1	+ 1	+ 1

SOLAR TABLE (W. H.). Correction of the Heights for the Sun's Effect, in *tenths of feet.*

☽'s Transit.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
h m												
0 30	− 1	− 1	0	0	− 2	− 5	− 6	− 4	0	+ 4	+ 5	+ 1
1 30	− 3	− 1	+ 1	+ 1	− 2	− 6	− 7	− 4	+ 1	+ 5	+ 4	− 1
2 30	− 3	− 1	+ 1	+ 1	− 2	− 6	− 6	− 3	+ 3	+ 6	+ 3	− 3
3 30	− 3	− 1	+ 0	0	− 2	− 5	− 4	− 2	+ 3	+ 5	+ 2	− 4
4 30	− 2	− 1	− 1	− 1	− 2	− 3	− 2	− 1	+ 2	+ 3	+ 1	− 2
5 30	0	− 1	− 3	− 3	− 2	0	+ 1	0	− 1	− 1	− 1	0
6 30	+ 1	− 1	− 6	− 6	− 2	+ 2	+ 3	+ 1	− 2	− 4	− 3	+ 2
7 30	+ 2	− 1	− 7	− 7	− 2	+ 3	+ 5	+ 2	− 3	− 4	− 2	+ 3
8 30	+ 3	− 1	− 6	− 6	− 2	+ 2	+ 4	+ 2	− 3	− 2	− 1	+ 4
9 30	+ 4	− 1	− 4	− 4	− 2	+ 1	+ 2	+ 1	− 3	− 2	0	+ 4
10 30	+ 2	− 1	− 3	− 3	− 2	− 1	− 1	− 1	− 2	− 1	+ 1	+ 3
11 30	+ 0	− 1	0	0	− 2	− 3	− 3	− 3	0	+ 1	+ 3	+ 2

TABLE (B. T.). The Lunar Effects rejected.

Residual Quantities.—*Times.*

☽'s Transit.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
Means.....	+·2	+·9	−·5	−·6	−1·4	−·8	+·1	+·5	−·5	−·8	−1·0	−1·5
	Remainders.											
h m	m	m	m	m	m	m	m	m	m	m	m	m
0 30	−1·5	−1·6	− ·4	+1·0	+3·7	+2·2	− ·6	−2·0	+ ·9	+ ·9	+5·4	+1·9
1 30	−1·7	+2·3	+ ·4	+1·7	+2·3	− ·7	−3·3	−1·0	−6·5	+1·5	+2·2	−1·7
2 30	−1·2	−1·2	+2·4	+2·0	+1·0	−1·4	−3·9	+ ·1	+1·3	+1·1	+2·1	− ·7
3 30	−3·1	−2·6	+ ·8	+4·0	− ·5	−2·6	−1·5	−4·1	+2·0	+ ·5	−1·9	−2·8
4 30	− ·1	+ ·4	+1·8	− ·2	−2·6	−2·3	+2·0	+2·7	+1·8	+ ·5	−4·3	−2·6
5 30	+ ·6	+3·4	− ·3	−3·6	−4·8	−1·6	+1·1	+5·2	+ ·4	−3·0	−6·4	− ·9
6 30	+5·1	+2·0	−2·3	−3·6	−3·7	+1·2	+6·5	+2·9	+ ·6	−6·1	−3·6	+1·5
7 30	+3·9	−2·9	−2·2	−7·4	−1·0	+2·9	− ·9	+ ·1	−2·7	−3·2	− ·7	+1·9
8 30	−3·4	−1·5	+1·2	− ·4	+1·8	− ·2	−1·2	+ ·9	− ·2	+ ·9	+1·9	+2·2
9 30	−2·2	+3·1	+ ·3	+1·9	+ ·9	− ·0	−2·9	−1·1	+ ·8	+1·8	+1·8	·3
10 30	−2·7	+ ·1	− ·6	+2·8	+2·3	+ ·6	− ·5	−1·1	+ ·9	+3·5	+1·7	+1·2
11 30	+6·0	−1·4	−1·1	+1·6	+ ·6	+1·7	− ·9	−2·4	+ ·7	+1·1	+1·9	+ ·2

TABLE (C. T.). Periodical part of the Solar Effect.

The remainders in Table (B. T.) corrected by interpolation.

☽'s Transit.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
h m												
0 30												
1 30	−2	0	+2	+3	+2	0	−3	−4	−2	0	+2	0
2 30	−2	0	+3	+3	+2	0	−3	−2	0	+2	+2	0
3 30	−3	0	+3	+4	+3	0	−4	−4	0	+1	+1	0
4 30												
5 30	+3	0	−3	−4	−3	0	+4	+4	0	−4	−4	0
6 30	+4	0	−4	−5	−4	0	+5	+5	0	−5	−5	0
7 30	+4	0	−4	−5	−4	0	+2	+2	0	−3	−3	0
8 30												
9 30												
10 30												
11 30												

SOLAR TABLE (W. T.). Correction of the Times for the Sun's Effect.

For this correction we may use Table (C. T.), omitting the correction for the numbers there omitted till further investigations have made the Table more correct: and introducing also the interpolated nonperiodical correction, viz.

Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
	m		m	m	m		m		m	m	m
0	+1	0	−1	−1	−1	0	+1	0	−1	−1	−1

XIII. *On the Action of Light upon Plants, and of Plants upon the Atmosphere.* By Charles Daubeny, *M.D. F.R.S., Professor of Chemistry and of Botany in the University of Oxford.*

Received November 3,—Read December 17, 1835.

THE researches of Priestley, Ingenhousz, Senebier, Ellis, and above all of the younger Saussure, have long put us in possession of the leading facts appertaining to the influence of light upon the green parts of plants; and Professor Decandolle has embodied the substance of all that had been ascertained on this subject, up to the year 1831, in his admirable work on Vegetable Physiology. But there appear, by the confession of this latter naturalist, to remain certain subordinate questions respecting this same function, which, though perhaps occasionally touched upon by the above-cited experimentalists and by others, can scarcely be said to have as yet obtained a satisfactory reply.

The first of these questions relates to the *nature* of the influence which, in the cases alluded to, is assignable to light. As this agent often produces chemical changes by its direct action upon inorganic bodies, decomposing saline solutions, discolouring oils, and reducing metallic oxides, so it may be supposed to operate directly upon the air, and to possess the power of decomposing carbonic acid, when this substance is presented to it within the pores of the vegetable tissue. And, on the other hand, as light appears to be a specific stimulus to the vital functions of animals, so it may be imagined to act in a similar manner on those of plants, thus enabling them to secrete from the carbonic acid presented to them the carbon required for their nutrition.

Another point as yet undecided relates to the *extent* of the influence it exerts over the vegetable kingdom; or, in other words, the degree in which certain processes attributed to its presence are capable of counteracting others that are going on at all times, whether light be absent or not. Thus, although it may be conceded, as a fact already well established, that plants purify the air in the sunshine, it still remained to be proved by more decided experiments than had hitherto been instituted, whether the quantity of oxygen given out by them during the day exceeded that absorbed during the night; and moreover, supposing this latter question answered in the affirmative, whether the probable excess was likely to be such, as would

afford a counterpoise to the effects produced by animal respiration, combustion, and the like*.

After considering therefore the *mode* in which light appears to affect the functions of plants, I shall naturally proceed to examine the *extent* of the changes produced by the latter upon the air through its influence.

Part I.—*On the Influence of Light upon Plants.*

If of the two modes of considering the operation of light above noticed we adopt the second, that is, if light be supposed to affect plants by a specific stimulus, such as it exerts on animals, and not in the first instance the air as a chemical agent, it ought to follow, that those portions of the spectrum which possess the strongest illuminating power, should exercise upon them the most powerful influence, and produce the most decided effects.

Senebier, however, has stated, that the green colour of leaves, which is supposed to be connected with the decomposition of carbonic acid and the evolution of oxygen, is produced most rapidly under the action of the violet ray†; and as the latter, from the feeble light and heat it communicates, seems almost inert, with reference to the functions *of animals*, such a circumstance, if substantiated, would seem strongly to favour the contrary hypothesis.

This latter view of the mode in which carbonic acid is decomposed within the vessels of the plant, would likewise be somewhat confirmed, if it should appear, that whilst the above process was most favoured by violet light, other functions, which are affected by the presence of this agent, but which evidently depend upon a process taking place in the vessels of the plant, are influenced in proportion to the luminousness of the ray; whilst, if the same law were found to prevail in both these cases, and if all the processes alluded to could be proved to go on most rapidly under the influence of the darkest and most refrangible portion of the solar spectrum, a curious difference between the *mode* of its operation upon the vegetable and animal kingdoms might then be suggested.

* I am aware it may be urged, that the quantity, of carbonic acid added, and of oxygen subtracted by these latter means within any moderate period of time, is in itself so small compared with the entire bulk of the atmosphere, that we must not argue, because the constitution of the latter has appeared to continue uniform ever since accurate methods have been devised for determining it, therefore that no change can be taking place insensibly in the proportions of its ingredients.

Still, however, when we recollect, how many ages have elapsed since the present races of animals were created, and how many more since the existence of others, which, although extinct, appear from the analogies of their structure to have carried on the same respiratory process which those now in existence fulfill, and therefore could not have endured an atmosphere much more highly charged with oxygen than the present one, we cannot help feeling, that nature must have some means at her disposal, by which the purity of the atmosphere is restored, and its constitution thus maintained without alteration.

† Mém. de Phys. Chim., tom. ii. p. 55.

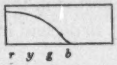
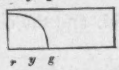
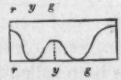
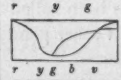
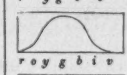
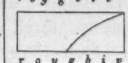
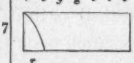
I felt therefore desirous of putting to the test of experiment the two following questions: 1st, whether the several solar rays act upon plants with equal or with different degrees of energy; and secondly, whether all the functions of plants that seem dependent on the presence of light are affected in the same ratio by similar rays.

Now the following functions are found to depend in great measure on the influence of light.

1. The decomposition of carbonic acid, and the consequent evolution of oxygen, already spoken of.
2. The green colour of leaves, and other analogous parts.
3. The expansion or unfolding of the leaves in certain species, the folding of which on the close of day constitutes what has been called "the sleep of plants."
4. The irritability belonging to certain other plants, such as the *Mimosa pudica*.
5. The exhalation of water from the leaves.
6. The absorption of the same by the roots.

The difficulty, however, of comparing the relative intensity of the light transmitted by the various coloured media, which were employed in my experiments, induced me to content myself with showing, that the effect of light upon plants corresponds with its illuminating, rather than with its chemical, or its calorific influence; and to wave the more difficult inquiry, whether its operation upon the vegetable kingdom exactly keeps pace with the increase in its own intensity. And in order to show that the former is the case, I will in the first place set down the order of sequence of the several media, with reference one to the other, in respect to their illuminating, their calorific, and their chemical power; stating at the same time, by means of a diagram, the rays intercepted and transmitted by each, as was determined for me by Professor Powell; and afterwards proceed to a statement of their respective influence upon plants.

With regard to the means adopted for estimating these points, I need perhaps only remark, that the relative illuminating power of the several media was ascertained by the number of thicknesses of wire gauze, which produced a certain definite degree of obscurity or indistinctness when interposed; their relative calorific power, by the number of degrees which a thermometer with a blackened bulb was raised in a given time by the light transmitted through each; their relative chemical influence, by the time required to reduce paper moistened with a solution of nitrate of silver to a certain standard point of discoloration. The figures in the annexed Table therefore must be understood to express nothing more than the *order of sequence*, and not to indicate the ratio between the several media in any of the above respects.

Nature of the media.	Its type.	Its illuminating influence.	Its calorific influence.	Its chemical influence.
Transparent glass	.....	7	7	7
Orange, No. 5.	 r y g b	6	6	4
Red, No. 4.	 r y g	4	5	0
Blue, No. 3.	 r y g	4	3	6
Purple, No. 2.	 r y g b v	3	4	6
Green, No. 1.	 r o y g b i v	5	2	3
Bottle containing a solution of ammonio-sulphate of copper, No. 6.	 r o y g b i v	2	1	5
Bottle containing port wine, No. 7	 r	1	3	0

Now it remained to be seen, with which of the above scales the power of occasioning a decomposition of carbonic acid in the vessels of the plant, and of forwarding those other functions of vegetable life which depend upon light, most nearly corresponded.

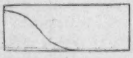
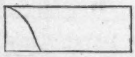
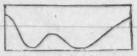
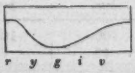
For this purpose, a certain number of fresh leaves, which presented in each case an extent of surface as nearly as possible equal, and had been previously ascertained to give out equal quantities of oxygen, were introduced severally into jars, filled with water impregnated with carbonic acid gas, placed on the surface of a pneumatic trough, and exposed for a certain time to the influence of the solar rays.

The jars, in which the leaves thus selected stood, were severally covered over by a wooden screen, which intercepted all light from the included jar, excepting in front, where a frame was fitted to it of a nature calculated to support, either a circular pane of glass, or a flat bottle of corresponding dimensions.

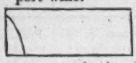
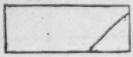
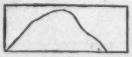
By fixing into the frame the various coloured media with which it was intended to operate, I was enabled to ascertain the influence which the light transmitted through each might exert upon the plant included.

From a variety of experiments, which it seems unnecessary to detail, in as much as they merely tend to confirm the statements of preceding observers, it appeared, that the kind of leaf selected made but little difference in the result. I therefore contented myself with selecting such, as could be procured most readily, and in the freshest condition.

The first plant operated on was the common Cabbage (*Brassica oleracea*); and as some of the results obtained in this instance appeared anomalous, in as much as they indicated an evolution of pure nitrogen, which I have detected in none of my other experiments, three or more trials with each kind of glass were in this instance had recourse to. The following were the results obtained.

Media through which the light was transmitted.	State of the weather.	Proportion between the whole quantity of gas obtained in the two jars.	Proportion per cent. between the oxygen and nitrogen in the two jars.	Proportion between the oxygen in the two jars.	Proportion between the nitrogen in the two jars.
Jar 1. Transparent window-glass Jar 2. Glass No. 5. (Orange.) Type  r y g	*Exp.* 1. Bright sunshine.	Jar 1. 100 Jar 2. 80	O. 44 N. 56 O. 40 N. 60	100 77·5	100 86
	Exp. 2. Bright sunshine.	Jar 1. 100 Jar 2. 82	O. 33 N. 66 O. 33·4 N. 66·6	100 80	100 83
	Exp. 3.* Bright sunshine.	Jar 1. 100 Jar 2. 83	O. 0 N. 100 O. 0 N. 83		
Jar 1. Same as before. Jar 2. Glass No. 4. (Crimson-coloured.) Type  r y g	*Exp.* 1. Bright sunshine with a few fleecy clouds.	Jar 1. 100 Jar 2. 45	O. 37·5 N. 62·5 O. 33·3 N. 66·6	100 40	100 48
	Exp. 2. Strong bright sunshine.	Jar 1. 100 Jar 2. 54	O. 39 N. 61 O. 38 N. 62	100 52·5	100 55
Jar 1. Same as before. Jar 2. Glass No. 3. (Dark blue.) Type  r y g	*Exp.* 1. Sunshine with partial clouds.	Jar 1. 100 Jar 2. 57·5	O. 33 N. 66 O. 34·6 N. 65·4	100 56	100 56·5
	Exp. 2. Bright sunshine without clouds.	Jar 1. 100 Jar 2. 57·5	O. 33·3 N. 66·6 O. 30·2 N. 69·8	100 70·5	100 53·5
Jar 1. Same as before. Jar 2. Glass No. 2. (Purple.) Type  r y g i v	*Exp.* 1. Sunshine without clouds.	Jar 1. 100 Jar 2. 47	O. 38·4 N. 61·6 O. 30 N. 70	100 35·4	100 51
	Exp. 2. Sunshine more obscured than in Exp. 1.	Jar 1. 100 Jar 2. 31·4	O. 25 N. 75 O. 22 N. 78	100 27·7	100 26·6

* N.B. The leaves in this latter case were quite fresh, and had not been previously immersed in water.

Media through which the light was transmitted.	State of the weather.	Proportion between the whole quantity of gas obtained in the two jars.	Proportion per cent. between the oxygen and nitrogen in the two jars.	Proportion between the oxygen in the two jars.	Proportion between the nitrogen in the two jars.
Jar. 1. Same as before. Jar. 2. Bottle No. 7. Filled with port wine. Type  r y g b i v	*Exp.* 1 & 2. Sunshine of different degrees of brightness.	Jar 1. 100 Jar 2. No Gas.			
Jar 1. Same as before. Jar 2. Bottle No. 6. Filled with the ammonio-sulphate of copper. Type  r y g b i v	*Exp.* 1. Sunshine of feeble intensity, though without dense clouds.	Jar 1. 100 Jar 2. 18	O. 47 N. 53 O. 37·8 N. 62·2	100 14·5	100 20·6
	Exp. 2. Bright and cloudless sunshine.	Jar 1. 100 Jar 2. 49	O. 32·7 N. 67·3 O. 37 N. 63	100 55	100 46
Jar 1. Same as before. Jar 2. Glass No. 1. (Green.) Type  r y g b i v	*Exp.* 1. Bright sunshine with occasional clouds.	Jar 1. 100 Jar 2. 32·8	O. 44 N. 56 O. 34·7 N. 65·3	100 26	100 38
	Exp. 2. Sunshine, for the most part bright.	Jar 1. 100 Jar 2. 25	O. 34 N. 66 O. 26·7 N. 73·3	100 18	100 25·6
	Exp. 3. Sunshine feeble and intermitting.	Jar 1. 100 Jar 2. no gas collected.			

The *second series* of experiments undertaken was with the leaves of the *Salicornia herbacea*, and the following were the results obtained. Temperature 65° Fahr.

Media through which the light was transmitted.	State of the weather.	Proportion between the whole quantity of gas obtained in the two jars.	Proportion per cent. between the oxygen and nitrogen in the two jars.	Proportion between the oxygen in the two jars.	Proportion between the nitrogen in the two jars.
Jar 1. Transparent glass .... Jar 2. No. 5. Glass (Orange)..	Feeble sunshine.	Jar 1. 100 Jar 2. 67	O. 25 N. 75 O. 16·4 N. 83·6	100 44	100 75
Jar 1. Transparent ..... Jar 2. No. 4. (Red).....	Feeble sunshine.	Jar 1. 100 Jar 2. 60	O. 10 N. 90 O. 0 N. 60	100 0	100 66·5
Jar 1. Transparent ..... Jar 2. No. 3. (Blue)	Feeble sunshine.	Jar 1. 100 Jar 2. 20	O. 25 N. 75 O. 0 N. 20	100 0	100 26·7

Media through which the light was transmitted.	State of the weather.	Proportion between the whole quantity of gas obtained in the two jars.	Proportion per cent. between the oxygen and nitrogen in the two jars.	Proportion between the oxygen in the two jars.	Proportion between the nitrogen in the two jars.
Jar 1. Transparent Jar 2. No. 2. (Purple)	Bright sunshine, with a few clouds.	Jar 1. 100 Jar 2. 16	O. 46 N. 64 O. 15 N. 85	100 5·25	100 21·4
Jar 1. Transparent Jar 2. No. 7. Bottle filled with port wine	Bright sunshine.	Jar 1. 100 Jar 2. 0			
Jar 1. Transparent Jar 2. No. 6. Bottle (with ammonio-sulphate of copper)	Bright sun, though with a few clouds.	Jar 1. 100 Jar 2. 42	O. 33 N. 66 O. 0 N. 42	100 0	100 63
Jar 1. Transparent Jar 2. No. 1. Glass (Green)	Bright sunshine.	Jar 1. 100 Jar 2. 43	O. 46 N. 64 O. 0 N. 43	100 0	100 67

The *third series* undertaken was with common Sea Wrack (*Fucus digitatus*), immersed in water of temperature 65°, and the following were the results obtained.

Media through which the light was transmitted.	State of the weather.	Proportion between the whole quantity of gas obtained in the two jars.	Proportion per cent. between the oxygen and nitrogen in the two jars.	Proportion between the oxygen in the two jars.	Proportion between the nitrogen in the two jars.
Jar 1. Transparent glass Jar 2. No. 5. (Orange)	*Exp.* 1. Feeble sunshine.	Jar 1. 100 Jar 2. 31	O. 55·5 N. 44·5 O. 14·2 N. 85·8	100 7·9	100 59
	Exp. 2. Bright sunshine.	Jar 1. 100 Jar 2. 19·2	O. 60 N. 40 O. 44 N. 56	100 14	100 27
Jar 1. Transparent Jar 2. No. 4. (Red)	Bright sunshine.	Jar 1. 100 Jar 2. 10·8	O. 75 N. 25 O. 46 N. 54	100 6·65	100 23
Jar 1. Transparent Jar 2. No. 3. Glass (Blue)	*Exp.* 1. Bright sunshine.	Jar 1. 100 Jar 2. 13	O. 69 N. 31 O. 31·5 N. 68·5	100 6	100 28·7
	Dull day.	Jar 1. 100 Jar 2. 7·3	O. not determined. O. 0 N. 7·3		
Jar 1. Transparent Jar 2. No. 2. Glass (Purple)	Feeble sunshine.	Jar 1. 100 Jar 2. 12·5	O. 60 N. 40 O. 8·8 N. 91·2	100 1·8	100 28·5
Jar 1. Transparent Jar 2. No. 6. Bottle (containing a solution of ammonio-sulphate of copper)	Bright sunshine.	Jar 1. 100 Jar 2. 8	O. 70 N. 30 O. 19 N. 81	100 2	100 21·7
Jar 1. Transparent Jar 2. No. 1. (Green)	*Exp.* 1. Weak sunshine.	Jar 1. 100 Jar 2. 7·6	O. 40 N. 60	100	100 7·6
	Exp. 2. Strong sunshine.	Jar 1. 100 Jar 2. 5·2	O. 65 N. 35	100	100 5·2

Bottle No. 7 with port wine tried without effect.

The *fourth series*, with leaves of *Tussilago hybrida*, in water of temp. 70°, gave the following results.

Media through which the light was transmitted.	State of the weather.	Proportion between the whole quantity of gas obtained in the two jars.	Proportion per cent. between the oxygen and nitrogen in the two jars.	Proportion between the oxygen in the two jars.	Proportion between the nitrogen in the two jars.
Jar 1. Transparent glass. Jar 2. Glass No 5. (Orange.)	Sultry day.	100 56	O. 45 N. 55 O. 41 N. 59	100 51	100 60
Jar 1. Transparent. Jar 2. Glass No. 4. (Red.)	Ditto.	100 49	O. 44·5 N. 55·5 O. 61 N. 59	100 67·5	100 39
Jar 1. Transparent. Jar 2. Glass No. 3. (Blue.)	Ditto.	100 41	O. 62 N. 38 O. 33 N. 66	100 22	100 72
Jar 1. Transparent. Jar 2. Glass No. 2. (Purple.)	Ditto.	100 10·7	O. 54 N. 46		
Jar 1. Transparent. Jar 2. No. 1. (Green.)	Ditto.	100 14	O. 53 N. 47 O. none, or barely any.		
Jar 1. Transparent. Jar 2. Bottle No. 6. containing the copper solution.	Ditto.	100 21	O. 50 N. 50 O. 13 N. 87	100 5·5	100 20·5

Fifth Series, with leaves of *Cochlearea Armoracia* immersed in water having a temperature of 72°, gave the following results.

Media through which the light was transmitted.	State of the weather.	Proportion between the whole quantity of gas obtained in the two jars.	Proportion per cent. between the oxygen and nitrogen in the two jars.	Proportion between the oxygen in the two jars.	Proportion between the nitrogen in the two jars.
Jar 1. Transparent glass .... Jar 2. No. 5. (Orange)	Bright sunshine. Th. 80°.	100 75	O. 57 N. 43 O. 54 N. 46	100 71	100 80
Jar 1. Transparent ..... Jar 2. No. 2. (Blue)	Ditto.	100 24	O. 57 N. 43 O. 27 N. 73	100 11·4	100 40
Jar 1. Transparent ..... Jar 2. No. 5. (Green).....	Ditto.	100 14·5	O. 57 N. 43 O. 20 N. 80	100 5·25	100 26·8
Jar 1. Transparent ..... Jar 2. No. 4. (Red).....	Ditto.	100 15	Not measured. O. 31 N. 69		
Jar 1. Transparent ..... Jar 2. No. 3. (Purple) ..... Jar 3. No. 1. (Green)..... Jar 4. No. 6. (Bottle with copper solution) ..	Ditto.	100 36 20 12	O. 52 N. 48 O. 35 N. 65 O. 16 N. 84 O. 5 N. 95	100·0 24·2 6·2 1·2	100 48 35·2 22·5

N.B. In another experiment with No. 6. no gas at all was collected.

Sixth series, with sprigs of *Mentha viridis* immersed in water of the temperature of 74° Fahr.

Media through which the light was transmitted.	State of the weather.	Proportion between the whole quantity of gas obtained in the two jars.	Proportion per cent. between the oxygen and nitrogen in the two jars.	Proportion between the oxygen in the two jars.	Proportion between the nitrogen in the two jars.
Jar 1. Transparent glass	Cloudless sky.	100	O. 59 N. 41	100	100
Jar 2. No. 5. (Orange)		70	O. 37 N. 63	44	108
Jar 3. No. 2. (Blue)	Thermometer 80°.	22·5	O. 8 N. 92	30	50·5
Jar 4. No. 6. (Bottle with copper solution)		20·0	O. 4 N. 96	13·5	47·0
Jar 1. Transparent glass	Ditto.	100	O. 47 N. 53	100	100
Jar 2. No. 1. (Green).....		20	O. 7 N. 93	3	34
Jar 3. No. 3. (Purple)		20	O. 0 N. 100	0	40
Jar 4. No. 4. (Red)		30	O. 12 N. 88	7·7	49

Similar experiments made upon leaves of *Rheum Rhaponticum*, of *Allium ursinum*, and of various species of Meadow-grass, corroborated the same conclusions; as likewise did some on plants confined in atmospheric air, containing about six per cent. of carbonic acid, and exposed to these several media.

In the above experiments the proportion of oxygen was ascertained, by heating the air in a bent graduated tube with phosphorus, and observing the diminution of capacity thereby occasioned; two per cent. being allowed for the expansion caused in nitrogen by phosphorous vapour. This method, which I have always found to give very uniform results, I adopted in preference to that of exploding the gas with hydrogen, as being less troublesome and more expeditious, in a climate so damp as ours, than a process requiring the aid of electricity.

The constant presence of more or less nitrogen in the air emitted by the plant, is a circumstance which, although often before observed, deserves, nevertheless, here to be briefly adverted to.

Its quantity appeared to be relatively smaller in proportion to the intensity of the solar influence, being always least under transparent glass; and where the light transmitted was not energetic enough to cause any emission of oxygen at all, still some portion of nitrogen would frequently be given out. Perhaps this circumstance may admit of explanation, by considering the emission of gas from leaves, when exposed to light under water, as derived from two sources; the first, the disengagement of a portion of atmospheric air which it had previously absorbed, and whose place within the tissue of the plant is probably supplied, either by the water with which it is surrounded, or by the carbonic acid with which this water is impregnated; the latter, the emission of pure oxygen, derived from a decomposition of the carbonic acid in contact with it.

Hence in Experiment 1. with cabbage-leaves, where we obtained 100 parts of a gas consisting of oxygen 44, and nitrogen 56 parts, we may suppose that the leaves had emitted,

Of atmospheric air which had been previously absorbed	68 parts:	consisting of	nitrogen 56 oxygen 12
together with excess of oxygen . .	32	Excess of oxygen . .	32
Total of gas obtained	100	Total of oxygen . .	44

whereas, when orange-coloured glass No. 5. was employed, we obtained,

Of atmospheric air	60·5	Oxygen	12·6
Excess of oxygen	19·4	Excess of oxygen .	19·4
Total of gas . .	79·9	Total of oxygen .	32·0

The two most difficult cases to explain seem to be, first, the evolution of pure nitrogen, and secondly, that of the same gas accompanied with a smaller proportion of oxygen than that present in the atmosphere.

In the instance in which the former was observed, no incipient putrefaction could be suspected by way of accounting for its occurrence, for the plants were fresh and healthy; and the circumstance that gas is not disengaged at all in the dark, proves the evolution of nitrogen to be in both cases a process connected with the same kind of action as that to which the emission of oxygen is to be ascribed.

Perhaps the phenomenon may be better understood by reference to the experiments of the younger Saussure, which go to prove; that oxygen becomes fixed in the plant in a condition, such as renders it incapable of being withdrawn from the vegetable tissue by the air-pump, or by other mechanical means; that it there unites with the carbon, so as to bring the latter into a fit state for the plant to assimilate it; and that it is then again disengaged from its combination, by a process not unaptly compared by the late Professor Burnett to the digestion of animals.

Now for this latter function to be discharged, the stimulus of the more luminous rays may be requisite, whilst that of the duller portions of the spectrum may suffice for the mere respiration of the plant, or for the elimination of the residuary air. Hence when rays of the latter description are alone transmitted, the composition of the gas evolved may even indicate a smaller amount of oxygen than that present in atmospheric air, because a portion of this element had become combined with certain of the carbonaceous principles present in the vegetable tissue, or been fixed in some manner within the plant.

The other processes, enumerated as under the influence of solar light, appear to be subjected to the same law, as that by which we have seen the decomposition of carbonic acid in the green parts of plants to be regulated.

From a few experiments I have made on the secretion of green matter in the leaves, I should be led to infer, in contradiction to the results of Senebier, that the most luminous rays were most influential; the orange glass, whose chemical influence was as 4, whilst its illuminating power was as 6, quickly imparting to the primordial leaves of beans which had just appeared above ground a bright green hue, whereas

under the ammonio-sulphate, whose illuminating power was as 2, whilst its chemical influence was as 5, they continued of a pale yellow, scarcely indeed of a shade darker than in another case where light was completely excluded.

I have made some experiments with similar results on the colours of flowers, the intensity or depth of which appeared also to depend on the brightness of the kind of light that had been allowed admission to them.

The irritability of the Sensitive Plant was likewise found to be dependent on the influence of bright rays, and not of those which act chemically. Six healthy sensitive plants were introduced in the beginning of August into an oblong box, with partitions so arranged, that each pot was in contact with a differently coloured light. In five weeks' time, that which had been exposed to the full light of the sun, transmitted through transparent glass, was still excitable, as was the one covered with the orange-coloured; but those which had received merely the portions of light transmitted through the copper solution, through port wine, and through a solution of green muriate of copper, as well as one which had been kept in entire darkness, lost all their irritability. Yet in each case the temperature was kept up to the same point by means of a hot-bed.

The exhalation of moisture from the leaves, and the absorption of it by the roots, are the last processes dependent on the action of light to which my attention has been directed. The results of my experiments on these two points confirm the same general inference as that to which the foregoing ones point; but having met with some apparent anomalies, I shall forbear at present to report the numerical results of the respective trials made with various glasses. It will be sufficient to lay before the Society a statement of the plan on which the experiments were conducted, and to particularize one or two which tend to show, that the processes above alluded to are probably dependent on the combined action of heat and light, coupled with those mechanical influences, which operate upon dead, as well as upon living organic matter.

The method whereby I proposed to estimate the degree, in which the exhalation of moisture depended on the quality of the light admitted to the plant, was in itself sufficiently simple. It consisted, in placing some plant growing in a pot, in a square tin vessel, the margin of which received in a groove a cucumber frame of sufficiently large dimensions to inclose it. All communication with the external air was cut off, by means of a little oil introduced into the groove into which the edges of the frame dipped, and the moisture exhaled was absorbed by concentrated sulphuric acid, placed in shallow earthen pots, along with the plant, in the interior of the tin vessel.

By weighing these vessels, just before the plant was introduced, and immediately after it had been taken out, I hoped to ascertain the amount of water that had been evolved, and after deducting from the sum total the quantity which had been previously found to be given off by the plant in the dark, I concluded that the remainder ought to represent the quantity to be set down to the action of light. But as it was impos-

sible to command an uniform intensity of solar radiation during the whole period occupied by any one series of experiments, another plant of the same kind and size was placed under transparent glass; and from the comparative amount of moisture emitted by it, I calculated what might be the difference in the amount of solar influence during the period at which the experiments were carried on.

Now although the experiments conducted on the above plan in general tended to show, that the extrication of moisture, *cæteris paribus*, was most abundant in proportion to the intensity of the light admitted, (orange glass in general causing more moisture to be exhaled than red or green,) yet in some instances blue and purple glasses, and still more remarkably, bottles filled with the cupreous solution, would cause a more abundant exhalation than orange or even transparent glass. Here, however, another principle seems to come into play, namely, the influence of heat radiated from the surface of the screen. This I infer, first, because the quantity of water exhaled under the influence of the copper solution became greatest, when the state of the weather was such as to elevate the temperature of the liquid considerably above that of the surrounding atmosphere; and, secondly, because a bottle filled with water blackened with ink to such a degree, as to transmit just as much light, so far as could be measured by the eye, as that filled with the copper solution was found to do, caused an equally considerable amount of water to be evolved by the plant.

Thus I selected two plants of the Tree Mallow (*Lavatera arborea*), which, by a previous experiment, had been found to exhale in the open air equal quantities of moisture, and placed the one under a frame, into which were inserted the bottles of ink and water, and the other under one with the solution of ammonio-sulphate of copper. Both fluids soon acquired in the sun a temperature from 110° to 120° Fahr.; and at the end of two hours the sulphuric acid in connexion with each of the plants was successively weighed, and the increase found to be nearly uniform; that under the ink and water having gained 150 grains per hour, that under the cupreous solution 162 grains in the same time.

Now as water with the addition of a little ink is known to absorb the rays proceeding from all parts of the spectrum in an equal ratio, it follows, that the effect produced in either instance must be ascribed to the heat radiated, and not to any peculiar virtue of the violet extremity in stimulating the vegetative functions.

Yet it is curious that the presence of some light seems essential to the due continuance of this process. The same plants which had been employed in the preceding experiment were placed out in the sun on a bright day; one, as before, under the influence of the light transmitted through the cupreous solution, the other under a frame covered over with blue tiles, which, together with the liquid, soon became heated by the sun's rays to the temperature of 110° or 120° of Fahrenheit. At the end of a certain time the sulphuric acid contained in the tin vessel which had inclosed the plant exposed to the action of the violet end of the spectrum had gained at the rate of 159 grains in the hour, (which is within 3 grains of the amount ob-

tained in the previous experiment,) whilst that in connexion with the one covered with the tiles had only increased by 32 grains.

Thus it would appear, that although heat assists the process, some degree of light is essential to its activity.

I was desirous, likewise, of ascertaining whether the brightest kind of light attainable by artificial means contributed in any degree to the process under consideration; Professor DECANDOLLE having found that the leaves of plants placed in a cellar became green on exposure to a strong light from lamps, and that their flowers even reversed their natural periods of opening, when the cellar was illuminated by night, and kept dark during the day.

In my own experiments, the light employed was that produced by a jet of mixed oxygen and hydrogen directed upon a ball of quick-lime, a kind of light, which I have found capable, like that from the sun, of passing through, and being concentrated by, a lens. Nevertheless, in two or three experiments, each lasting nearly an hour, in which the rays proceeding from the incandescent lime were directed towards, and thrown back upon the plant, by concave metallic reflectors, no increase in the quantity of moisture exhaled could be detected, beyond what the same individual had given out whilst in the dark.

The last function which it was left for me to consider, namely, the absorption of water by the roots, is so related to the preceding one, that it might almost be inferred *à priori* to be subject to the same laws.

It seems indeed evident, that, *cæteris paribus*, in proportion to the velocity with which the sap ascends, will the extremities of the roots absorb moisture from the ground; since, unless the former operation continued, the latter organs would very soon become fully charged with humidity, and thus the absorption be put a stop to.

In order to ascertain the quantity of water absorbed under different circumstances by the roots of plants, I made the following experiments.

Two small plants of *Helianthus annuus*, in pots marked A and B, were immersed in tin vessels nearly full of water, the height of which within was measured by glass tubes cemented into them below, and rising on the outside nearly to the top. These vessels were severally provided with tin covers, each of which had a circular aperture at its centre, through which the stem of the plant passed, and another small one at the side, through which water might be introduced. Being elsewhere closely attached to the vessel, little or no evaporation could take place from the surface of the included water.

Things being thus prepared, the two plants were placed for twenty-four hours in the open air, part of the time exposed to a bright sun, with the thermometer at 75° in the shade.

At the expiration of that time it was found that the vessels had lost, as nearly as could be ascertained, the same quantity of water, which amounted in each case to four ounces.

The next day, the thermometer being as on the preceding occasion, and the sun bright, sunflower A was placed under a frame glazed with orange-coloured glass the same as No. 5, and sunflower B under one glazed with blue glass as No. 2.

In the evening it was found, that the tin vessel containing plant A had lost three ounces of water, and that containing B one ounce. Now in another experiment with the same plants, the tin containing A, placed under a frame glazed with blue glass No. 2, had lost six ounces; that containing B, under one glazed with orange, No. 5, had lost $9\frac{1}{2}$. So that although the ratio between the two was very different (being in the one case as 1 to 3, in the second as 1 to $1\frac{1}{2}$), still in both there was a manifest superiority in the orange over the blue glass with respect to its power of producing absorption. Now orange-coloured glass seemed to act with about half the energy which belonged to transparent; for in another experiment upon the same plants, whilst the *former* had caused an absorption of four ounces, the *latter* had occasioned one of only two.

The same plants being next tried, one with a covering of transparent, the other with one of red glass, it was found that the former had absorbed $4\frac{1}{2}$, the latter 2 ounces, in equal times, the ratio being as 2·00 to ·89.

Hence the following may be stated as representing the relative amount of absorption in these several cases:

Under transparent	2·00
Under orange	1·00
Under blue, varying from	·33 to ·66
Under red	·89

Similar experiments were tried with Vines, and with the *Sagittaria sagittifolia*, which latter being an aquatic plant, continued for a longer period in a healthy condition when immersed in the water. But in either instance the same exception was found with respect to the influence of light on the rate of absorption, as had been observed in that of exhalation, those glasses which radiated most heat, appearing to act upon the plant with an energy quite disproportionate to their illuminating power.

The heat of the weather was very great during these experiments, the thermometer being frequently above 80°, and the liquor in the bottles often mounting as high as 105°, so that it would seem, that the heat radiated from the coloured liquid assisted in promoting absorption in the roots, as it appeared to have done in the former one in increasing the exhalation from the leaves. The same might have been the case, though in a lesser degree, when the blue or red glasses were the media employed, and thus certain irregularities observed in the results may perhaps be explained, by supposing that the joint action, of the light transmitted, and of the heat radiated from these screens, caused a greater exhalation from the leaves, and thereby produced a more abundant absorption by the roots to supply the deficiency.

Upon the whole, then, I am inclined to infer, from the general tenor of the experi-

ments I have hitherto made, that both the exhalation and the absorption of moisture by plants, so far as they depend upon the influence of light, are affected in the greatest degree by the most luminous rays, and that all the functions of the vegetable economy, which are owing to the presence of this agent, follow in that respect the same law. And this is just what we ought to expect, if we suppose that light acts upon the vegetable, as it does upon the animal kingdom, in the character of a specific stimulus; for we all are sensible, that the vivifying influence of light upon ourselves is in proportion to its brightness; nor is it uninteresting to remark, that rays of the very description which most abound in solar light, are at once the most cheering to the animal creation, and the most conducive to the growth and well-being of the vegetable.

We have already seen, that even that most intense form of artificial light which is emitted from incandescent lime, produces no sensible influence upon plants; and we are reminded, that the same holds good with respect to animals, by the fact said to have been observed by the exhibitors of the oxyhydrogen microscope, namely, that animalcules of kinds which used to be speedily destroyed by the too stimulating action of solar light, appear to suffer much less from that now substituted, provided the water in which they are immersed does not become heated thereby.

Part II. *On the Action of Plants upon the Atmosphere.*

Having now considered the mode in which light may be supposed to operate upon plants, I shall next proceed to examine the extent of the changes wrought in the constitution of the atmosphere by the latter, under the influence of this agent.

I say the extent of the changes produced, because no one denies the nature of this operation, or questions the fact, that carbonic acid is really decomposed, and oxygen given out, by the green parts of plants under certain circumstances. But between the original views of Priestley, who saw in this process a counterpoise to the effects of animal respiration and the like, and those of Ellis, who did not admit it even as an equivalent to the opposite tendency of vegetable respiration, as carried on during the absence of solar light, a wide difference exists, and hence some fresh investigations seem necessary with reference to this question.

On perusing the account, which Mr. Ellis, in the Second Part of his Researches on Air, has given of the experiments by which he attempts to establish this latter opinion, several circumstances occurred to me, of a nature calculated to throw doubts upon their conclusiveness. I may mention, for example, in the first place, the smallness of the volume of air in which his plants were confined; in the second, the length of time that was suffered to elapse before the air underwent examination, owing to which it is even stated occasionally that the leaves had begun to fade and drop off; in the third place, the removal in some cases, and the neglect of a due supply in all, of that carbonic acid, the decomposition of which would have constituted the very source of the oxygen which it was expected to discover.

Accordingly I kept constantly in view these three essential conditions, and

contrived an apparatus, in which the quantity of air should be so large, that the healthy functions of the plant might be as little as possible interfered with; in which the constitution of the air could be examined as frequently as I pleased; and in which a regular supply of carbonic acid could be kept up, without disturbing the plant, or suspending the progress of the experiment by its introduction.

The apparatus consisted of a large bell-glass jar, containing in one case 600, in the other 800 cubic inches of air*, and suspended by pulleys. Its edges dipped into quicksilver, contained in a double iron cylinder of corresponding dimensions to the jar, which being closed at bottom, constituted a well of about six inches in depth, calculated to receive a fluid, and to admit of the glass vessel moving freely in it. The inner margin of this hollow cylinder was cemented air-tight, according to circumstances, either to a plate or a pot of iron, upon which the plant operated on might be placed; and the jar was then let down upon it, until its edges were sunk a little beneath the surface of the mercury.

Thus all communication with external atmosphere was cut off, and the effect of the plant upon the air inclosed in the jar was readily measured, by simply pressing down the latter, and thus expelling a portion of its contents through a tube, communicating with its interior, and introduced at its outer extremity under a pneumatic trough, wherein the air might be collected and examined. By connecting this extremity with a vessel containing a measured quantity of carbonic acid, and raising the jar a little in the well of mercury, it was easy to draw in any proportion of that gas, with which it was thought proper that the plant should be supplied. A portion of the air was always tested, immediately after the introduction of every fresh portion of carbonic acid, and again after an interval of some hours, and the proportion of this gas and of oxygen present was carefully registered. The amount of carbonic acid was determined by a solution of potass, that of oxygen by the rapid combustion of phosphorus with a portion of it in a bent tube.

Such was the mode of procedure, when an entire plant became the subject of experiment; but some of the most satisfactory trials were with branches of certain shrubs, themselves too large to be admitted under the jar. These branches, without being detached from the parent trunk, were introduced through a hole in the centre of two corresponding semicircular plates of iron, which were cemented air-tight, to the inner margin of the iron cylinder on the one hand, and to the stem of the branch on the other. In this manner, when the jar came to be placed over them, and to dip beneath the surface of the mercury, the external air was as effectually excluded, as it had been when the whole of the plant was inclosed.

The results of several experiments conducted after this plan will be given in a tabular form; but it may be well in the first instance to specify one of the most satisfactory of those undertaken. In this case the jar itself contained about 600 cubic inches of air, and the plant experimented on was the common Lilac (*Syringa vulgaris*).

* Larger jars, containing from 1200 to 1300 cubic inches, were latterly employed.

The proportion of carbonic acid in the jar was each morning made equivalent to 5 or 6 per cent. by additions through the tube.

The first day no great alteration in the air was detected, but on the second day, by eight in the evening, the oxygen had risen to 26·5 per cent. In the morning it had sunk to 26, but by 2 P.M. it had again risen to no less than 29·75, and by sunset it had reached 30 per cent. At night it sunk one half per cent.; but the effect during the following day was not estimated, as the sickly appearance which the plant now began to assume induced me to suspend the experiment.

In a second trial, however, the branch of a healthy Lilac growing in the garden was introduced into the same jar, where it was suffered to remain until its leaves became entirely withered.

The first day the increase of oxygen in the jar was no more than 0·25 per cent., but on the second it rose to 25·0. At night it sunk to nearly 22 per cent., but the next evening it had again risen to 27 per cent. This was the maximum of its increase, for at night it sunk to 26, and in the morning exhibited signs of incipient decay. Accordingly in the evening the oxygen amounted only to 26·5; the next evening to 25·5; the following one to 24·75; and the one next succeeding it had sunk to the point at which it stood at the commencement, or to 21 per cent.

The reason of this decrease was, however, very manifest in the decay and falling off of the leaves; so that this circumstance does not invalidate the conclusion which the preceding experiments concur in establishing, namely, that in fine weather, at least so long as the plant continues healthy, it adds considerably to the oxygen of the air when carbonic acid is freely supplied.

In the last instance quoted, the exposed surface of all the leaves inclosed in the jar, which were about fifty in number, was calculated at no more than three hundred square inches, and yet there must have been added to the air of the jar as much as twenty-six cubic inches of oxygen, in consequence of the action of this quantity upon the carbonic acid introduced.

But there is reason to believe, that even under the circumstances above stated (which were more favourable to the due performance of the functions of the plant than those to which Mr. Ellis's were subjected,) the amount of oxygen evolved was much smaller than it would have been in the open air, for by introducing several plants into the same jar of air in pretty quick succession, I have succeeded in raising the amount of oxygen contained from 21 to 39 per cent., and probably had not even then attained the limit to which the increase of this constituent might have been brought.

How great then must be the effect of an entire tree in the open air under favourable circumstances! and we must recollect that, *cæteris paribus*, the circumstances will be favourable to the exertion of the vital energies of the plant, within certain limits at least, in proportion as animal respiration and animal putrefaction furnish to it a supply of carbonic acid.

Neither is this influence exerted exclusively by plants of any particular kind or description. I have found it alike in the monocotyledonous and the dicotyledonous, in such as thrive in sunshine and those which prefer the shade, in aquatic as well as in terrestrial, in cryptogamous and imperfect, such as Ferns and Algæ, as well as in those of a more complicated organization. How low in the scale of vegetable life this power extends is not yet exactly ascertained, but Professor Marcet of Geneva, in a late paper, has shown that it does not prevail amongst Fungi.

The disappearance of carbonic acid in my experiments always more than kept pace with the addition to the quantity of oxygen; but the shortness of time during which the plant could be retained in a sufficiently healthy condition, prevented my ascertaining, whether after the carbonic acid had been absorbed by it, a part was not at some subsequent period given out again unchanged.

A small portion might perhaps have been taken up by a thin film of water, which I was compelled to keep continually upon the surface of the mercury, in order to prevent the latter from destroying, by a disengagement of its vapour, the plant confined underneath it. This quantity, however, must have been inconsiderable compared with the amount introduced.

I shall now conclude, by placing in a tabular form some of the principal, or the more illustrative experiments which I have carried on, appending some remarks immediately suggested by the particular phenomena observed in each.

Experiments concerning the influence of plants on atmospheric air mixed with various proportions of carbonic acid; the plants being exposed to the sun, and confined in jars containing from 600 to 800 cubic inches of air, and which rested upon mercury covered by a thin film of water.

Experiment 1.

Date.	Circumstances of the experiment.	Proportion per cent. of carbonic acid added.	Absorption per cent. caused by solution of potash.	Residuum per cent. after burning phosphorus in a portion of the air.	Allowance for phosphorus vapour.	Calculated amount per cent. of	
						nitrogen.	oxygen.
April 10.	12 A. M. A small Cypress in a garden-pot was introduced into the jar, its stem being cemented air-tight into the hole in the two hemispherical iron plates, that fit the inner margin of the hollow cylinder	0	0	81	2	79	21
	6 P. M. After a cloudy and gloomy day, with only occasional gleams of sunshine	0	0	81	2	79	21
April 11.	8 A. M.	0	0	79	2	77	23
	$8\frac{1}{2}$	8	8				
	5 P. M. Stormy and cloudy day, much like that preceding it	0	0	79·5	2	77·5	22·5
April 12.	12 A. M. No sun during the morning, but a settled rain throughout	0	0	81·5	2	79·5	20·5
	1 P. M.	4	4				
April 13.	8 A. M.	0	0	82	2	80	20
	12 A. M. A fine bright day, with occasional storms.....	3	3				
	4 P. M.	0	1	79	2	77	23
April 14.	The unfavourable state of the weather induced me to suspend the experiment.						

Remarks.—The circumstances most worthy of remark in this experiment appear to be, 1st, the emission of 2 per cent. of oxygen the second day, when the proportion of carbonic acid in the air of the jar was too small to be detected by my apparatus; and 2ndly, the absorption of carbonic acid afterwards when no oxygen was evolved. In the first case we must suppose that the plant had imbibed from the atmosphere, previously to its confinement, the carbonic acid, which under the influence of sunshine it decomposed within the jar; in the second, that it absorbed a large quantity of carbonic acid, which in the unfavourable state of the weather it did not decompose. This latter supposition must be adopted as applicable to most of the succeeding experiments; for my apparatus was proved to be sufficiently perfect to prevent any such escape of carbonic acid, within a corresponding period of time, introduced into the jar, when no plant was present in it, and thereby to obviate any suspicion that it might arise from a defect in the union of the joints.

Experiment 2.

April 19.	2 P. M. A dull day, with a tendency to rain. Stem and branches of a Persian Lilac (*Syringa persica*) introduced into the jar in the manner above described	9	9	81	2	79	21
	7 P. M.	0	2·75	78	2	76	24
April 20.	8 A. M. Air examined before the sun had acquired any power	0	2·75	80	2	78	22
	11 A. M.	3·25	6·00				
	7 P. M. After a dull day, with occasional storms until 2 P. M., when the sun broke out.....	0	4·25	80	2	79	21
April 21.	Observed the leaves to be altered and faded, and this, together with the unfavourable state of the weather, induced me to discontinue the experiment.						

Observations.—It appears from this experiment, that when a plant is in a perfectly healthy and fresh condition, it may add considerably to the amount of oxygen even in dull weather.

Experiment 3.

Date.	Circumstances of the experiment.	Proportion per cent. of carbonic acid added.	Absorption per cent. caused by solution of potash.	Residuum per cent. after burning phosphorus in a portion of the air.	Allowance for phosphorus vapour.	Calculated amount per cent. of	
						nitrogen.	oxygen.
April 26.	12 A. M. Dull day, with occasional gleams of sunshine, but no rain. A (*Pelargonium*) Geranium with its pot introduced into the jar	5	5	81	2	79	21·0
	7 P. M.	0	1	79·5	2	77·5	22·5
April 27.	8 A. M.	0	0	79·25	2	77·25	22·75
	The unfavourable state of the weather led me to discontinue this experiment.						

Experiment 4.

April 29.	12 A. M. Dull and stormy day, with occasional gleams of sunshine. A Geranium with its pot introduced into the jar, occupying together a capacity of about 60 cubic inches	5	5	81	2	79	21
	6 P. M.	0	1	83·6	2	81·6	18·4
April 30.	10 A. M.	5	5	82·5	2	80·5	19·5
	5 P. M. After a bright day, with clouds and showers occurring occasionally	0	0	78·5	2	76·5	23·5
May 1.	8 A. M. Access of the morning sun prevented by an opake screen covering the jar	0	2				
	A diminution in the quantity of oxygen was detected, but its amount was not registered, as the apparatus was found not to be perfectly air-tight. The experiment was therefore suspended in order to repair the defect.						

Experiment 5.

May 8.	12 A. M. Bright day. A Geranium in a pot was introduced into the jar	5	5	81	2	79	21·00
	1½ P. M.	0	2·5	79·75	2	77·75	22·25
	2½	0	2·25	79·25	2	77·25	22·75
	3½	0	0·05	79·00	2	77·00	23·00
May 9.	8 A. M. During the morning screened from the sun	0	0·0				
	10 A. M. Fine bright day	10	10·0				
	11 A. M.	0	9·50	82	2	80·00	20·00
	12 A. M.	0	6·25	82	2	80·00	20·00
	2 P. M.	0	6·5	80	2	78·00	22·00

Experiment 6.

May 10.	Fine bright day. At 12 A. M. a fresh Geranium was introduced	10·5	10·5	81	2	79	21
	5½ P. M.	0	1·0	76	2	74	26

Experiment 7.

Date.	Circumstances of the experiment.	Proportion per cent. of carbonic acid added.	Absorption per cent. caused by solution of potash.	Residuum per cent. after burning phosphorus in a portion of the air.	Allowance for phosphorus vapour.	Calculated amount per cent. of	
						nitrogen.	oxygen.
May 11.	11 A.M. Dull and sultry day. Fresh Geranium introduced	8·5	8·5	81	2	79	21
	11 P.M.	0	1	81	2	79	21

Observations.—These two latter experiments show decidedly that the disappearance of carbonic acid is not at all in proportion to the disengagement of oxygen. In Experiment 6, in a bright sun, the Geranium, which absorbed 9·5 per cent. of carbonic acid, emitted 5 per cent. of oxygen; in Experiment 7. a similar plant, which absorbed 7·5 of the former in a dull day, seemed to have made no addition at all to the amount of oxygen in the jar.

Experiment 8.

May 13.	2 P.M. Bright sunshine all the day. A young Lilac (*Syringa vulgaris*) had its stem introduced under the jar, its roots being in a pot outside	8	8	81	2	79	21
	7½ P.M.	0	0·05	82·5	2	80·5	19
May 14.	8 A.M.	0	6·00	not estimated.			
	8½. Day as that preceding it	2	8				
	8 P.M.	0	2	75·5	2	73·5	26·5
May 15.	9 A.M. Jar during the preceding part of the morning covered with a screen	0	3·5	76·0	2	74	26·0
	9½ A.M.	6·5	10·0				
	2 P.M.	0	3·0	72·25	2	70·25	29·75
	8½ P.M.	0	0·0	72·00	2	70·00	30·00
May 16.	8 A.M. Jar during the former part of the morning having been covered with a screen ..... The plant beginning to look unhealthy was now removed.	0	2·75	72·50	2	70·50	29·50

Observations.—The remarkable clearness of the sky, and the warmth of the weather during these three days, was peculiarly favourable to the experiments, and will account for the large quantity of oxygen obtained, which was equal to 8·5 per cent., or 8·5 × 6 = 51 cubic inches. Owing to the longer time that the plant continued in a state of health, the quantity of carbonic acid that disappeared did not so much exceed that of the oxygen given out as in the preceding experiments.

Experiment 9.

May 20	4 P.M. Day dull, but free from rain: in the evening a little sunshine. Branch of a Lilac, having on it about fifty healthy leaves, each leaf on an average presenting a surface of about six square inches (50 × 6 = 300 square inches to the whole)	5	5	81	2	79	21
	7 P.M.	0	3·3	80·75	2	78·75	21·25
May 21.	7½ A.M. Jar during the former part of the morning covered by an opake screen	0	2·25	80	2	78	22
	11½ A.M. Bright sunny day. Thermometer 65°	0	2·25	79·5	2	77·5	22·5
	3½	0	0·00	77	2	75·0	25·0

Experiment 9.—Continued.

Date.	Circumstances of the experiment.	Proportion per cent. of carbonic acid added.	Absorption per cent. caused by solution of potash.	Residuum per cent. after burning phosphorus in a portion of the air.	Allowance for phosphorus vapour.	Calculated amount per cent. of	
						nitrogen.	oxygen.
May 22.	8 A. M. Before uncovering the jar..	0	0·5	79·75	2	77·75	22·25
	9 A. M. Fine bright cloudless day. Thermometer 68°	8·5	9·0	....	....	....	....
	1½ P. M.	0	6·0	75·75	2	73·75	26·25
	5 P. M.	0	3·0	74·5	2	72·5	27·5
May 23.	8 A. M. Before the screen had been removed	0	2	75	2	73	27
	8 P. M. After a fine bright day	0	0	76	2	74	26
May 24.	8 A. M. Before removing the screen..	0	0	75·5	2	73·5	26·5
	4 P. M. After a fine bright day	0	0	76·5	2	74·5	25·5
May 25.	8 A. M. Before removing the screen..	0	2	76·75	2	74·75	25·25
	6 P. M. Fine bright day, like yesterday	0	0	77·25	2	75·25	24·75
May 27.	10 A. M.	0	1	80·75	2	78·75	21·25
	Experiment discontinued, as most of the leaves had dropped off, and the remainder looked decayed and withered. The decay of the leaves began to date from the 24th, at which time the amount of oxygen will be seen to have begun to diminish.						

Observations.—This experiment shows how erroneous a conclusion we might deduce, if we contented ourselves with examining the air after an interval of some days, and how soon, even under the most favourable circumstances, confinement interferes with the healthy functions of a plant.

Experiment 10.

Aug. 20.	12 A. M. Day dull and cloudy, with a few occasional gleams of sunshine. Therm. 70°. A young Cedar with its pot was introduced under a jar having a capacity of about 800 cubic inches	7·5	7·5	81	2	79	21
	10 P. M.	....	2·75	80	2	78	22
Aug. 21.	10 A. M. Day overcast, but with occasional gleams of sunshine. Thermometer 70°	0	2·0	82·5	2	80·5	19·5
	10½	7·5	9·5	....	....	....	....
	7 P. M.	0	5·0	80	2	78	22·0
Aug. 22.	11 A. M. Windy day, but with a bright sun till about 4 P. M. Thermometer 70°	0	4·0	80	2	78	22·0
	7 P. M.	0	0·0	78·75	2	76·75	23·25
Aug. 23.	8 A. M. Windy and cloudy day, with occasional showers, and few gleams of sunshine. Therm. 64°	0	2·0	80	2	78	22
	10 A. M.	7·5	9·5	80	2	78	22
	7 P. M.	0	4·0	78·50	2	76·50	23·50
Aug. 24.	10 A. M. Cloudy day, without sun. Therm. 66°	0	1	77·25	2	75·25	24·75
	12 A. M.	7·5	....	....	....	....	....
	7 P. M.	0	5	78·20	2	76·20	23·80
	Experiment discontinued.						

Experiment 11.

Date.	Circumstances of the experiment.	Proportion per cent. of carbonic acid added.	Absorption per cent. caused by solution of potash.	Residuum per cent. after burning phosphorus in a portion of the air.	Allowance for phosphorus vapour.	Calculated amount per cent. of	
						nitrogen.	oxygen.
Aug. 20.	12 A. M. Bright day, with occasional clouds. Therm. 72°. Another Cedar like the former, with its pot, was introduced into a jar containing about 1300 cubic inches of air; size of the plant being 8 inches in diameter, branches and all, and 16 inches from the pot to the topmost branch	7·50	7·50	....	....	79	21
	10 P. M.	....	2·00	80	2	78	22
Aug. 21.	10 A. M. Windy day, often overcast.	....	1·75	79·50	2	77·50	22·50
	10½ A. M.	7·50	9·25	....	....	....	....
	7 P. M.	....	5·00	79·50	2	77·50	22·50
Aug. 22.	11 A. M. Windy day, but with a bright sun	....	2·50	77·0	2	75·00	25·00
	7 P. M.	0	1·0	75·5	2	73·5	26·5
Aug. 23.	8 A. M. Windy and cloudy day, as above stated	0	1·0	78·0	2	76	24·0
	10 A. M.	7·5	....	....	....	....	....
	7 P. M.	0	6·0	75·25	2	73·25	26·75
Aug. 24.	10 A. M. Cloudy day, without sun.	0	5·0	77·25	2	75·25	24·75
	12 A. M.	2·5	7·5	....	....	....	....
	7 P. M. Experiment was here discontinued.	0	6·0	75·25	2	73·25	26·75

Observations.—These two latter experiments tend to show that the leaves of evergreens purify the air as well as those of deciduous plants, although they may not act upon the carbonic acid with the same energy as the latter do.

Experiment 12.

May 27.	Bright sunny day. Therm. 60°. *Crassula lactea* with its pot introduced into the jar at 12 A. M.	11	11	....	....	79	21
	6 P. M.	0	6·5	81·25	2	79·25	20·75
May 28.	8 A. M. Jar previously covered with a screen.	0	6·0	81·25	2	79·25	20·75
	5 P. M. After a bright day	0	6·5	80·00	2	78·00	20·00

Observations.—This experiment, and another made on another species of Crassula, show that these succulent plants do not act with much energy on the air, at least under confinement; but the following experiment evinces that even this tribe act in the same manner, though not with the same intensity.

Experiment 13.

June 30.	12 A. M. Plant of *Mesembryanthemum verrucosum* introduced into jar. (N.B. No record of the weather: probably it was fine)	6·6	....	....	....	79	21
	8 P. M.	....	4·9	78·6	2	76·6	23·4
July 1.	10 A. M.	....	2·0	....	....	....	....
	10½ A. M.	4·5	6·5	....	....	....	....

Experiment 13.—Continued.

Date.	Circumstances of the experiment.	Proportion per cent. of carbonic acid added.	Absorption per cent. caused by solution of potash.	Residuum per cent. after burning phosphorus in a portion of the air.	Allowance for phosphorus vapour.	Calculated amount per cent. of	
						nitrogen.	oxygen.
July 1.	5 P. M.	0	5·0	82·5	2	80·5	19·5
July 2.	1 P. M.	0	5·0	79·5	2	77·5	22·5
July 3.	12 A. M.	0	2·0	80·75	2	78·75	21·25
July 4.	12 A. M.	3·25	2·25	81·75	2	79·75	20·25
	8 P. M.	....	5·00	81·5	2	79·5	20·50
July 5.	1 P. M.	....	2·00	81·25	2	79·25	20·75
July 7.	12 A. M.	....	1·00	83·75	2	81·75	18·25

Observations.—Before the experiment was brought to a close, the plant had assumed an unhealthy appearance; yet the length of time during which it was confined, and the smallness of the change it produced upon the included air, show the more languid manner in which the functions of vegetable life are carried on through the medium of the metamorphosed leaves of this tribe of plants. A similar experiment was tried with a specimen of *Aloe mitræformis*, in which case also 2 per cent. was added to the amount of oxygen by its presence.

Experiment 14.

Aug. 30.	12 A. M. Healthy Dahlia in a pot, containing one full-blown flower and twenty leaves, were introduced into the jar	7·5	....	....	....	79	21
	8 P. M. After a day, bright till 3 P. M. afterwards overcast	....	3·0	79·25	2	77·25	22·75
Aug. 31.	10 P. M. Bleak and boisterous weather throughout the day, with much rain. Therm. 65°	0	2·0	77·25	2	75·25	24·75
Sept. 1.	Weather cold, stormy, and bleak, but a bright sun occasionally. Thermometer 55° at 9 A. M.	7·5	....	....	....	....	....
	At 10 P. M.	....	2·0	78·25	2	76·25	23·75
Sept. 2.	Fine day but cloudy: sun at intervals bright. Therm. 62° at 10 P. M.	0	1·0	79·25	2	77·25	22·75
Sept. 3.	Dull stormy day at 9 A. M.	7·5	....	....	....	....	....
	At 4 P. M.	....	4·0	77·25	2	75·25	24·75
Sept. 4.	Fine day at 1 P. M.	....	0·0	75·50	2	73·50	26·50
	Experiment suspended, though the plant seemed to have suffered but very little from its confinement.						

Experiment 15.

Aug. 30.	Weather reported above. Healthy Dahlia, with about as many leaves as the former one, but without any flower, was introduced into the jar at 12 A. M.	7·5	....	....	....	79	21
	8 P. M.	0	3·75	....	....	....	....
Aug. 31.	10 A. M.	0	3·75	82·25	2	80·25	20·75
	10 P. M.	0	2·00	82·75	2	80·75	19·25
Sept. 1.	10 A. M.	7·5	9·50	....	....	....	....
	10 P. M.	....	7·50	78·00	2	76·00	24·00
Sept. 2.	10 P. M.	....	5·00	79·00	2	77·00	23·00
Sept. 3.	4 P. M.	....	4·00	78·25	2	76·25	23·75
Sept. 4.	1 P. M.	....	1·00	78·00	2	76·00	24·00
	Experiment was here suspended, as I was on the point of leaving home.						

Observations.—These two last experiments show that even plants which are in flower may purify the atmosphere, the influence of the accompanying leaves more than counterbalancing that of the parts of fructification: and, indeed, a comparison of the results of the first experiment with those of the second seems to indicate, that the presence or absence of a flower does not make so great a difference as might have been anticipated.

The same experiment was tried with a plant of *Helianthus annuus* in full flower, and an increase of 1 per cent. of oxygen was detected.

In the following experiments a succession of different plants were introduced at intervals of from four to twelve hours, into a jar containing about 240 cubic inches of common air, together with generally about ten cubic inches of carbonic acid. The following Table will show the increase of oxygen caused.

Experiment 16.

Date.	Circumstances of the experiment.	Oxygen.	
		Proportion per cent.	Calculated number of cubic inches.
May 23.	Before any plant was introduced	21	50
	After a pot of *Sinapis alba* had continued in contact with the air during six hours of bright sunshine	25	60
	After a pot containing *Hydrangea hortensis*, only in leaf, had continued in contact with the air during twelve hours of bright sunshine	26·5	63·5
May 24.	After a pot, containing a *Crassula lactea*, had continued in contact with the air during four hours of bright sunshine	32·75	77
	After a pot of *Lepidium sativum* had continued in contact with the air during four hours of bright sunshine	33·5	80·5
May 25.	After a pot containing a Geranium (Pelargonium) had continued in contact with the air during four hours' bright sunshine	36·5	87·0
	After the same Hydrangea as before had continued in contact with the air during five hours' bright sunshine	33·25	80·0
May 27.	After a Geranium had continued in contact with the air during five hours' bright sunshine	37·0	88·0
	After a healthy Myrtle had continued in contact with the air during four hours' bright sun	39·0	93
May 28.	After a young but apparently healthy plant of the Sweet Pea (*Lathyrus odoratus*) had continued in contact with the air for four hours	33	78·5
May 30.	After a Geranium had continued in contact with the air for five hours	36·5	87·0

Observations.—Here, owing to an accident, the experiment was suspended; but it was carried far enough to show how much oxygen may be added to the air during the day by plants in a healthy condition. The increase was progressive until I employed the same plant a second time: it probably had suffered from its previous confinement, or from being passed backwards and forwards through the water of the pneumatic trough on which the jar rested. The only other plant which diminished the amount of oxygen was the Sweet Pea; all the others added somewhat to its quantity.

Experiment 17.

Capacity of jar 620 cubic inches (600 atmospheric air, 20 carbonic acid).

August 7.	Before any plant was introduced	21	126
	6 P.M. After a Geranium had continued in contact with the air during five hours' strong sun	23	138
August 8.	10 A.M. After a second Geranium had remained in contact with the air during the previous night and morning	22	132
	5 P.M. After a Myrtle had been in contact with the air during the period since the previous experiment made	24·5	147
August 9.	11 A.M. After a second Myrtle had continued in contact with the air ever since the last experiment	26	156
	5 P.M. A common garden Lettuce growing in a pot, having continued in contact with the air since the last experiment: day fine	29	174
August 10.	11 A.M. A second Lettuce, having continued in contact with the air since the preceding experiment: day wet	30	180
	5 P.M. After a Geranium had been introduced since the preceding experiment: day wet	35	210
August 11.	11 A.M. After a second Geranium had continued in contact with the air during the interval since the last experiment	28	168
	Finding a temporary difficulty in procuring healthy plants, I here suspended the experiments.		

Observations.—This latter series of experiments was undertaken in order to show, that even when the plants were confined in the air of the jar, by night, as well as by day, the balance was still greatly in favour of their purifying influence.

With respect to the comparative influence of similar plants in direct and in diffused light, I have as yet made only a few experiments, but the results of these few seem to indicate that even under the latter circumstances an increase in the amount of oxygen will be sometimes produced by them.

	Per cent.
Thus a jar, in which a Geranium had been confined, contained, after exposure for four hours to direct light, of oxygen	25
Whilst another, in which a similar plant had been placed for the same time in diffused light, contained	19·5
On the other hand,	
Two jars, containing Myrtles, after being placed in direct light for eight hours, had acquired of oxygen,	
No. 1.	23·75
No. 2.	26·00
Two similar jars with Myrtles in diffused light, exposed for the same time, contained,	
No. 1.	20·0
No. 2.	23·0
Two jars with *Polypodium dryopteris*, the one in direct, the other in diffused light, each contained of oxygen	22·0

In conclusion, it may perhaps be well to present in a tabular form the different questions which this inquiry embraces, pointing out how far each may be considered as answered, either by the experiments of preceding investigators, or by those detailed in the present memoir.

Scheme of Experiments.

PART I.—ON THE ACTION OF LIGHT UPON PLANTS.

I. Solar light.
A. direct.

1. In causing the leaves to emit oxygen, and to decompose carbonic acid .....
   - Immersed in water. — Tried with 1. *Brassica oleracea*; 2. *Salicornia herbacea*; 3. *Fucus digitatus*; 4. *Tussilago hybrida*; 5. *Cochlearia Armoracia*; 6. *Mentha viridis*; 7. *Rheum rhaponticum*; 8. *Allium ursinum*; and several species of *Gramineæ*.
   - In atmospheric air. — Tried with Geraniums.
2. To become green when etiolated.... — Tried with Beans.
3. To maintain their irritability ..... — Tried with the Sensitive Plant (*Mimosa pudica*).
4. To exhale water by their leaves ..... — Tried with Vines, Dahlias, Helianthuses, *Lavatera arborea*, &c.
5. To absorb the same by their roots .... — Tried with plants of *Helianthus annuus*, *Sagittaria sagittifolia*, Vines, &c.

B. diffused or reflected. — Its influence compared with that of direct solar light in the above particulars. — Tried with the Geranium, Myrtle, and Polypodium, so far as regards its relative influence in causing the emission of oxygen.

II. Artificial light, obtained A. from lamps. Tried by Professor DECANDOLLE.
B. from incandescent lime. Tried by myself, but no influence detected.

PART II.—ON THE ACTION OF PLANTS UPON THE ATMOSPHERE.

I. Proportion between the effects attributable to their action during the night and during the day.

1. During fine weather, and in bright sunshine.
   - Of plants without flowers, and with leaves alone, viz.:

	Maximum increase per cent. of oxygen.
Cupressus	2
Cedrus	3·75
Syringa vulgaris	8·75
Ditto	6·50
Pelargonium	2·00
Ditto	5·00
Crassula, 2 sp.	0·00
Mesembryanthemum	2·40
Dahlia	3·00

   - Of plants with flowers and leaves.

Dahlia	3·75
Helianthus	1·00

2. During bad weather, or in diffused light.
   - *Syringa persica* 3 per cent.
   - Geranium, Myrtle, Fern, as noticed above.

II. Proportion between the carbonic acid absorbed and oxygen evolved. — My experiments show that when plants are confined the former is always greatest at first; but this may not continue to be the case after a certain interval.

III. Greatest amount of oxygen that can be added to the air of a jar by the influence of a plant. — My experiments show that at least 18 per cent. of oxygen may be so added.

IV. At what stage in the scale of vegetable life the function of purifying air stops. — Probably where there cease to be leaves.—I have shown that it exists in dicotyledonous and monocotyledonous, in evergreens and deciduous, in terrestrial and aquatic plants, in the green parts of succulents as well as in ordinary leaves, in Algæ and in Ferns as well as in phanerogamous families.
Prof. MARCET has shown that it does not take place in Fungi.

XIV. *Researches in the Integral Calculus.—Part* I. *By* H. F. Talbot, *Esq. F.R.S.*

Received and read March 10, 1836.

§ 1. *Brief Historical Sketch of the Subject.*

THE first inventors of the integral calculus observed that only a certain number of formulæ were susceptible of exact integration, or could be reduced to a finite number of terms involving algebraic, circular, or logarithmic quantities. When this result could not be attained, they were accustomed to develop the integral in an infinite series. But this method, although useful when numerical values are to be computed, is entirely inadequate, in an analytical point of view, to supply the place of the exact integral; for the progress of analysis has shown many instances of exact relation between different integrals which cannot by any means be inferred from the infinite series in which they are developed.

The first great improvement beyond this was made by Fagnani about the year 1714. This most acute and ingenious mathematician proposed the following question to the scientific world in an Italian journal*: "Given a biquadratic parabola whose equation is $x^4 = y$, and an arc of it, to find another arc, so that *their difference* may be rectifiable."

No answer appearing, he published a solution of the problem in the year 1715†, and extended it in a nearly similar manner to other curves whose equation is $x^n = y$, viz. to those cases where n equals one of the numbers 3, $\frac{4}{3}$, $\frac{4}{2}$, $\frac{4}{3}$, $\frac{4}{1}$, $\frac{4}{4}$.

In the year 1718 and afterwards he published a variety of important theorems respecting the division into equal parts of the arcs of the lemniscate, and respecting the ellipse and hyperbola, in both of which he showed how two arcs may be determined of which the difference is a known straight line. These discoveries justify us in regarding Fagnani as the founder of a new and very curious branch of analysis.

Euler, who enriched almost every department of science with new discoveries, exhibited the complete algebraic integral of the equation

$$\frac{dx}{\sqrt{\alpha + \beta x + \gamma x^2 + \delta x^3 + \varepsilon x^4}} + \frac{dy}{\sqrt{\alpha + \beta y + \gamma y^2 + \delta y^3 + \varepsilon y^4}} = 0;$$

a remarkable theorem, which long continued to be the *ne plus ultra* of this branch of science, little success having attended the endeavours of mathematicians to arrive at results of greater generality.

* Giornale de' Letterati d'Italia, tom. xix. p. 438. † tom. xxii. p. 229.

The excellent work of LEGENDRE* was destined to arrange, classify, and distinguish the properties of elliptic integrals which are implicitly contained in EULER's theorem above mentioned. In this treatise he has thoroughly examined the nature of these transcendents, and presented the results of his inquiries in a luminous and well-arranged theory.

The extensive tables which accompany his work will enable future mathematicians to make as frequent and convenient use of elliptic integrals as they have hitherto done of circular and logarithmic ones.

In the year 1828 Mr. ABEL, of Christiania in Norway, published a very remarkable theorem, which gives the sum of a series of integrals of the form $\int \frac{P\,dx}{\sqrt{R}}$, where P and R are entire functions of x, of the form $x^n + a\,x^{n-1} + b\,x^{n-2} + \ldots\ldots$; n being any whole positive number, and a, b, &c. constant coefficients.

This theorem extends much further than EULER's, in as much as the latter is limited to those forms of R which contain no higher powers of x than the fourth. It departs still more widely from EULER's theorem, in exhibiting the sum, not of two only, but of many integrals of the same form. And it must be observed that this plurality of terms is in general necessary; for if we give to the expression $\int \frac{dx}{\sqrt{R}}$ its utmost generality, it does not appear possible to find the sum of only two such integrals in finite algebraic, or logarithmic terms; but it is requisite to combine a greater number of them, below which number the problem cannot be reduced.

ABEL's theorem in general furnishes a multitude of solutions for each particular case of the problem: notwithstanding which it is possible to find other solutions which appear not to be comprised in his theorem, nor deducible from it†.

On the publication of this theorem the illustrious LEGENDRE, who at an advanced age still cultivated his favourite science with all the ardour of youth, was one of the first to feel its extent and importance. And accordingly, with a degree of zeal almost unequalled in the annals of science, he devoted a large portion of time to the verification and elucidation of the theorem by numerical examples. The result of these calculations was amply confirmatory of its truth, and it therefore undoubtedly stands upon the basis of rigorous demonstration.

There can be little doubt that the ingenious mathematician to whom we are indebted for this theorem would have arrived at fresh discoveries, of not inferior value,

* Exercices de Calcul Intégral. Paris, 1811. Traité des Fonctions Elliptiques. Paris, 1825.

† For instance, if $\frac{dx}{\sqrt{1+x^4}} + \frac{dy}{\sqrt{1+y^4}} = 0$, his theorem gives the integral $xy=1$; but, apparently, it does not give this other integral $y^2 = \frac{\sqrt{1+x^4}+x\sqrt{2}}{\sqrt{1+x^4}-x\sqrt{2}}$, which was discovered by FAGNANI (Produzioni Matematiche, vol. ii. p. 369.).

if a premature death had not terminated his career, to the irreparable loss of science, at the early age of twenty-seven.

Before concluding this slight historical sketch of the subject, I ought not to omit mention of a valuable recent memoir by M. POISSON*, in which he has considered various forms of integrals which are not comprehended in ABEL's formula.

It has already been stated that the integrals to which ABEL's theorem relates are those comprised in the general expression $\int \frac{P\,dx}{\sqrt{R}}$, where P and R are entirely polynomials in x. Next in order of succession to these there naturally presents itself the class of integrals whose general expression is $\int \frac{P\,dx}{\sqrt[3]{R}}$, where the polynomial R is affected with a cubic instead of a quadratic radical.

But ABEL's theorem has no reference to these, and consequently affords us no assistance in their solution. The same may be said with regard to the succeeding classes of integrals, $\int \frac{P\,dx}{\sqrt[4]{R}}, \int \frac{P\,dx}{\sqrt[5]{R}}$, and generally $\int \frac{P\,dx}{\sqrt[n]{R}}$. Still less does it enable us to find the sum of such integrals as $\int \varphi(R)\,dx$, R being as before an entire polynomial†, and φ *any function whatever*. This is the problem to the solution of which the following pages will be dedicated.

I may be here permitted to mention, that ABEL's theorem was unknown to me until some years after its publication, and that these Researches were nearly completed before I was acquainted with it. I have, however, made no alteration in them, but have chosen to present the subject in the manner in which it originally occurred to me.

I am not aware that Mr. ABEL has left any memorial of the successive steps of reasoning by which he arrived at his theorem. Probably they were very different from those which I have employed, and therefore I have detailed at some length my method of investigation, beginning with the first rudiments of the theory at which I afterwards arrived.

§ 2.

It was remarked by the earliest inventors of the integral calculus, that there was a mutual dependence between the two integrals $\int y\,dx$ and $\int x\,dy$, so that if the one were given the other became known, by virtue of the equation

$$\int x\,dy + \int y\,dx = xy + C.$$

If therefore one of these forms happened to be more easy of integration than the other, they directed it to be substituted for it.

* CRELLE's Journal, vol. xii. p. 89. Berlin, 1834.

† By "polynomial" I here understand an expression containing at least two different powers of x.

There is, however, one case in which no alteration is produced by this substitution, and that is when the variable x is the same function of y that y is of x; or when $x = \varphi y$, $y = \varphi x$. For then the integral $\int y\,dx$ or $\int \varphi x\,dx$ has the same form with $\int x\,dy$ or $\int \varphi y\,dy$.

In this case therefore

$$\int \varphi x\,dx + \int \varphi y\,dy = xy + C.$$

This equation holds good whether $\int \varphi x\,dx$ can be integrated in finite terms, or whether it cannot.

The equations $x = \varphi y$ and $y = \varphi x$, manifestly imply that a *symmetrical equation* exists between x and y, and its symmetry is the only requisite condition. In other respects it may be any whatever.

Notwithstanding the simplicity of this reasoning, it does not appear that any mathematician before FAGNANI clearly perceived the important consequences which might be deduced from it. But he has obtained from it the following important theorem respecting the arcs of the hyperbola.

If x be the abscissa of a hyperbola whose principal semi-axis $= 1$, its arc

$$= \int dx \sqrt{\frac{e^2x^2-1}{x^2-1}},$$

where e is the eccentricity, or the distance between the centre and focus.

Let y be another abscissa, so related to the former that

$$ey = \sqrt{\frac{e^2x^2-1}{x^2-1}},$$

whence

$$e^2x^2y^2 = e^2(x^2+y^2) - 1.$$

This equation being symmetrical with respect to x and y, it follows that those letters may be permuted,

$$\therefore ex = \sqrt{\frac{e^2y^2-1}{y^2-1}}.$$

Multiplying these equations respectively by dx and dy, and then adding them,

$$ey\,dx + ex\,dy = dx\sqrt{\frac{e^2x^2-1}{x^2-1}} + dy\sqrt{\frac{e^2y^2-1}{y^2-1}}.$$

$$\therefore exy + C = \int dx\sqrt{\frac{e^2x^2-1}{x^2-1}} + \int dy\sqrt{\frac{e^2y^2-1}{y^2-1}},$$

which is the theorem in question.

Since the arc of an ellipse may also be expressed by the formula $\int dx\sqrt{\frac{e^2x^2-1}{x^2-1}}$, it may be asked whether the theorem applies to the ellipse, or to the hyperbola, or to both curves?

Let us therefore return to the equation

$$e^2 x^2 y^2 = e^2 (x^2 + y^2) - 1,$$

whence

$$-(x^2 + y^2) + x^2 y^2 = -\frac{1}{e^2}$$

$$(1 - x^2)(1 - y^2) = 1 - \frac{1}{e^2}.$$

In the ellipse the abscissæ x, y are necessarily both less than 1, and in the hyperbola they are both greater than 1. Therefore in either case the product $(1 - x^2)(1 - y^2)$ is a positive quantity, $\therefore 1 - \frac{1}{e^2}$ must be a positive quantity, which gives $1 > \frac{1}{e^2}$, or $e > 1$. This condition obtains in the hyperbola but not in the ellipse, therefore the theorem is not applicable to the latter. An analogous theorem, however, exists for the ellipse, which I shall not now stop to examine.

In imitation of the above proceeding, let us make the more general supposition

$$e y = \left(\frac{e^n x^n - 1}{x^n - 1}\right)^{\frac{1}{n}},$$

whence

$$e^n x^n y^n = e^n (x^n + y^n) - 1,$$

a symmetrical equation;

$$\therefore e x = \left(\frac{e^n y^n - 1}{y^n - 1}\right)^{\frac{1}{n}}.$$

Proceeding as before, we find

$$e x y + C = \int \left(\frac{e^n x^n - 1}{x^n - 1}\right)^{\frac{1}{n}} dx + \int \left(\frac{e^n y^n - 1}{y^n - 1}\right)^{\frac{1}{n}} dy$$

$$= S \int \left(\frac{e^n x^n - 1}{x^n - 1}\right)^{\frac{1}{n}} dx,$$

where the notation $S\int$ is employed to express with brevity the sum of two (or any number) of similar integrals.

The sums of many other integrals might be found in the same manner; but I proceed to more general inquiries.

§ 3.

The first idea of a more extended method occurred to me about fifteen years ago, when pursuing mathematical studies at Cambridge; and it was suggested by an attentive consideration of the process by which FAGNANI had rectified the hyperbola, as mentioned in the preceding section. The question occurred to me, whether it might not be possible to combine *three* integrals in a similar manner, by supposing two symmetrical equations to exist between three variables?

Since we have

$$xyz + C = \int yz\,dx + \int xz\,dy + \int xy\,dz,$$

if we suppose any two equations to exist between the variables, then y and z are functions of x which assume definite values when x is given. Therefore also the product yz is a function of x, which may be called φx.

If now the two equations are symmetrical, it follows that the letters x, y, z may be permuted; which gives $xz = \varphi y$, and $xy = \varphi z$;

$$\therefore xyz + C = \int \varphi x\,dx + \int \varphi y\,dy + \int \varphi z\,dz$$

$$= S\int \varphi x\,dx.$$

It is evident that this reasoning may be extended to any number n of variables between which there exist $n-1$ *symmetrical* equations, which circumstance renders them all similar functions of each other.

Let r designate their product $xyz\ldots$, and therefore $\frac{r}{x} = yz\ldots$, or the product of all except x.

$$\therefore r + C = \int \frac{r}{x}\,dx + \int \frac{r}{y}\,dy +, \&c.$$

But if $\frac{r}{x} = \varphi x$, we have by merely permuting the letters,

$$\frac{r}{y} = \varphi y, \frac{r}{z} = \varphi z, \&c.$$

Therefore

$$\left.\begin{aligned} r + C &= \int \varphi x\,.\,dx + \int \varphi y\,.\,dy +, \&c. \\ &= S\int \varphi x\,.\,dx. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (A.)$$

This equation I first obtained in the year 1821, but not having leisure at that time to pursue the subject much further, I contented myself with making a note of it as being a subject that deserved to be further examined into. I afterwards found it to be the key, as it were, of the whole method.

In the year 1825 I resumed this investigation, and endeavoured, by the trial of various forms of symmetrical equations between the variables, to see whether this method would lead to new results, or whether, on the contrary, it would turn out to be a mere variation of the methods in common use.

I here give the results of some of these early trials, just as I find them in the original papers.

Ex. 1. Let the 2 symmetrical equations be

$$(1.)\quad x + y + z = a$$

$$(2.)\quad x^2 + y^2 + z^2 = b^2,$$

a and b^2 being constants.

These give

$$\varphi x \text{ or } y z = \frac{a^2 - b^2}{2} - a x + x^2.$$

And theorem A. gives

$$(3.)\quad x y z + \mathrm{C} = \int \varphi x \, dx + \int \varphi y \, dy + \int \varphi z \, dz.$$

Now here it is easy to verify the theorem, because $\int \varphi x \, dx$ is known, viz.

$$\int \varphi x \, dx = \frac{a^2 - b^2}{2} x - \frac{a}{2} x^2 + \frac{x^3}{3},$$

and similarly with respect to $\int \varphi y \, dy$ and $\int \varphi z \, dz$; $\therefore$ by addition,

$$\mathrm{S} \int \varphi x \, dx = \frac{a^2 - b^2}{2} (x + y + z) - \frac{a}{2} (x^2 + y^2 + z^2) + \frac{x^3 + y^3 + z^3}{3},$$

or, by help of equations (1.) and (2.),

$$\mathrm{S} \int \varphi x \, . \, dx = \text{const.} + \frac{x^3 + y^3 + z^3}{3}.$$

Although this result differs at first sight from that given by equation (3.),

$$\mathrm{S} \int \varphi x \, dx = \text{const.} + x y z,$$

yet it is easy to see that they are identical. For since

$$x + y + z = \text{const., by equation (1.),}$$

and

$$x^2 + y^2 + z^2 = \text{const., by equation (2.),}$$

it follows as a necessary consequence that

$$\frac{x^3 + y^3 + z^3}{3} = x y z + \text{const.},$$

which verifies the theorem in this case.

In the examples which follow next I shall suppose one of the given equations to be

$$x + y + z = 0.$$

Ex. 2. Let the other equation be $(x^2 - 1)(y^2 - 1)(z^2 - 1) = -1$, we find

$$\varphi x, \text{ or } y z = -1 + \sqrt{\frac{1 + x^2 - x^4}{1 - x^2}}.$$

Ex. 3. Let $x^4 + y^4 + z^4 = 2 x y z$, we find

$$2 y z = 2 x^2 + x + \sqrt{4 x^3 + x^2} = 2 \varphi x.$$

Ex. 4. Let $x^5 + y^5 + z^5 = -5$, we find

$$y z = \frac{x^2}{2} + \sqrt{\frac{1}{x} + \frac{x^4}{4}} = \varphi x.$$

In each of these cases therefore we find the sum of three integrals of the form $\int \varphi x \, . \, dx$ to be equal to $x y z + \mathrm{C}$.

Before going further it may be well to adopt, for the sake of brevity, the following notation.

Let there be any number of variables, three for example; then the sum of their nth powers or $x^n + y^n + z^n$ may be briefly written $\mathrm{S}\,x^n$, and similarly if $f\,x$ be any function of x, $\mathrm{S}f\,x$ stands for the sum of $f\,x + f\,y + f\,z$. Also $\mathrm{S}\,x\,y$ means $x\,y + x\,z + y\,z$. And in general

$$\mathrm{S}f(x,y) = f(x,y) + f(y,x) + f(x,z) + f(z,x) + f(y,z) + f(z,y),$$

being the sum of all the permutations of the letters. A few examples will render this notation familiar.

Let there be 3 variables, then

$$\mathrm{S}\,x = x + y + z$$
$$\mathrm{S}\,x\,y = x\,y + x\,z + y\,z$$
$$\mathrm{S}\,x^2\,y = x^2\,y + y^2\,x + x^2\,z + z^2\,x + y^2\,z + z^2\,y$$
$$\mathrm{S}\,x^2\,y^2 = x^2\,y^2 + x^2\,z^2 + y^2\,z^2.$$

Let $r = x\,y\,z$.

$$\mathrm{S}\,\frac{r}{x} = y\,z + x\,z + x\,y = \mathrm{S}\,x\,y$$

$$\mathrm{S}\,\frac{1}{x} = \frac{1}{r}\,\mathrm{S}\,x\,y$$

$$\mathrm{S}\,\frac{1}{x^3} = \frac{1}{r^3}\,\mathrm{S}\,x^2\,y^2$$

$$\mathrm{S}\,\frac{1}{x^n} = \frac{1}{r^n}\,\mathrm{S}\,x^n\,y^n, \text{ \&c. \&c.}$$

Let there be 5 variables, u, v, x, y, z, then

$$\mathrm{S}\,u\,v\,x\,y = u\,v\,x\,y + u\,v\,x\,z + u\,v\,y\,z + u\,x\,y\,z + v\,x\,y\,z, \text{ \&c. \&c. \&c.}$$

The greater the number of the variables, the greater is the advantage of this abbreviated notation.

To resume our examples:

Ex. 5.—Let $\mathrm{S}\,x^2\,y^2 + a\,.\,x\,y\,z + b\,\mathrm{S}\,x\,y = c$. Then supposing, as before, that $x + y + z = 0$, we find

$$2\,y\,z = 2\,x^2 - a\,x - b + \sqrt{(4\,c + b^2) + 2\,a\,b\,x + a^2\,x^2 - 4\,a\,x^3}.$$

By properly determining the constants a, b, c, this radical may be made to agree with any proposed cubic $\sqrt{x^3 + \alpha\,x^2 + \beta\,x + \gamma}$.

Ex. 6.—Let $x^6 + y^6 + z^6 = 0$, or $\mathrm{S}\,x^6 = 0$. Here $y\,z$ or $\varphi\,x$ is an implicit function of x, only determinable by the solution of the cubic equation

$$(m - x^2)^3 = \frac{\mathrm{S}}{2}\,x^2\,m^2,$$

where $m = y\,z$.

Notwithstanding the complicated nature of the function φx, we still have

$$S\int \varphi x \,.\, dx = xyz + C.$$

But the most interesting result was obtained by combining the equations

$$x + y + z = 0$$
$$xy + xz + yz = -\frac{x^2 y^2 z^2}{4};$$

or, in the compendious notation, putting $xyz = r$,

$$Sx = 0 \qquad Sxy = \frac{-r^2}{4}:$$

whence

$$yz = \frac{2\sqrt{1+x^4}}{x^2} - \frac{2}{x^2} = \varphi x,$$

and by the general theorem A

$$xyz + C = S\int \varphi x \,.\, dx. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

But $\int \varphi x \,.\, dx$ consists of two parts, of which the latter is $\int \frac{-2\,dx}{x^2} = \frac{2}{x}$. Therefore the sum of three such portions $= \frac{2}{x} + \frac{2}{y} + \frac{2}{z} = 2S\frac{1}{x} = \frac{2Sxy}{r} = \frac{-r}{2}$; since by hypothesis $Sxy = \frac{-r^2}{4}$.

Hence if we put

$$\int \frac{\sqrt{1+x^4}}{x^2} \,.\, dx = \int \psi x \,.\, dx,$$

equation (1.) becomes

$$r + C = 2S\int \psi x \,.\, dx - \frac{r}{2};$$

whence

$$S\int \psi x \,.\, dx = \frac{3}{4} r + \text{const.}$$

Now this result is highly deserving of attention; for the integral which we have here called $\int \psi x \,.\, dx = \int \frac{\sqrt{1+x^4}}{x^2} \,.\, dx$, is no other than the arc of an equilateral hyperbola whose abscissa is x, the equation of the curve being referred to its asymptotes. When I arrived at this result, I immediately perceived that (provided there were no error in the reasoning, of which I at first entertained some doubts,) it was an entirely new and undiscovered property of the hyperbola. I therefore proceeded to verify it by calculating numerical examples.

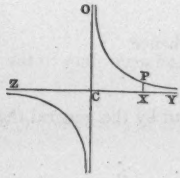
The theorem may be stated thus: If three abscissæ of an equilateral hyperbola verify the equations $Sx = 0$, $Sxy = \frac{-r^2}{4}$, the sum of the arcs subtended by those abscissæ $= \frac{3}{4} r +$ const. In order to eliminate the constant, the value of which was unknown, I supposed one of the abscissæ x to assume some other value x', and there-

fore a corresponding change to take place with respect to y and z (since they are functions of x). These new values may be called y' and z'.

At the same time the product $x\,y\,z = r$ is changed to $x'\,y'\,z' = r'$; and the first set of arcs, which may be denoted by A, are changed for a second set, which may be denoted by B.

Now the original equation gives, sum of arcs $\mathrm{A} = \frac{3}{4}\,r + \mathrm{C}$, and the changed equation gives, sum of arcs $\mathrm{B} = \frac{3}{4}\,r' + \mathrm{C}$; $\therefore$ by subtraction, sum of arcs A $-$ sum of arcs $\mathrm{B} = \frac{3}{4}\,(r - r')$, in which result the constant is eliminated.

Fig. 1.



The accompanying figure 1. represents two opposite equilateral hyperbolas, with their asymptotes. C is the centre, and origin of the abscissæ $\mathrm{C\,X} = x$, $\mathrm{C\,Y} = y$, $\mathrm{C\,Z} = z$, of which the latter must be negative (supposing the two former positive), by reason of the equation $x + y + z = 0$. Therefore it belongs to the opposite hyperbola. If X P is the ordinate corresponding to the abscissa C X, the equation of the curve is $\mathrm{C\,X} \,.\, \mathrm{X\,P} = 1$, and the arc subtended by C X is the infinite arc O P.

Fig. 2.

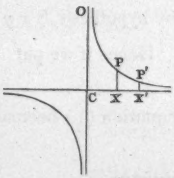


Fig. 2. represents the abscissa $\mathrm{C\,X} = x$, both in its original and in its altered state when it has become $\mathrm{C\,X'} = x'$. In the former case it subtends the infinite arc O P, and in the latter case the infinite arc O P'. But in taking the difference there remains the portion of abscissa X X' subtended by the arc P P', which is a finite quantity, and thus the embarrassing consideration of the infinite arcs is avoided. Now the sum of arcs A $-$ sum of arcs B $=$ sum of three limited arcs, of which P P' is one, and the others subtend the portions of abscissæ Y Y' and Z Z'. Denoting these arcs by K, we have this equation in finite terms:

$$\text{Sum of arcs } \mathrm{K} = \tfrac{3}{4}\,(r - r').$$

Now in order to put this equation to the test of numerical computation, it is requisite to find three quantities that verify the equations

$$\mathrm{S}\,x = 0 \qquad \mathrm{S}\,x\,y = \frac{-r^2}{4}$$

Suppose, therefore,

$$\begin{aligned} x &= 1 \\ y &= 1{\cdot}7535 \\ z &= -2{\cdot}7535, \end{aligned}$$

whence

$$x\,y\,z = -4{\cdot}8281 = r.$$

The equations are satisfied by these values, and also by the following:

$$\begin{aligned} x' &= 1{\cdot}1 \\ y' &= 1{\cdot}5826 \\ z' &= -2{\cdot}6826, \end{aligned}$$

whence

$$x'\,y'\,z' = -4{\cdot}670 = r'.$$

[4.] $$\therefore\ x' - x = \ \ 0{\cdot}1 \qquad\qquad r' - r = 0{\cdot}1581.$$
$$y' - y = -0{\cdot}1709$$
$$z' - z = \ \ 0{\cdot}0709$$

Now we can, without difficulty, calculate the approximate value of the arc (P P' in fig. 2.) subtended by the portion of abscissa $x' - x$ (X X' in the figure). Calling this the arc (x), we have

$$\text{Arc } (x) = 0{\cdot}1351$$
$$\text{Arc } (y) = 0{\cdot}1817$$
$$\text{Arc } (z) = 0{\cdot}0715.$$

And according to the theorem we ought to have

$$\text{Sum of arcs} = \tfrac{3}{4}(r' - r) = 0{\cdot}118.$$

But in this example, arc (x) is to be accounted negative. Therefore we have

$$\text{Arc } (y) + \text{Arc } (z) = 0{\cdot}253$$
$$-\text{Arc } (x) = 0{\cdot}135$$
$$\text{Sum} = \overline{0{\cdot}118}$$

which is in accordance with the theorem.

Second example.—Suppose, as before,

$$x = \ \ 1$$
$$y = \ \ 1{\cdot}7535$$
$$z = -2{\cdot}7535,$$

whence

$$x\,y\,z = -4{\cdot}8281.$$

And also

$$x' = \ \ 2$$
$$y' = \ \ 0{\cdot}8875$$
$$z' = -2{\cdot}8875,$$

whence

$$x'\,y'\,z' = -5{\cdot}1253;$$

both of which systems of values satisfy the given equations of condition.

[5.] $$\therefore\ x' - x = \ \ 1 \qquad\qquad r' - r = -0{\cdot}297.$$
$$y' - y = -0{\cdot}866$$
$$z' - z = -0{\cdot}134$$

By calculation we find

$$\text{Arc } (x) = 1{\cdot}1319$$
$$\text{Arc } (y) = 1{\cdot}0443$$
$$\text{Arc } (z) = 0{\cdot}1350.$$

But in this example both arc (x) and arc (z) are negative. Therefore we have

$$\begin{aligned} -\operatorname{Arc}(x) - \operatorname{Arc}(z) &= -1{\cdot}267 \\ +\operatorname{Arc}(y) &= +1{\cdot}044 \\ \text{Sum} &= -0{\cdot}223 \end{aligned}$$

Also

$$\begin{aligned} r' - r &= -0{\cdot}297 \\ \therefore \tfrac{3}{4}(r' - r) &= -0{\cdot}223 \end{aligned}$$

in accordance with the theorem.

I had at first some difficulty to perceive the reason why some of the arcs were to be considered negative rather than the others. This question was one of a novel nature, which had not hitherto occurred to analysts, and therefore no solution of it was to be met with in books. On the other hand, to leave so essential a point without any demonstration was unsatisfactory. But the following considerations appeared to afford an explanation of this fact.

In the first example, since $x + y + z = 0$, and both x and y are positive quantities, z must be negative. Therefore xy is positive, but xz and yz are negative.

Now we have

$$\frac{yz}{2} = \frac{\sqrt{1+x^4}-1}{x^2},$$

which therefore must be negative: $\therefore$ also $\sqrt{1+x^4}-1$ must be negative. But this quantity would necessarily be positive if the radical had a positive sign. Therefore the radical must have a negative sign.

Reasoning in the same manner, because $\frac{xz}{2} = \frac{\sqrt{1+y^4}-1}{y^2}$, this radical has necessarily a negative sign; and because $\frac{xy}{2} = \frac{\sqrt{1+z^4}-1}{z^2}$, this radical has a positive sign. Attributing, therefore, these signs to the radicals, the three hyperbolic arcs are respectively,

$$-\int\frac{dx}{x^2}\sqrt{1+x^4}, \qquad -\int\frac{dy}{y^2}\sqrt{1+y^4}, \qquad +\int\frac{dz}{z^2}\sqrt{1+z^4}.$$

On the other hand, during the change of x, y, z to x', y', z' respectively, we have seen that x and z increase while y diminishes (see equations [4.]); $\therefore dx$ and dz are to be accounted positive, and dy negative. This consideration renders it necessary to write

$$[6.] \quad -\int\frac{dx}{x^2}\sqrt{1+x^4}, \qquad +\int\frac{dy}{y^2}\sqrt{1+y^4}, \qquad +\int\frac{dz}{z^2}\sqrt{1+z^4}.$$

And thus the assertion that arc (x) is to be considered negative in this example is justified.

Second example.—Here the reasoning remains the same as far as regards the signs of the different radicals; but it appears from the equations [5.], that while x increases, both y and z diminish, $\therefore dx$ is positive, and dy and dz are negative. Therefore the only difference between this example and the former respects the sign of dz. Therefore equation [6.] must be written

$$-\int\frac{dx}{x^3}\sqrt{1+x^4}, \qquad +\int\frac{dy}{y^3}\sqrt{1+y^4}, \qquad -\int\frac{dz}{z^3}\sqrt{1+z^4},$$

which justifies the assertion, that in this example arcs (x) and (z) are to be accounted negative, and arc (y) positive.

Perhaps this reasoning may not be altogether free from objection: I wish it, therefore, to be remembered that I am not giving it here as being the most convenient method of determining the signs of the arcs, but merely as being the reasoning I employed at the time * when I first met with this theorem.

This theorem shows that three hyperbolic arcs may be determined in an infinity of ways, so that their sum may be an algebraic quantity. At the same time it shows that one of these arcs cannot be supposed always to be 0, so that FAGNANI'S theorem respecting the sum of *two* arcs is not an instance or particular case of this. I have dwelt at some length on this theorem, because the theory of the conic sections has always been regarded as so important by mathematicians that any considerable addition to it is thought deserving of attention.

I now proceed to other results which presented themselves in the course of this inquiry.

Still continuing to suppose the variables to be three in number, it is allowable to suppose between them any two symmetrical equations whatever; and thence if we can deduce the value of yz or φx in terms of x alone, we may apply the general theorem A.

Ex. 1. Let $Sx = a$, and $Sxy = \left(\frac{xyz}{2}\right)^2$, we find

$$\frac{yz}{2} = \frac{1}{x^2} + \frac{\sqrt{1-x^4+ax^3}}{x^3}.$$

Ex. 2. Let $Sx = a$, and $Sxy = \sqrt{2b.xyz}$, we find

$$yz = x^2 - \overline{a-b}.x + \sqrt{2bx^3 + (b^2 - 2ab)x^2}.$$

Ex. 3. Let $\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{a}$ (or $S\sqrt{x} = \sqrt{a}$), and let $Sx = b$, we find

$$\sqrt{yz} = \frac{a-b}{2} - \sqrt{ax} + x = fx,$$

whence

$$yz = \varphi x = [fx]^2.$$

A great variety of different suppositions of this sort may be made; but if the resulting function φx should become too complicated, little practical advantage would be derived from the knowledge of its properties. I therefore thought of another method of obtaining this function, by means of what may be termed "changing the conditions." Thus let the original equations of condition be

$$x+y+z=0, \quad \text{and} \quad xy+xz+yz=-1,$$

* 1825.

whence this equation results,

$$y z = x^2 - 1.$$

Now for x, y, z write their cubes (both in the original and in the resulting equation), and they become

$$[7.] \quad x^3 + y^3 + z^3 = 0$$
$$x^3 y^3 + x^3 z^3 + y^3 z^3 = -1$$

and

$$y^3 z^3 = x^6 - 1.$$

Taking the cube root of this, we have

$$y z = \sqrt[3]{x^6 - 1}.$$

Whence it follows that the sum of three integrals

$$\int d x \sqrt[3]{x^6 - 1} + \int d y \sqrt[3]{y^6 - 1} + \int d z \sqrt[3]{z^6 - 1} = x y z + C,$$

whenever x, y, z satisfy the two given equations of condition [7.], which may be briefly written

$$S x^3 = 0 \qquad S x^3 y^3 = -1.$$

Here we *changed the conditions*, by writing x^3 for x. We might have written x^n for x, and thereby obtained a more general result*. Even values of n must, however, be excluded, because the equation $x^n + y^n + z^n = 0$ would otherwise be impossible.

Ex. 4. Let $S x = a$, and $S \frac{1}{x} = \frac{1}{b}$, whence

$$y z = \frac{a b x - b x^2}{x - b}.$$

Now if we write for x, y, z, a, b, their square roots, these three equations become

$$\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{a},$$
$$\sqrt{\frac{1}{x}} + \sqrt{\frac{1}{y}} + \sqrt{\frac{1}{z}} = \sqrt{\frac{1}{b}},$$

and

$$\sqrt{y z} = \frac{\sqrt{a b}\, x - \sqrt{b} \,.\, x}{\sqrt{x} - \sqrt{b}} = f x,$$

whence

$$y z = \phi x = [f x]^2.$$

Many interesting theorems may be obtained by this method of "*changing the original conditions*," but these examples of it will suffice for the present.

I now perceived that the hypothesis upon which my method was grounded, viz. that $n - 1$ symmetrical equations existed between n variables, was the same thing as to suppose that these variables were the roots of an equation of n dimensions, *one of whose coefficients at least was variable*, the others being either constants, or functions of the variable one. This consideration introduced a great degree of clearness and

* Viz. the sum of three integrals, like $\int d x \sqrt[3]{x^{3n} - 1}$.

simplicity into the subject, besides facilitating in no ordinary degree the progress of research. For instance, suppose there are 3 variables, and let $x+y+z=p$, $xy+xz+yz=q$ and $xyz=r$, then x, y, z are the roots of the equation

$$u^3-pu^2+qu-r=0,$$

where the variable u denotes *indifferently* either of the variables x, y, or z. This new letter u is only introduced for the sake of clearness, since we may equally well say that x, y, z are the roots of the equation

$$x^3-px^2+qx-r=0,$$

or of

$$y^3-py^2+qy-r=0, \text{ \&c.}$$

This latter mode of expression is often more convenient. Now the function φx, which we wish to determine,

$$=yz=\frac{xyz}{x}=\frac{r}{x};$$

and since p, q are here supposed to be *given functions of* r, we may find the value of $\frac{r}{x}$ in terms of x, provided we can solve the algebraic equation

$$x^3-px^2+qx-r=0,$$

with respect to r.

Example. Let us resume the question concerning the sum of 3 arcs in the equilateral hyperbola. The equations of condition were

$$x+y+z=0,$$

$$xy+xz+yz=\frac{-x^2y^2z^2}{4},$$

or

$$p=0, q=\frac{-r^2}{4},$$

$\therefore x, y, z$ are the roots of

$$x^3-\frac{r^2}{4}x-r=0.$$

This equation (arranged according to the powers of r) is

$$\frac{-x}{4}.r^2-r+x^3=0,$$

or

$$r^2+\frac{4}{x}.r-4x^2=0,$$

whence

$$r=\frac{-2}{x}+2\sqrt{x^2+\frac{1}{x^2}},$$

and

$$\frac{r}{x}=\frac{-2}{x^2}+\frac{2\sqrt{1+x^4}}{x^2}=\varphi x,$$

which agrees with the former result. But now we are able to point out with clearness the limits of the possibility of the theorem. For the cubic equation

$$x^3-\frac{r^2}{4}.x-r=0$$

being compared with the form

$$x^3 - a\,x + b = 0,$$

must have impossible roots when $\frac{b^2}{4} > \frac{a^3}{27}$, or when $\frac{r^2}{4} > \frac{r^6}{64 \times 27}$, or $1 > \frac{r^4}{16 \times 27}$, or $16 \cdot 27 > r^4$. Hence it appears that there are impossible roots whenever r is less than

$$\sqrt[4]{16 \times 27} = 2\sqrt[4]{27} = \pm 4{\cdot}559.$$

Accordingly in our numerical examples it will be seen that the values* of r are not contained within these limits.

Another example. We have found (page 183. *Ex.* 3.) that the suppositions $\mathrm{S}\,x = 0$, $\mathrm{S}\,x^4 = 2\,r$, give

$$2\,\varphi\,x = 2\,x^2 + x + \sqrt{4\,x^3 + x^2}.$$

Here x, y, z are roots of $x^3 + q\,x - r = 0$, and we easily find from the doctrine of equations that

$$\mathrm{S}\,x^4 = 2\,q^2, \quad \therefore 2\,q^2 = 2\,r, \quad \therefore q^2 = r.$$

Therefore $x + q\,x - q^2 = 0$. Solving this quadratic equation with respect to q we have

$$q = \frac{x}{2} + \frac{1}{2}\sqrt{x^2 + 4\,x^3}.$$

But since

$$x^3 + q\,x - r = 0, \frac{r}{x} = x^2 + q.$$

$$\therefore \frac{2\,r}{x} = 2\,x^2 + x + \sqrt{x^2 + 4\,x^3} = 2\,\varphi\,x,$$

which agrees with the former result.

Another example. Let $x^3 + q\,x - r = 0$, which gives for the first condition

$$\mathrm{S}\,x = 0.$$

And let the second condition be

$$r^2 + c\,r = q^2 + a\,q + b,$$

a, b, c being constants. We find

$$\frac{r}{x} = \frac{x^2 + \frac{c}{2} \cdot x - \frac{a}{2} + \sqrt{\mathrm{X}}}{1 - x^2},$$

where X is a polynomial of 6 dimensions.

Now let the n variables $x, y, z \ldots\ldots$ be roots of $x^n - p\,x^{n-1} + \ldots\ldots \pm r = 0$, where I continue to denote the product of all the roots by r; we have still $\varphi\,x = \frac{r}{x}$. Let the coefficients p, &c. &c. be replaced by their values in terms of r (which are supposed given, by means of $n - 1$ equations of condition). Then let the equation be arranged according to the powers of r, and the solution of it will give the value of r in terms of x, and therefore the value of $\frac{r}{x} = \varphi\,x$.

* These were $r = -4{\cdot}83$, $r = -4{\cdot}67$, $r = -5{\cdot}13$.

Now if it be considered that this method extends to any number whatever of variables, and that the coefficients of the equation may be any functions of each other that we please to make them, it will appear at once how wide a field of inquiry here opens before us. It was the wish to reduce these extensive but rather complicated results to something like a clear and connected system which obliged me to defer the publication of them longer than I should otherwise have wished, by which means I lost the priority which at one time was in my power of announcing the existence of this new branch of analysis; for the results hitherto mentioned, together with many others, which for the sake of brevity I omit, were obtained in the years 1825 and 1826, and consequently two or three years previously to the publication of ABEL's theorem. And it will be observed that they comprise large classes of integrals which are not contained in his formula

$$\int \frac{P\,dx}{\sqrt{R}}.$$

Of this I have given an instance in the integral,

$$\int dx \sqrt[3]{x^6 - 1},$$

and the more general one,

$$\int dx \sqrt[n]{x^{2n} - 1}.$$

But an unlimited number of such forms may be found by the method I have pointed out of "changing the conditions" at first established between the variables. We may conclude therefore that if $x, y, z \ldots\ldots$ are the roots of an equation of n dimensions, having at least one variable coefficient, and if we can find the function $\varphi x = \frac{r}{x}$ in terms of x, we may thence deduce the algebraic sum of the n integrals,

$$\int \varphi x \,.\, dx + \int \varphi y \,.\, dy + \ldots\ldots$$

But the inverse problem still remains. *Given the function* φx, *or the integral* $\int \varphi x \,.\, dx$, *to find the equation*

$$x^n - p x^{n-1} + \ldots\ldots \pm r = 0,$$

of which $x, y, z \ldots\ldots$ *must be roots, in order that* $S \int \varphi x \,.\, dx$ *may have an algebraic sum?*

This is evidently the most important part of the subject, for in applying the method to practice the form of the function φx is given beforehand. That this research requires methods of its own will appear at once from a simple example.

Let $\int \sqrt{1 + x^n} \,.\, dx$ be the proposed integral, where n is any whole number. Let us first suppose

$$\sqrt{1 + x^n} = \varphi x = \frac{r}{x}.$$

This gives

$$r = x\sqrt{1+x^n},$$

or

$$x^{n+2} + x^2 - r^2 = 0,$$

an equation, the product of whose roots must be $\pm r^2$. But by the hypothesis this product is always represented by r. Whence it follows that no solution of the required problem is effected by the supposition $\sqrt{1+x^n} = \frac{r}{x}$. And at the same time we see that in order for any supposition to be successful, it is necessary that the resulting equation, arranged according to the powers of x, should have r for its last term.

Now let us remark, that if $S\int \psi x \,.\, dx$ has an algebraic sum, then $S\int (\psi x + x^a)\, dx$ has likewise an algebraic sum. For it equals the former sum, with the addition of $S\int x^a\, dx$, or $\frac{1}{a+1}\,(x^{a+1} + y^{a+1} + z^{a+1} + \ldots\ldots)$, which is an algebraic quantity. In the same way we see that $S\int (\psi x + m\,x^a + n\,x^b + \ldots)\, dx$ has an algebraic sum if m, n, a, b, &c. are constants, and if there are any number of such simple terms of the form $m\,x^a$. Hence if the proposed integral be $\int \psi x \,.\, dx$, and the supposition φx or $\frac{r}{x} = \psi x$ does not succeed, we are led to try the suppositions

$$\frac{r}{x} = \psi x + x,\ \frac{r}{x} = \psi x + x + x^2, \text{ \&c. \&c.}$$

Example. Let the integral $\int \psi x \,.\, dx = \int \sqrt{1+x^n} \,.\, dx$ as before. Suppose $\sqrt{1+x^n} = \frac{x}{2} + \sqrt{1+r}$, whence we deduce

$$x^n - \frac{x^2}{4} - \sqrt{1+r}\,.\,x - r = 0,$$

an equation which has n roots, whose product is r. We also find

$$1 + r = 1 + x^n - x\sqrt{1+x^n} + \frac{x^2}{4},$$

whence

$$\frac{r}{x} = x^{n-1} + \frac{x}{4} - \sqrt{1+x^n},$$

or

$$\varphi x = x^{n-1} + \frac{x}{4} - \psi x.$$

And therefore since $S\int \varphi x\, dx$ has an algebraic sum $= r +$ const., it follows that $S\int \psi x\, dx$ has an algebraic sum also, viz.

$$S\int x^{n-1}\,dx + \tfrac{1}{4}\,S\int x\,dx - (r + \text{const.})$$

or

$$\tfrac{1}{n}\,S\,x^n + \tfrac{1}{8}\,S\,x^2 - (r + \text{const.})$$

But if the form of the proposed integral $\int \psi x \,.\, dx$ is complicated, no doubt it would be difficult to find an equation like

$$\frac{r}{x} = \pm\psi x + m x^a + n x^b + \ldots\ldots,$$

such that when developed and arranged according to the powers* of x its last term should be r. Probably this is not possible in general. And yet the proposed integrals $S\int \psi x\,dx$ may have an algebraic sum. For hitherto we have tacitly supposed that this algebraic quantity, if it existed, was the product of the variables $= r + \text{const.}$, since we have derived all our reasonings from the theorem

$$xyz\ldots = \int yz\ldots.dx + \int xz\ldots.dy + \&c.$$

But it is evident that the algebraic sum may as well have any other form as the one in question. It may be a constant or any symmetrical combination of the variables. The foundation of our reasoning has therefore hitherto been too limited, and requires to be extended. Let us therefore direct our inquiries to the attainment of a more general method.

§ 4. *Exposition of a more general method.*

If $x, y, z \ldots\ldots$ are the roots of any equation,

$$x^n - p\,x^{n-1} + p'\,x^{n-2} \ldots\ldots = 0,$$

then not only the coefficients themselves p, p', p'', &c., but also all combinations of them, are symmetrical functions of the roots. Let v be a general symbol denoting any one of these coefficients or of these combinations. Then v may be considered either as a function of all the roots, or of only one of them. And in the latter case this root may be changed for any of the others without causing any alteration in the value of v.

Example. Let there be two variables x and y, roots of $x^2 - v\,x + 1 = 0$, which may be also written $y^2 - v\,y + 1 = 0$. Then v if considered as a function of both x and y, is equal to $x + y$, the sum of the roots. But if considered as a function of x alone, it is $= \frac{1 + x^2}{x}$. And if considered as a function of y alone, it is $= \frac{1 + y^2}{y}$.

$$\therefore \frac{1 + x^2}{x} = \frac{1 + y^2}{y},$$

* The coefficient of the highest power of x being always supposed $= 1$.

or x may be permuted for y. Hence also

$$\varphi . \frac{1+x^3}{x} = \varphi . \frac{1+y^3}{y},$$

φ being any function.

Quantities which $\left(\text{like } \varphi . \frac{1+x^3}{x} \text{ in this example}\right)$ are not changed in value by permuting the roots, may be termed "symmetrical functions" of the variables x, y, z, &c. or simply "symmetricals" of the equation whose roots are $x, y, z \ldots\ldots$ Thus the quantity $\varphi\left(\frac{1+x^2}{x}\right)$ is a symmetrical of the equation

$$x^2 - v x + 1 = 0.$$

But the quantity $\varphi\left(\frac{1-x^3}{x}\right)$ is not a symmetrical of it, because $\frac{1-x^3}{x}$ is not equal to $\frac{1-y^3}{y}$.

Ex. 2. Let x, y, z, be roots of $x^3 - v x + 1 = 0$, which may also be written $y^3 - v y + 1 = 0$, or $z^3 - v z + 1 = 0$, whence

$$v = \frac{1+x^3}{x} = \frac{1+y^3}{y} = \frac{1+z^3}{z},$$

whence also

$$\varphi . \frac{1+x^3}{x} = \varphi . \frac{1+y^3}{y} = \varphi . \frac{1+z^3}{z}.$$

Therefore the quantity $\varphi . \frac{1+x^3}{x}$ is a symmetrical of the equation $x^3 - v x + 1 = 0$: but $\varphi . \frac{1+x^2}{x}$ is not a symmetrical of it, because $\frac{1+x^2}{x}$ is not equal to $\frac{1+y^2}{y}$.

These things being premised, it is evident that the same quantity may be a symmetrical of one equation and not so of another. Therefore the problem arises: *Any quantity being given, to find the equation with respect to which it is symmetrical?*

Ex. 1. Let $\frac{1+x^2}{x}$ be the given quantity. Put $\frac{1+x^2}{x} = v$, v being a general symbol for any symmetrical quantity:

$$\therefore x^2 - v x + 1 = 0$$

is the required equation, and the indeterminate v is thereby determined to be the sum of the roots.

Ex. 2. Let $\frac{1+x^3}{x}$ be the given quantity. Put $\frac{1+x^3}{x} = v$:

$$\therefore x^3 - v x + 1 = 0$$

is the required equation, and $-v$ is thereby determined to be the sum of the products of every two roots *.

* *Another example.*—Let $\frac{1+x+x^2}{1-x}$ be the given quantity. Put it $= v$:

$$\therefore x^2 + (1+v)x + (1-v) = 0$$

is the required equation, and $(1+v)$ is determined to be the sum of the roots $x+y$, and $1-v$ their product xy. Whence, by eliminating v, we find the following relation between x and y: $xy - (x+y) = 2$. This example will be referred to hereafter.

Ex. 3. Let $x + \sqrt{1-x^2}$ be the given quantity. Put $x+\sqrt{1-x^2}=v$:

$$\therefore (v-x)^2 = 1 - x^2$$

$$2x^2 - 2vx + (v^2 - 1) = 0$$

$$x^2 - vx + \frac{v^2-1}{2} = 0$$

is the required equation, and v is determined to be the sum of its roots. Also $\frac{v^2-1}{2}$ is equal to their product xy. In other words, the equation must be such, that its roots x, y, answer the condition $xy = \frac{(x+y)^2-1}{2}$.

Ex. 4. Let $x + \sqrt{\frac{1}{x}}$ be the given quantity. Put $x + \sqrt{\frac{1}{x}} = v$:

$$\therefore (v-x)^2 = \frac{1}{x}$$

$$\therefore x^3 - 2vx^2 + v^2x - 1 = 0$$

is the required equation, which is thereby determined to be a cubic, the product of whose roots $= 1$, and v is found to be half the sum of the roots.

Case of exception.—It is essential to remark, that when the given quantity contains only one power of x, it cannot be a symmetrical. *Ex.* $\sqrt{1+x^n}$ cannot be a symmetrical; for if it could, we should have $\sqrt{1+x^n} = \sqrt{1+y^n} = \sqrt{1+z^n} =$ &c., whence $x = y = z =$ &c.; whereas we suppose the roots to be in general all different from one another. With this exception the required equation may be easily found in most cases by putting the given quantity, or $fx = v$. And if the roots of the equation thus found are denoted by $x, y, z \ldots\ldots$ it is an immediate consequence of the hypothesis that $fx = fy = fz =$ &c. Thus in the last example we have

$$x + \sqrt{\frac{1}{x}} = y + \sqrt{\frac{1}{y}} = z + \sqrt{\frac{1}{z}}.$$

Let us now suppose that $\mathrm{S}\,dx$, the sum of the differentials of the roots, or $dx + dy + dz$ &c., is multiplied by a symmetrical, that is, by one of the above-mentioned quantities fx. The product is $fx \,.\, dx + fx \,.\, dy + fx \,.\, dz +$ &c. But in consequence of the equality $fx = fy = fz =$ &c. the result is the same, if the first term is multiplied by fx, the second by fy, the third by fz, and so on. So that the product is $fx \,.\, dx + fy \,.\, dy + fz \,.\, dz +$ &c., which is our abbreviated notation $= \mathrm{S}fx \,.\, dx$.

$$\therefore fx \,.\, \mathrm{S}\,dx = \mathrm{S}fx \,.\, dx. \quad \ldots\ldots\ldots\ldots \quad (1.)$$

This theorem is of the greatest importance, and will be of constant use in the sequel. It must not be forgotten that it is only true when fx is a symmetrical, and therefore capable of being represented by v. Replacing fx by v, it becomes $v\,\mathrm{S}\,dx = \mathrm{S}v\,dx$. In this form it is self-evident, because v remains the same, however the letters $x, y, z, \ldots$ are permuted.

More generally, if the quantity $S\psi x \,.\, dx$, which means $\psi x.dx + \psi y.dy + \psi z.dz +$ &c., is multiplied by fx, the product may be exhibited in the form

$$fx\psi x\,.\,dx + fy\psi y\,.\,dy + fz\psi z\,.\,dz + \&c.,$$

which in our notation is $Sfx\psi x\,.\,dx$. Whence the theorem

$$fx\,S\psi x\,.\,dx = Sfx\psi x\,.\,dx, \quad \ldots\ldots\ldots\ldots \quad (2.)$$

which may also be put in the self-evident form

$$v\,S\psi x\,.\,dx = Sv\psi x\,.\,dx.$$

Equation (1.) is a corollary from (2.) when $\psi x = 1$.

These results may be comprised in a general rule, viz. that whatever be the nature of the differential $\psi x\,.\,dx$, if we multiply the sum of a series of such quantities, or $S\psi x\,.\,dx$, by fx any function of x, the multiplication is effected by introducing fx within the sign S, provided (and this is the essential condition) *that fx is a symmetrical of that equation of which all the variables are roots.* It is upon this principle that the method which I am about to explain chiefly reposes.

Suppose $\int X\,.\,dx$ to be the proposed integral, X being any function of x. In the first place we have to determine the number of the other variables y, z, &c., and also the nature of the equation $x^n - px^{n-1} + \&c. = 0$, of which they are roots. And this may in general be accomplished by the following process: *Assume* X *to be a symmetrical of this unknown equation*, or that $X = v$; then if this equation $X = v$ can be cleared of radicals, &c., (as in examples 1, 2, 3, 4,) it may be ultimately reduced to the form

$$x^n - px^{n-1} + p'x^{n-2} \ldots\ldots = 0,$$

where p, p', p'', the coefficients, are either constants or functions of v. The index n of this equation determines the number of the variables.

Let Y be a quantity containing the variable y, in the same manner that X contains x, and let Z contain z in the same manner, and so on for the other roots. Then $v = X = Y = Z = \&c.$, in consequence of the hypothesis that X is a symmetrical of this equation. Therefore the sum of the following series of differentials,

$$X\,.\,dx + Y\,.\,dy + Z\,.\,dz + \&c.,$$

is equal to $X\,(dx + dy + dz + \&c.)$

$$= X\,.\,S\,dx = v\,S\,dx.$$

Now $S\,dx = dp$, where p is the coefficient of the second term of the equation

$$x^n - px^{n-1} + p'x^{n-2} \ldots\ldots = 0;$$

and, as we have before remarked, p is either a constant or a function of $v = \varphi\,.\,v$.

First let it be a constant: then

$$dp = S\,dx = 0,$$

which gives

$$\begin{aligned} &X\,dx + Y\,dy + Z\,dz + \ldots = v\,S\,dx \\ &\qquad = 0, \end{aligned}$$

whence we deduce the very important consequence

$$\int X.dx + \int Y.dy + \int Z.dz + \ldots = \text{const.},$$

which is true, whatever be the nature of the function X, provided only that the coefficient p is constant.

Secondly, let $p = \varphi v$,

$$\therefore dp = d.\varphi v,$$

$$\therefore X.dx + Y.dy + Z.dz + \&c. = v.d\varphi v,$$

$$\therefore \int X.dx + \int Y.dy + \int Z.dz + \&c. = \int v.d\varphi v;$$

and therefore the sum of the integrals, or $S\int X.dx$, is known, whenever the formula $\int v\,d\varphi v$ is capable of integration; or, which is equivalent, when the form $\int \varphi v.dv$ is capable of integration. These consequences flow, with respect to the proposed integral $\int X.dx$, from the supposition that X is a symmetrical of the equation whose roots the variables are. But a much more general method is attainable, by putting the proposed integral under the form $\int \frac{X}{\psi x}.\psi x\,dx$, and then assuming the quantity $\frac{X}{\psi x}$ to be a symmetrical of the said equation *.

Therefore $\frac{X}{\psi x}$ may be represented by the general symbol v, and the proposed integral by $\int v.\psi x.dx$.

The series of differentials

$$X\,dx + Y\,dy + Z\,dz + \&c.$$

may therefore be written

$$\begin{aligned} &v\psi x\,dx + v\psi y\,dy + v\psi z\,dz + \&c. \\ &= v\,(\psi x\,dx + \psi y.dy + \&c.), \end{aligned}$$

which we write

$$= v\,S\psi x.dx.$$

Therefore, whenever it happens that $S\psi x.dx = 0$, we have $v\,S\psi x.dx = 0$,

$$\therefore \int X\,dx + \int Y\,dy + \int Z\,dz + \&c. = \text{const.}:$$

* Under this new supposition the quantity X of course ceases, in general, to be a symmetrical of the equation.

and since ψx is arbitrary, this condition $S \psi x \, dx = 0$ may frequently be realized. But it will be observed, that every change in the form of ψx changes the equation between the variables. But if $S \psi x \, dx$ is not $= 0$, as it generally is not, it must be a quantity symmetrically composed with respect to all the variables, and therefore a function of v, since all the coefficients of the equation are functions of v, or constants. Therefore it may be represented by $d \cdot \varphi v$,

$$\therefore X\,dx + Y\,dy + Z\,dz + \&c. = v\,S\,\psi x\,dx$$
$$= v \cdot d \cdot \varphi v$$
$$\therefore \int X\,dx + \int Y\,dy + \&c. = \int v \cdot d \cdot \varphi v :$$

and therefore the sum of the integrals is known in all those cases in which $\int v \cdot d \cdot \varphi v$ can be integrated.

The most direct and advantageous method of treating any proposed integral $\int X\,dx$, is to make one of the two suppositions above mentioned, viz. $X = v$, or $X = v \cdot \psi x$. But the supposition $X = v + \psi x$, also, often leads to simple and satisfactory results. Our choice, however, is not limited to these forms, but may include others that are comprehended under the general formula $X = f(v, x)$, each of which may perhaps find its application in special cases.

This process, in all its generality, constitutes the method which I now propose. The use and application of it will be best shown by examples.

§ 5.

Direct integration of the formula $\int X \cdot dx$, when that is possible, confirms and illustrates the above results, of which it will be convenient to adduce a few simple examples.

Let there be two variables x, y, roots of $x^2 - vx + 1 = 0$. Therefore the quantity $\frac{1+x^2}{x} = \frac{1+y^2}{y} = v$ is a symmetrical of this equation. And we find

$$x + y = v \quad \therefore dx + dy = dv \quad \ldots\ldots\ldots\ldots (1.)$$

$$xy = 1 \quad \therefore \frac{dx}{x} + \frac{dy}{y} = 0 \quad \ldots\ldots\ldots\ldots (2.)$$

$$x^2 + y^2 = v^2 - 2 \quad \therefore x\,dx + y\,dy = v\,dv ; \quad \ldots\ldots (3.)$$

which results may be thus written:

$$S\,dx = dv \quad \ldots\ldots\ldots\ldots (1.)$$

$$S\,\frac{dx}{x} = 0 \quad \ldots\ldots\ldots\ldots (2.)$$

$$S\,x\,dx = v\,dv. \quad \ldots\ldots\ldots\ldots (3.)$$

Multiply each of these equations by the equation $\frac{1+x^2}{x} = v$, and we find

$$\frac{1+x^2}{x} \mathrm{S}\, dx = v\, dv \quad \ldots \quad (1.)$$

$$\frac{1+x^2}{x} \mathrm{S}\frac{dx}{x} = 0 \quad \ldots \quad (2.)$$

$$\frac{1+x^2}{x} \mathrm{S}\, x\, dx = v^2\, dv. \quad \ldots \quad (3.)$$

But because $\frac{1+x^2}{x}$ is a symmetrical, we may, according to the general principle, introduce it within the sign S.

$$\therefore \mathrm{S}\frac{1+x^2}{x} dx = v\, dv \quad \ldots \quad (1.)$$

$$\mathrm{S}\frac{1+x^2}{x^2} dx = 0 \quad \ldots \quad (2.)$$

$$\mathrm{S}\,(1+x^2)\, dx = v^2\, dv. \quad \ldots \quad (3.)$$

The integrals of these equations are

$$*\mathrm{S}\int\frac{1+x^2}{x} dx = \frac{v^2}{2} + \text{const.} \quad \ldots \quad (1.)$$

$$\mathrm{S}\int\frac{1+x^2}{x^2} dx = \text{const.} \quad \ldots \quad (2.)$$

$$\mathrm{S}\int(1+x^2)dx = \frac{v^3}{3} + \text{const.} \quad \ldots \quad (3.)$$

And we propose to verify these three results by direct integration. First then we have

$$\int\frac{1+x^2}{x} dx + \int\frac{1+y^2}{y} . dy = \frac{x^2+y^2}{2} + \log x + \log y + \text{const.}$$

But

$$xy = 1 \therefore \log x + \log y = 0,$$

and

$$x^2 + y^2 = v^2 - 2.$$

$$\therefore \int\frac{1+x^2}{x} . dx + \int\frac{1+y^2}{y} dy = \frac{v^2}{2} + \text{const.} \quad \ldots \quad (1.)$$

Secondly we have

* It is indifferent whether we write $\mathrm{S}\int$ or $\int\mathrm{S}$, and we may remark that the signs $\mathrm{S}\int d$ may often be permuted. Thus if there are two variables,

$$\mathrm{S}\int dx = \int\mathrm{S}\, dx = x + y$$

$$d\,\mathrm{S}\, x = \mathrm{S}\, dx = dx + dy$$

$$\frac{1}{2} d\,\mathrm{S}\, x^2 = \mathrm{S}\, d\frac{x^2}{2} = x\, dx + y\, dy.$$

$$\left.\begin{aligned}\int\frac{1+x^2}{x^2}\,dx+\int\frac{1+y^2}{y^2}\,dy &= (x+y)-\left(\frac{1}{x}+\frac{1}{y}\right)+\text{const.}\\ &= (x+y)-\left(\frac{x+y}{1}\right)+\text{const.}\\ &= \text{const.}\end{aligned}\right\}\quad .\quad .\quad (2.)$$

Thirdly we have

$$\int(1+x^2)\,dx+\int(1+y^2)\,dy=(x+y)+\frac{x^3+y^3}{3}+\text{const.}$$

$$=v+\frac{x^3+y^3}{3}+\text{const.}$$

But the formula makes it

$$=\frac{v^3}{3}+\text{const.}\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad (3.)$$

It is necessary therefore to show that these two results are in accordance, or that

$$\frac{v^3}{3}+\text{const.}=v+\frac{x^3+y^3}{3},$$

or that

$$x^3+y^3=v^3-3\,v+\text{const.}$$

This may be shown by multiplying together the equations

$$x^2+y^2=v^2-2,$$
$$x+y=v,$$

which gives

$$x^3+y^3+x\,y\,(x+y)=v^3-2\,v,$$

and since

$$x\,y=1,\text{ and } x+y=v$$
$$x^3+y^3=v^3-3\,v$$

in accordance with the formula.

I will now apply the method to another example, which conducts to a new and interesting property of the cubic parabola. Since $\varphi\frac{1+x^3}{x}$ is a symmetrical of the equation $x^2-v\,x+1=0$, (as has been already remarked) we have, as a particular instance of this,

$$\sqrt{\frac{1+x^3}{x}}=\text{a symmetrical}=\surd v.$$

Multiply this equation by $\mathrm{S}\,dx=dv$,

$$\therefore\sqrt{\frac{1+x^3}{x}}\,\mathrm{S}\,dx=\sqrt{v}\,.\,dv,$$

or

$$\mathrm{S}\sqrt{\frac{1+x^3}{x}}\,.\,dx=\sqrt{v}\,.\,dv.$$

The sign S being thus transposed because $\sqrt{\frac{1+x^2}{x}}$ is a symmetrical.

The integral of this last equation is

$$S\int\sqrt{\frac{1+x^2}{x}}.dx=\frac{2}{3}v^{\frac{3}{2}}+C,$$

which means that

$$[1.]\int\sqrt{\frac{1+x^2}{x}}.dx+\int\sqrt{\frac{1+y^2}{y}}.dy=\frac{2}{3}(x+y)^{\frac{3}{2}}+C,$$

provided that $x\,y=1$. For since x, y, are roots of $x^2-v\,x+1=0$, $x+y=v$, a variable quantity, and $x\,y=1$ is the only condition which the variables must satisfy. Now assume $x=u^2$, $y=t^2$, and the relation between the new variables will be $u^2\,t^2=1$, or $u\,t=1$; and equation (1.) becomes, when divided by 2,

$$\frac{1}{3}(u^2+t^2)^{\frac{3}{2}}+C=\int\sqrt{1+u^4}.du+\int\sqrt{1+t^4}.dt,$$

whence the following theorem.

If u, t, two ordinates of the cubic parabola, are reciprocals, so that $u\,t=1$, then the sum of the two corresponding arcs of the curve* $=\frac{1}{3}(u^2+t^2)^{\frac{3}{2}}+$ const.

The reader may wish to see this result also verified by direct integration. Since then $u\,t=1$, let us write $\frac{1}{u}$ instead of t in the equation

$$\frac{1}{3}(u^2+t^2)^{\frac{3}{2}}+C=\int\sqrt{1+u^4}.du+\int\sqrt{1+t^4}.dt,$$

and it becomes

$$\frac{1}{3}\left(u^2+\frac{1}{u^2}\right)^{\frac{3}{2}}+C=\int\sqrt{1+u^4}.du-\int\sqrt{1+u^4}.\frac{du}{u^4},$$

or

$$\frac{1}{3}\frac{(1+u^4)^{\frac{3}{2}}}{u^3}+C=\int\sqrt{1+u^4}.du\left(1-\frac{1}{u^4}\right),$$

which ought to be identically true, whatever be the value of u. To see that it is so in reality, we have only to differentiate the first part of the equation, and we find its differential to be

$$2\sqrt{1+u^4}.du-\frac{(1+u^4)^{\frac{1}{2}}}{u^4}.du$$

$$=\sqrt{1+u^4}.du\left(2-\frac{1+u^4}{u^5}\right)=\sqrt{1+u^4}\,du\left(1-\frac{1}{u^4}\right),$$

which is the differential of the second part of the equation.

Let us now show the application of the method to formulæ containing cubic radicals.

* The equation to the cubic parabola, whose coordinates are u, u', being $3\,u'=u^3$, it follows that the arc $=\int du\sqrt{1+u^4}$.

Resuming the former equations

$$x^2 - v\,x + 1 = 0, \quad v = \frac{1+x^3}{x},$$

we have

$$\sqrt[3]{v} = \sqrt[3]{\frac{1+x^3}{x}}.$$

Multiply this equation by $\mathrm{S}\,dx\,(=dv)$

$$\therefore \sqrt[3]{v}\,.\,dv = \sqrt[3]{\frac{1+x^3}{x}}\,.\,\mathrm{S}\,dx.$$

But we may introduce $\sqrt[3]{\frac{1+x^3}{x}}$ (since it is a symmetrical) within the sign S,

$$\therefore \sqrt[3]{v}\,.\,dv = \mathrm{S}\sqrt[3]{\frac{1+x^3}{x}}\,.\,dx,$$

and integrating

$$\frac{3}{4}v^{\frac{4}{3}} + \text{const.} = \mathrm{S}\int\sqrt[3]{\frac{1+x^3}{x}}\,.\,dx.$$

It is plain that the sum of two integrals of the form $\int\sqrt[n]{\frac{1+x^3}{x}}\,.\,dx$ may be found by a similar process, provided always that $x\,y = 1$.

Resuming the last example we have

$$\sqrt[3]{v} = \sqrt[3]{\frac{1+x^3}{x}}.$$

If we multiply this equation by $\mathrm{S}\frac{dx}{x}$ instead of $\mathrm{S}\,dx$, we have

$$\sqrt[3]{v}\,.\,\mathrm{S}\frac{dx}{x} = \sqrt[3]{\frac{1+x^3}{x}}\,\mathrm{S}\frac{dx}{x} = \mathrm{S}\sqrt[3]{\frac{1+x^3}{x}}\,.\,\frac{dx}{x} = \mathrm{S}\sqrt[3]{\frac{1+x^3}{x^4}}\,.\,dx.$$

But $\mathrm{S}\frac{dx}{x} = 0$ in this example *,

$$\therefore 0 = \mathrm{S}\sqrt[3]{\frac{1+x^3}{x^4}}\,.\,dx$$

$\therefore$ integrating we find the sum of two integrals of the form $\int\sqrt[3]{\frac{1+x^3}{x^4}}\,.\,dx$ is a constant, if $x\,y = 1$.

Since nothing tends more to elucidate a subject than a frequent recurrence to first principles, I will remark that this result also follows at once from the supposition $x\,y = 1$. For if we write $\frac{1}{x}$ for y,

$$\int\sqrt[3]{\frac{1+x^3}{x^4}}\,.\,dx + \int\sqrt[3]{\frac{1+y^3}{y^4}}\,.\,dy$$

becomes

$$\int\sqrt[3]{\frac{1+x^3}{x^4}}\,.\,dx - \int\sqrt[3]{x^4+x^2}\,.\,\frac{dx}{x^3}$$

$$= \int\sqrt[3]{\frac{1+x^3}{x^4}}\,.\,dx - \int\sqrt[3]{\frac{1+x^3}{x^4}}\,.\,dx = \int 0 = \text{const.}$$

* See page 200; or page 208, note.

We are now in possession of principles which enable us to attack the general problem, "To find an algebraic relation between n variables $x, y, z \ldots\ldots$, such that

$$\int \varphi(X)\, dx + \int \varphi(Y)\, dy + \int \varphi(Z)\, dz + \&c. = \text{const.},$$

X being a polynomial of n dimensions with constant coefficients of the form

$$x^n - a x^{n-1} + b x^{n-2} + \&c. + h x + k,$$

and containing at least two distinct powers of x; and φ being any function whatever of the said polynomial.

It does not appear that any mathematician has hitherto proposed this problem. The principles of our method lead to the following solution:

Let X, or $x^n - a x^{n-1} + \ldots. + h x + k = v$, v being a variable quantity susceptible of any value *.

$$\therefore x^n - a x^{n-1} + \ldots\ldots + h x + (k - v) = 0.$$

This equation has only one variable coefficient, viz. $(k - v)$. Therefore the values of its n roots depend upon v, so far, at least, that when v changes its value, each root (generally speaking) undergoes a corresponding change. Also the sum of the roots $x + y + z + \ldots. = a$ is constant.

$$\therefore dx + dy + dz + \ldots\ldots = 0,$$

or

$$S\, dx = 0.$$

Since $v = X$, $\varphi v = \varphi X$. Multiply this equation by $S\, dx = 0$,

$$\therefore \varphi v\, S\, dx = \varphi X \,.\, S\, dx$$

$$\therefore 0 = \varphi X \,.\, S\, dx = S\, \varphi X \,.\, dx$$

(because φX is a symmetrical of this equation)

$$\therefore \text{const.} = S \int \varphi X \,.\, dx,$$

which therefore is the required solution of the problem.

Example.—Let $\int \sqrt[3]{x^3 + x + 1} \,.\, dx$ be the proposed integral. Assume $x^3 + x + 1 = v$,

$$\therefore x^3 + x + (1 - v) = 0.$$

* To suppose $X = v$ is the same as to suppose X to be a symmetrical of the equation between the variables (as recommended at pages 198, 200). Whence also ϕX is a symmetrical of the same equation. The symbol v retains the same meaning as before, viz. that of a quantity independent of x, or which continues to have the same value when x is permuted for any other root of the equation. I shall give it this meaning throughout the present memoir.

The solution given in the text may be expressed in other words, by saying that any two of the variables, as for instance x and y, are mutually connected by the equation

$$x^n - a x^{n-1} + \ldots\ldots + h x = y^n - a y^{n-1} + \ldots\ldots + h y,$$

whence of course it follows that X, or $x^n - a x^{n-1} + \ldots\ldots + h x + k$, does not change its value when x is permuted for y, and therefore it may properly be denoted by v, according to the acceptation which we have hitherto given to that letter.

Attribute to v any numerical value, and let the three roots of the equation then be m, m', m''. And when v has some other value, let the roots be n, n', n''. So that while v has changed progressively from one value to the other, the root m has progressively changed its value to n, the root m' to n', and the root m'' to n''.

These things being thus understood, the meaning of the theorem is, that the value of the integral $\int d x \sqrt[3]{x^3 + x + 1}$ taken between the limits $x = m$, $x = n$,

$+$ its value between the limits m', n',
$+$ its value between the limits m'', n'',
$=$ a constant.

If the question be viewed geometrically, since the roots of an equation are the intersections of a curve with its axis, a progressive change in the value of $(k-v)$, the absolute term, is equivalent to a displacement of the axis parallel to itself, in consequence of which all the intersections change their places simultaneously.

In the case of two variables, we have simply

$$\mathrm{X} = x^2 - a x + b = v,$$

or

$$x^2 - a x + (b - v) = 0.$$

And if x, y, are the roots of this equation, the theorem becomes

$$\int \varphi \mathrm{X} \,.\, d x + \int \varphi \mathrm{Y} \,.\, d y = \text{const.},$$

φ being any function.

Now in this particular case the theorem admits of a very simple demonstration. For since $x + y = a$, $y = a - x$; and substituting this value in $\mathrm{Y} = y^2 - a y + b$, it becomes $(a - x)^2 - a (a - x) + b = x^2 - a x + b$: also $d y$ becomes $- d x$.

$\therefore \varphi (y^2 - a y + b) d y$ becomes $- \varphi (x^2 - a x + b) d x$.

Therefore

$$\varphi (x^2 - a x + b) d x + \varphi (y^2 - a y + b) d y = 0,$$

or

$$\varphi \mathrm{X} \,.\, d x + \varphi \mathrm{Y} \,.\, d y = 0$$

$$\therefore \int \varphi \mathrm{X} \,.\, d x + \int \varphi \mathrm{Y} \,.\, d y = \text{const.},$$

which was to be demonstrated.

Let $\mathrm{X} = x^n - a x^{n-1} + \ldots$. as before, it may be shown upon the same principles that $\mathrm{S} \int \varphi \mathrm{X} \,.\, x^m d x = \text{const.}$, provided $\mathrm{S}\, x^m d x = 0$, or $\mathrm{S}\, x^{m+1}$ is constant, that is to say, does not contain v; which depends on the relative values of m and n. Also we may obtain in a similar manner the solution of the following problem, viz.

$$\mathrm{S} \int \varphi \left(\frac{\mathrm{X}}{\mathrm{X}'} \right) d x = \text{const.},$$

where X is a polynomial of n dimensions, and X′ another polynomial of not more than $n - 2$ dimensions. For, putting

$$\frac{X}{X'} = v,$$

x, y, &c. &c. are roots of

$$X - vX' = 0,$$

which is of the form

$$x^n - a x^{n-1} + (b - v) x^{n-2} + \&c. = 0,$$

where

$$Sx = a = \text{const.},$$

and therefore

$$S\,dx = 0 \;\therefore S\varphi\left(\frac{X}{X'}\right) dx = \varphi\left(\frac{X}{X'}\right) S\,dx = 0,$$

$$\therefore S\int\varphi\left(\frac{X}{X'}\right) dx = \text{const.}$$

I will now add several examples, and I request the reader's attention to the *directness* with which their solutions are obtained by means of the foregoing principles. In the present paper I have avoided the use of transformations, except that of $x = u^n$, because they are unnecessary to the success of the method, and that I am here considering general principles rather than individual results.

§ 6. *Examples.*

Ex. 1. Let the proposed integral be

$$\int\frac{dx}{\sqrt{1-x^3}}.$$

This is Mr. LUBBOCK's first example in his paper on ABEL's theorem in the Philosophical Magazine*. The result which he finds is equivalent to this, that if x and y satisfy the equation

$$xy - (x + y) = 2,$$

then

$$\int\frac{dx}{\sqrt{1-x^3}} + \int\frac{dy}{\sqrt{1-y^3}} = \text{const.}$$

For the sake of comparison I will take this as the first example of my method, and supposing its solution to be unknown, proceed to investigate it as follows:

$\int\frac{dx}{\sqrt{1-x^3}}$ may be put under the form

$$\int\frac{dx}{(1-x)\sqrt{\frac{1+x+x^2}{1-x}}}.$$

Put

$$\frac{1+x+x^2}{1-x} = v$$

$$\therefore x^2 + (1+v)x + (1-v) = 0,$$

x and y must be the roots of this equation†,

* Vol. vi. p. 118.

† See the note in page 196.

$$\therefore x+y=-(1+v) \quad xy=1-v$$
$$\therefore xy-(x+y)=2,$$

which is the equation of condition found by Mr. LUBBOCK.

Again, the sum of the integrals

$$=\int\frac{dx}{1-x}\sqrt{\frac{1}{v}}+\int\frac{dy}{1-y}\sqrt{\frac{1}{v}}$$
$$=\int\sqrt{\frac{1}{v}}\left(\frac{dx}{1-x}+\frac{dy}{1-y}\right)=\int 0=\text{const.}$$

because, since

$$xy-(x+y)=2$$
$$(1-x)(1-y)=1+xy-(x+y)=3$$
$$\therefore \log(1-x)+\log(1-y)=\log 3$$
$$\therefore \frac{dx}{1-x}+\frac{dy}{1-y}=0$$

$\therefore$ sum of the integrals = const. *Q. E. D.*

Ex. 2. To find the sum of *three* integrals of the same form.

$\int\frac{dx}{\sqrt{1-x^3}}$ may be written $\int\frac{dx}{x}\sqrt{\frac{x^3}{1-x^3}}$. Put $\frac{x^3}{1-x^3}=\frac{1}{v}$,

$$\therefore x^3+vx^2- \quad =0.$$

The three variables x, y, z, must be roots of this equation,

$$\therefore xyz=1, \text{ and } xy+xz+yz=0.$$

Here we have

$$\sqrt{\frac{x^2}{1-x^3}}=\sqrt{\frac{1}{v}}.$$

Multiply this by $S\frac{dx}{x}=0$*,

$$\therefore\sqrt{\frac{x^2}{1-x^3}}S\frac{dx}{x}=0.$$

But

$$\sqrt{\frac{x^3}{1-x^3}}S\frac{dx}{x}=S\sqrt{\frac{x^3}{1-x^3}}\cdot\frac{dx}{x}=S\frac{dx}{\sqrt{1-x^3}}$$
$$\therefore S\frac{dx}{\sqrt{1-x^3}}=0 \quad \therefore S\int\frac{dx}{\sqrt{1-x^3}}=\text{const.}$$

* In any equation whose last term is constant, $S\frac{dx}{x}=0$. For

$$S\frac{dx}{x}=\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{z}+\&c.$$
$$=d\log x+d\log y+\&c.$$
$$=d.\log(xyz\ldots.)$$
$$=d.\text{const.}=0.$$

$\therefore$ the sum of the three integrals is constant, provided that $xy + xz + yz = 0$, and that $xyz = 1$. It will be observed that the solution is simpler in the case of three integrals than of two.

Ex. 3. Supposing the relation between x, y, z the same as in the last example, to find the sum of three integrals of the form $\int \frac{x\,dx}{\sqrt{1-x^3}}$. Since

$$\sqrt{\frac{1}{v}} = \frac{x}{\sqrt{1-x^3}},$$

or in other words, since $\frac{x}{\sqrt{1-x^3}}$ is a symmetrical of the equation

$$x^3 + vx^2 - 1 = 0,$$

if it be multiplied by $\mathrm{S}\,dx$ the result will be

$$\mathrm{S}\,\frac{x\,dx}{\sqrt{1-x^3}}.$$

Also

$$\mathrm{S}\,x = -v \quad \therefore \mathrm{S}\,dx = -dv$$

$$\therefore \mathrm{S}\,\frac{x\,dx}{\sqrt{1-x^3}} = \sqrt{\frac{1}{v}}\,\mathrm{S}\,dx = \frac{-dv}{\sqrt{v}}.$$

Therefore

$$\mathrm{S}\int \frac{x\,dx}{\sqrt{1-x^3}} = \text{const.} - 2\sqrt{v}.$$

Ex. 4. The same suppositions continuing, required the sum of three integrals of the form $\int \frac{x^2\,dx}{\sqrt{1-x^3}}$. As before,

$$\frac{x}{\sqrt{1-x^3}} = \sqrt{\frac{1}{v}}.$$

Multiply by $\mathrm{S}\,x\,dx$,

$$\therefore \mathrm{S}\,\frac{x^2\,dx}{\sqrt{1-x^3}} = \sqrt{\frac{1}{v}}\,\mathrm{S}\,x\,dx = dv\sqrt{v},$$

(because the equation $x^3 + vx^2 - 1 = 0$ gives $\mathrm{S}\,x^2 = v^2 \;\therefore \mathrm{S}\,x\,dx = v\,dv$)

$$\therefore \mathrm{S}\int \frac{x^2\,dx}{\sqrt{1-x^3}} = \frac{2}{3}v^{\frac{3}{2}} + \text{const.}$$

But since $\int \frac{x^2\,dx}{\sqrt{1-x^3}}$ is a form which is readily integrable *per se*, it will naturally be asked whether the result of direct integration is the same as that given by our formulæ. This example therefore affords a convenient opportunity of showing the close accordance between this branch of the integral calculus and the theory of algebraic equations.

By direct integration,

$$\mathrm{S}\int \frac{x^2\,dx}{\sqrt{1-x^3}} = -\frac{2}{3}\left(\sqrt{1-x^3} + \sqrt{1-y^3} + \sqrt{1-z^3}\right) + \text{const.}$$

And by our method,

$$S\int \frac{x^2\,dx}{\sqrt{1-x^3}} = \frac{2}{3}v^{\frac{1}{2}} + \text{const.}$$

$\therefore$ it remains to verify that

$$\sqrt{1-x^3}+\sqrt{1-y^3}+\sqrt{1-z^3} = -v^{\frac{1}{2}}.$$

In order to demonstrate this, let us resume the original equation $x^3+vx^2-1=0$, which gives $vx^2 = 1-x^3$,

$$\therefore \sqrt{v}\,.\,x = \sqrt{1-x^3},$$

and similarly,

$$\sqrt{v}\,.\,y = \sqrt{1-y^3}$$

$$\sqrt{v}\,.\,z = \sqrt{1-z^3}$$

$$\therefore \sqrt{1-x^3}+\sqrt{1-y^3}+\sqrt{1-z^3} = \sqrt{v}\,(x+y+z)$$

$$= \sqrt{v}\,(-v) = -v^{\frac{1}{2}}. \quad \textit{Q.E.D.}$$

Ex. 5. $\int \frac{dx}{\sqrt{(x^3+x^2)}+\sqrt[3]{x^3+x^2}}$.

This is a function of the binomial x^3+x^2, which being put $= v$, we have

$$x^3 + x^2 - v = 0.$$

The three roots of this equation are the variables that answer the problem.

Putting $\sqrt{v}+\sqrt[3]{v} = \varphi v$, the sum of the three integrals becomes

$$\int \frac{dx}{\varphi v}+\int \frac{dy}{\varphi v}+\int \frac{dz}{\varphi v} = \int \frac{1}{\varphi v}\,S\,dx;$$

but $S\,dx = 0$, because $x+y+z = -1$,

$$\therefore \text{ sum of integrals} = \int 0 = \text{const.}$$

Ex. 6. $\int dx\,\sqrt{1+x}$.

Here we cannot suppose $\sqrt{1+x^n} = v$ a symmetrical quantity, because that would amount to the supposition $\sqrt{1+x^n} = \sqrt{1+y^n} = \sqrt{1+z^n} =$ &c., which implies that $x = y = z =$ &c., whereas we suppose the roots to be in general all different from one another. This is the reason why it was remarked before, that it was requisite the polynomial should contain at least two distinct powers of x. When that is not the case, a second power of x must be introduced. There are several ways of doing this; the simplest is the following:

Put $\int dx\sqrt{1+x^n}$ under the form $\int x\,dx\sqrt{\frac{1+x^n}{x^2}}$. Assume $\frac{1+x^n}{x^2} = v$,

$$\therefore x^n - vx^2 + 1 = 0;$$

an equation of n dimensions, of which v is the only variable coefficient. The n roots of this equation answer the problem.

The sum of n integrals becomes

$$\int x\,dx\,\surd v + \int y\,dy\,\surd v + \&c. = \int \surd v\,.\,\mathrm{S}x\,dx.$$

If n is a number greater than 4, $\mathrm{S}x^2 = 0$:

$$\therefore \mathrm{S}x\,dx = 0 \quad \therefore \text{ sum of } n \text{ integrals} = \text{const.}$$

If $n = 4$, $\mathrm{S}x^2 = 2v$, $\therefore \mathrm{S}x\,dx = dv$,

$$\therefore \text{ sum of four integrals} = \int \surd v\,.\,dv = \tfrac{2}{3}v^{\frac{3}{2}} + \mathrm{C}.$$

If $n = 3$, $\mathrm{S}x^2 = v^2$, $\therefore \mathrm{S}x\,dx = v\,dv$,

$$\therefore \text{ sum of three integrals} = \int v^{\frac{1}{2}}\,dv = \tfrac{2}{5}v^{\frac{5}{2}} + \mathrm{C}.$$

But when n is greater than 4, the equation has impossible roots, therefore the solution is imaginary. Although, as LEGENDRE has demonstrated*, these imaginary cases do not cease to have a real analytical meaning; the sum of two imaginary integrals forming a real integral in a manner analogous to that in which two imaginary roots of an equation form a real sum and product.

But we may avoid these imaginary solutions by putting $\int dx\sqrt{1+x^n}$ in the form

$$\int dx\,(a + bx + cx^2 \ldots\ldots)\sqrt{\frac{1+x^n}{(a+bx+cx^2\ldots\ldots)^2}}.$$

Assuming, then,

$$\frac{1+x^n}{(a+bx+cx^2\ldots\ldots)^2} = v,$$

we may attribute to the polynomial any number of terms suitable to the exponent n†, and then it is in most cases possible to find such numerical values for the constant coefficients a, b, c, &c., that the resulting equation shall have all its roots real.

Each integral, then, has the form

$$\int dx\,(a + bx + cx^2 \ldots.)\sqrt{v},$$

and the sum of all

$$= \int \surd v\,\mathrm{S}\,dx\,(a + bx + cx^2 \ldots.),$$

where $\mathrm{S}\,dx\,(a + bx + cx^2 \ldots.)$ = the aggregate of the partial sums

$$a\,\mathrm{S}\,dx + b\,\mathrm{S}x\,dx + c\,\mathrm{S}x^2\,dx + \ldots.,$$

which is the differential of

$$a\,\mathrm{S}x + \frac{b}{2}\,\mathrm{S}x^2 + \frac{c}{3}\,\mathrm{S}x^3 + \ldots.,$$

and may therefore be expressed in terms of v, since the quantities $\mathrm{S}x$, $\mathrm{S}x^2$, $\mathrm{S}x^3$, &c. are readily found in terms of v by the usual doctrine of algebraic equations.

* Fonctions Elliptiques, vol. iii. p. 326.

† In general the number of its terms may be $\frac{n}{2}$ or $\frac{n+1}{2}$.

Ex. 7. $\int dx \sqrt[3]{1+x^n}$.

This may be put in the form

$$\int dx\,(a+bx\ldots\ldots)\sqrt[3]{\frac{1+x^n}{(a+bx+\ldots.)^3}};$$

and putting

$$\frac{1+x^n}{(a+bx+\ldots.)^3}=v,$$

the reasoning is the same nearly as in the preceding case. The same principles are applicable to the more general integral $\int dx \sqrt[m]{1+x^n}$, m being a whole number.

These solutions give the algebraic sum of n integrals of the proposed form. But this number n may be reduced by various methods to a lower number, which is the minimum that the problem admits of: ex. gr. the lowest number of integrals of the form $\int dx\ \sqrt{1+x^4}$ which have an algebraic sum is *two*; of the form $\int dx\ \sqrt{1+x^5}$ is *three*; of the form $\int dx \sqrt{1+x^{10}}$ is likewise *three*, &c. &c., which subject I shall treat of in a subsequent section.

Ex. 8. $\int \frac{dx}{\sqrt[3]{x^3-1}}$.

First solution. Put $x^3=t$, and the integral becomes

$$\frac{\frac{1}{3}t^{-\frac{2}{3}}dt}{\sqrt[3]{t-1}}=\frac{1}{3}\cdot\frac{dt}{\sqrt[3]{t^3-t^2}}.$$

Put $t^3-t^2=v$, or $t^3-t^2-v=0$. The three roots of this equation answer the problem.

$$\therefore \text{ the sum of three integrals } =\int\frac{\frac{1}{3}\mathrm{S}\,dt}{\sqrt[3]{v}}=\int 0=\text{const.}$$

(because $\mathrm{S}\,t=1$, being the coefficient of the second term of the equation $t^3-t^2-v=0$ taken negatively, whence $\mathrm{S}\,dt=0$).

Second solution. Put $x^3=t^2$, and the integral becomes

$$\frac{\frac{2}{3}t^{-\frac{1}{2}}dt}{\sqrt[3]{t^2-1}}=\frac{2}{3}\frac{dt}{\sqrt[3]{t^3-t}}.$$

Put $t^3-t=v$,

$$\therefore t^3-t-v=0,$$

and the roots of this equation answer the problem.

$$\therefore \text{ the sum of three integrals } =\int\frac{2}{3}\frac{\mathrm{S}\,dt}{\sqrt[3]{v}}=\int 0=\text{const.}$$

(because $\mathrm{S}\,t=0$ in the equation $t^3-t-v=0$ $\therefore \mathrm{S}\,dt=0$), and the sum of the integrals therefore reduces itself in this case also to a constant.

It will probably be satisfactory to the reader to see some one of these results verified by arithmetical computation. Let us therefore select this last example for that purpose.

§ 7. *Example of an arithmetical calculation of the sum of three Integrals.*

The preceding analysis shows that the sum of the three integrals

$$\int \frac{dx}{\sqrt[3]{x^3-1}} + \int \frac{dy}{\sqrt[3]{y^3-1}} + \int \frac{dz}{\sqrt[3]{z^3-1}} = \text{const.}$$

if $x^{\frac{1}{3}}, y^{\frac{1}{3}}, z^{\frac{1}{3}}$ are roots of the equation

$$t^3 - t - v = 0.$$

But the form of this equation shows that the sum of its roots $= 0$, the sum of the products of every two roots $= -1$, while the product of all the roots is a variable quantity $= v$; $\therefore$ the quantities x, y, z must satisfy the two following equations,

$$x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}} = 0$$

$$(xy)^{\frac{1}{3}} + (xz)^{\frac{1}{3}} + (yz)^{\frac{1}{3}} = -1.$$

And if they do so, we shall have

$$\int_x + \int_y + \int_z = \text{const.},$$

denoting by $\int_x$ the integral $\int \frac{dx}{\sqrt[3]{x^3-1}}$.

But in order to eliminate the constant, we may take three other variables x', y', z', satisfying the same two equations of condition, and thence deduce

$$\int_{x'} + \int_{y'} + \int_{z'} = \text{const.}$$

Whence by subtraction we eliminate the constant

$$\left(\int_x - \int_{x'}\right) + \left(\int_y - \int_{y'}\right) + \left(\int_z - \int_{z'}\right) = 0. \quad \ldots\ldots [1.]$$

Now by the usual methods we find that the equations of condition are satisfied by the values

$$x = \cdot 352342$$
$$y = \cdot 917532$$
$$z = 1\cdot 057860,$$

and also by the values

$$x' = \cdot 392456$$
$$y' = \cdot 900227$$
$$z' = 1\cdot 065602\text{*}.$$

---

* These values give

$x^{\frac{1}{3}} =$	$-0\cdot 209149$	$x'^{\frac{1}{3}} =$	$-0\cdot 245862$
$y^{\frac{1}{3}} =$	$-0\cdot 878885$	$y'^{\frac{1}{3}} =$	$-0\cdot 854138$
$z^{\frac{1}{3}} =$	$1\cdot 088034$	$z'^{\frac{1}{3}} =$	$1\cdot 100000$
Sum $=$	0	Sum $=$	0

This verifies the first equation of condition. The squares of these quantities are

It remains therefore to try by actual calculation whether these values satisfy the equation [1.].

$$\int_x = \int \frac{dx}{\sqrt[3]{x^3-1}} = x + \frac{1}{3}\cdot\frac{x^4}{4} + \frac{1.4}{3.6}\cdot\frac{x^7}{7} + \frac{1.4.7}{3.6.9}\cdot\frac{x^{10}}{10} + \&c.$$

$$\int_{x'} = \int \frac{dx'}{\sqrt[3]{x'^3-1}} = x' + \frac{1}{3}\cdot\frac{x'^4}{4} + \frac{1.4}{3.6}\cdot\frac{x'^7}{7} + \frac{1.4.7}{3.6.9}\cdot\frac{x'^{10}}{10} + \&c.$$

$\therefore$ putting $x' - x = \Delta x$, $x'^4 - x^4 = \Delta(x^4)$ &c.

$$\int_{x'} - \int_x = \Delta x + \frac{1}{3}\frac{\Delta(x^4)}{4} + \frac{1.4}{3.6}\cdot\frac{\Delta(x^7)}{7} + \frac{1.4.7}{3.6.9}\cdot\frac{\Delta(x^{10})}{10} + \&c.$$

and since $\Delta x = \cdot 04011$ is a small quantity, we readily find the sum of the series $= \cdot 04083$. Treating the other variables in the same manner, the result obtained is

$$X = \int_{x'} - \int_x = \cdot 040834$$

$$Y = \int_{y'} - \int_y = \cdot 027526$$

$$Z = \int_{z'} - \int_z = \cdot 013315.$$

With regard to the signs, it appears that the integral X has a sign opposed to that of the other two. We find therefore finally,

$$Y + Z = \cdot 040841$$
$$X = \cdot 040834$$
$$\therefore Y + Z - X = \cdot 000007.$$

On the other hand the formula gives $Y + Z - X = 0$, rigorously. Therefore the computation is only in error in the sixth place of decimals, which in consequence of the prolixity of these calculations may be considered to be a sufficient trial of its accuracy.

$x^3 = \cdot 043743$	$x'^3 = \cdot 060448$
$y^3 = \cdot 772439$	$y'^3 = \cdot 729552$
$z^3 = 1\cdot 183818$	$z'^3 = 1\cdot 210000$
Sum $= 2$	Sum $= 2$

Squaring the equation $x^{\frac{3}{7}} + y^{\frac{3}{7}} + z^{\frac{3}{7}} = 0$, we have

$$(x^3 + y^3 + z^3) + 2(\overline{xy}^{\frac{3}{7}} + \overline{xz}^{\frac{3}{7}} + \overline{yz}^{\frac{3}{7}} = 0,$$

and substituting the value just found of $x^3 + y^3 + z^3 = 2$, we have

$$\overline{xy}^{\frac{3}{7}} + \overline{xz}^{\frac{3}{7}} + \overline{yz}^{\frac{3}{7}} = -1,$$

which verifies the second equation of condition.

Note.—The integrals comprised in the formula $\int \frac{P\,dx}{\sqrt{R}}$ have been called ultra-elliptic by LEGENDRE. I think I have sufficiently shown that no line of distinction can be drawn between them and integrals in general; all of which, that are functions of a given polynomial, possess the property which was supposed to characterize the ultra-elliptic class.

METEOROLOGICAL JOURNAL,

KEPT BY THE ASSISTANT SECRETARY,

AT THE APARTMENTS OF THE

ROYAL SOCIETY,

BY ORDER OF

THE PRESIDENT AND COUNCIL.

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge.....=83 feet 2½ in.

_____ above the mean level of the Sea (presumed about) ....=95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House.....=79 feet.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR JULY AND AUGUST, 1835.

1835.		9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer.				Rain in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
							Fahrenheit.		Self-registering.				
		Barom.	Attach. Therm.	Barom.	Attach. Therm.		9 A.M.	3 P.M.	Lowest.	Highest.			
JULY	W 1	30.119	66.9	30.089	66.5	52	67.0	68.8	50.6	72.5		E	Fine and cloudless—light brisk wind.
	T 2	30.069	69.6	30.071	67.9	55	69.6	75.4	54.4	75.3		SE	A.M. Cloudy—light wind. P.M. Fine—light clouds. Evening, Cloudy.
	F 3	30.103	71.3	30.095	70.7	57	69.6	74.7	58.6	76.2		SW	Fine—nearly cloudless.
	S 4	30.115	67.0	30.059	70.7	55	65.4	73.4	57.5	74.8		S	A.M. Fine and cloudless—very light haze. P.M. Fine—light clouds. Evening, Cloudy.
	⊙ 5	29.849	66.0	29.848	70.4	57	60.9	67.3	58.6	70.6		SW	Overcast—very light rain. Evening, Cloudy.
	M 6	30.053	70.0	30.107	70.7	55	65.4	71.7	55.7	73.3	.105	WSW	Fine—light clouds. Evening, Fine and clear. Rain during the night.
	T 7	30.198	73.8	30.128	71.3	57	65.9	70.6	53.2	71.7		SSW	A.M. Fine—light clouds. P.M. Cloudy.
	W 8	30.004	69.5	30.030	70.4	54	66.3	70.4	55.6	70.7	.077	WSW	Cloudy—light wind. Evening, Fine and clear.
	T 9	30.010	65.9	29.924	68.8	53	63.5	66.3	55.0	70.8		S	Lightly overcast—light brisk wind.
	○ F 10	29.843	72.0	29.901	70.1	56	64.6	68.4	55.0	69.7		WSW	Fine—light clouds and wind.
	S 11	30.115	72.2	30.113	69.7	51	66.3	70.5	53.8	72.2		SW	Fine—light clouds and wind. Evening, Fine and clear.
	⊙ 12	30.055	74.5	29.950	70.7	51	66.7	70.8	53.3	71.6		SW	Fine—light clouds—light brisk wind. Evening, Overcast—brisk wind.
	M 13	29.895	74.0	29.889	71.0	57	65.4	65.6	58.3	70.7		SW	A.M. Fine—light clouds and wind. P.M. Lightly overcast—light rain. Evening, Cloudy.
	T 14	30.083	72.8	30.065	70.9	52	64.7	69.8	53.8	70.7	.047	SW	A.M. Lightly cloudy—light wind. P.M. Cloudy.
	W 15	29.950	70.6	29.883	71.7	56	67.8	72.6	54.8	73.6		S	Fine—light clouds and wind. Evening, Cloudy.
	T 16	29.962	73.7	29.962	72.0	58	67.9	74.8	60.8	75.7		SW	Fine—light clouds. Evening, Fine and clear.
	F 17	30.087	75.8	30.057	72.6	56	69.6	74.2	55.5	75.4		S	A.M. Fine—nearly cloudless. P.M. Fine and cloudless—high wind. Evening, Cloudy.
	S 18	30.025	73.6	29.968	73.8	60	69.8	77.7	58.4	78.5		S	A.M. Cloudy—light wind. P.M. Fine—light clouds.
	⊙ 19	30.186	75.7	30.186	73.6	58	67.3	75.2	56.7	75.7		SW	Fine—nearly cloudless.
	M 20	30.208	75.7	30.184	75.3	61	71.3	79.6	56.6	80.6		SW	Fine—light clouds and wind.
	T 21	30.237	76.6	30.212	75.5	65	75.7	78.0	66.9	76.2		E	A.M. Fine—light clouds and wind. P.M. Fine and cloudless—light brisk wind.
	W 22	30.275	75.9	30.245	74.7	60	69.4	73.4	59.9	73.8		W	Fine and cloudless—light brisk wind. Evening, Fine and clear.
	T 23	30.279	69.5	30.245	72.8	62	66.9	74.3	57.4	74.8		E	A.M. Fine—light clouds and wind. P.M. Fine and cloudless—light brisk wind.
	F 24	30.241	74.6	30.200	74.5	64	70.3	75.8	56.6	76.4		ESE	Fine and cloudless—light brisk wind.
	● S 25	30.212	76.3	30.184	74.9	62	70.8	74.7	58.6	75.2		E	Fine and cloudless—light brisk wind.
	⊙ 26	30.208	77.0	30.176	74.4	58	71.4	73.5	59.9	73.3		ESE	Fine and cloudless—light brisk wind.
	M 27	30.095	73.5	30.063	74.6	60	71.7	78.4	57.7	78.7		E	Fine—nearly cloudless—light wind. Evening, Fine and clear.
	T 28	30.097	76.7	30.073	77.2	61	75.0	83.0	63.2	84.2		SW	A.M. Fine and cloudless—light haze. P.M. Lightly cloudy—6 o'clock, P.M. heavy rain. Evening, Fine and clear.
	W 29	30.245	73.0	30.208	74.9	62	66.8	77.7	57.7	78.4	.277	E var.	Fine and cloudless—light haze and wind.
	T 30	30.212	76.7	30.132	75.9	56	72.3	80.9	61.8	81.3		SW	Fine and cloudless—light haze. Evening, Lightly cloudy.
	F 31	30.152	76.3	30.128	75.3	57	67.7	74.2	61.9	75.2		ENE	Fine—light clouds and wind. Evening, Fine and clear.
	MEANS..	30.103	72.8	30.077	72.4	57.4	68.2	73.6	57.3	74.8	Sum. .506	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.987 / 3 P.M. 29.962
AUGUST	S 1	30.162	75.2	30.083	74.9	57	67.9	75.6	57.8	75.8		SW	A.M. Fine and cloudless—light haze. P.M. Hazy. Evening, Cloudy.
	⊙ 2	30.010	72.6	29.972	75.0	62	67.2	70.8	60.9	73.6		E	A.M. Cloudy—light wind. P.M. Fine—light clouds and wind.
	M 3	30.055	69.0	30.045	72.3	56	64.8	68.4	57.9	70.6		E	A.M. Cloudy—light brisk wind. P.M. Fine—light clouds and wind.
	T 4	30.035	71.6	30.000	72.2	56	65.7	72.6	53.0	75.7		NE	Fine—nearly cloudless—light wind. Evening, Fine and clear.
	W 5	30.053	72.0	30.002	73.4	58	68.4	76.0	56.3	78.6		SW	A.M. Fine and cloudless—light haze. P.M. Lightly cloudy.
	T 6	30.049	72.4	30.055	74.7	63	69.6	74.5	64.0	75.7		SW	A.M. Cloudy—light wind. P.M. Fine—light clouds and wind.
	F 7	30.004	70.7	29.994	73.7	65	67.4	67.7	64.2	72.6		SSW	A.M. Lightly overcast—very light rain. P.M. Overcast—light continued rain. Evening, Fine and clear.
	○ S 8	30.281	70.5	30.280	71.0	52	61.2	70.2	51.9	70.2	.044	SW	Fine and cloudless—light wind.
	⊙ 9	30.380	70.7	30.338	70.5	56	62.7	73.3	52.0	73.6		W	Fine and cloudless.
	M 10	30.348	73.6	30.269	73.2	56	71.3	78.9	58.8	80.2		S	Fine and cloudless.
	T 11	30.128	73.7	30.002	76.3	62	75.3	82.7	63.0	84.4		ESE	Fine—light clouds—light brisk wind.
	W 12	29.954	74.6	29.958	76.6	63	69.8	78.4	61.2	79.7		SW	A.M. Cloudy. P.M. Fine—light clouds. Evening Cloudy.
	T 13	30.105	72.2	30.097	74.0	54	65.3	70.8	59.6	72.4		S	Cloudy—light brisk wind.
	F 14	30.241	70.0	30.216	72.6	54	63.9	69.7	55.4	70.6		N	Cloudy—light wind.
	S 15	30.198	70.0	30.156	72.4	57	63.6	74.5	57.4	75.5		E	A.M. Thick haze. P.M. Fine—light clouds. Evening, Cloudy.
	⊙ 16	30.188	71.9	30.174	73.6	60	68.3	76.4	62.5	76.2		SW	Lightly overcast. Evening, Cloudy.
	M 17	30.267	71.3	30.283	74.3	66	69.8	72.4	65.7	73.8		N	A.M. Overcast. P.M. Fine—light clouds. Evening, Cloudy.
	T 18	30.299	70.9	30.250	73.6	60	67.7	72.5	59.9	73.6		E	Fine—light clouds and wind. Evening, Fine and clear.
	W 19	30.233	70.7	30.150	73.7	64	67.8	76.3	59.2	76.6		E	Fine—nearly cloudless—light wind. Evening, Fine and clear.
	T 20	29.956	72.7	29.815	74.9	65	70.2	75.8	61.5	76.8		E	Fine and cloudless—light haze and wind. Evening, Fine & clear.
	F 21	29.659	73.2	29.566	76.0	63	71.0	79.3	61.4	80.6		SE	Fine—light clouds and wind. Evening, Overcast—very light rain.
	S 22	29.602	74.3	29.637	75.6	64	70.2	72.7	63.8	74.4		S	Fine—light clouds and wind.
	⊙ 23	29.771	74.3	29.780	74.4	59	67.4	72.7	58.2	73.2		SSW	A.M. Fine—light clouds and wind. P.M. Fine—light clouds and very light rain. Evening, Overcast.
	● M 24	29.594	68.9	29.562	73.2	62	65.3	70.4	61.4	71.6	.038	SE var.	A.M. Lightly overcast—light wind—rain early. P.M. Fine—light clouds and rain. Evening, Cloudy.
	T 25	29.600	68.7	29.602	71.6	61	63.2	68.8	58.0	71.2	.086	SW	A.M. Overcast—light wind. P.M. Lightly overcast—light shower. Evening, Overcast—light rain.
	W 26	29.679	64.8	29.695	68.0	55	57.3	63.3	55.8	64.4	.152	WSW	A.M. Overcast—very light rain & wind. P.M. Cldy.—light rain.
	T 27	29.754	63.3	29.812	68.0	55	57.2	65.3	53.0	68.0		W	A.M. Overcast. P.M. Fine—light clouds and wind. Evening, Fine and clear.
	F 28	30.010	65.9	30.027	68.9	60	64.8	68.2	56.8	69.4		N	A.M. Cloudy. P.M. Fine—light clds. & wind. Evening, Cloudy.
	S 29	30.069	65.0	30.033	68.3	56	62.7	68.2	53.4	70.6		E	Fine—light clouds and wind. Evening, Cloudy.
	⊙ 30	30.053	66.2	30.043	68.3	57	60.4	69.6	53.3	70.7		NE	A.M. Foggy. P.M. Fine—light clouds and wind. Evening, Cloudy.
	M 31	30.099	65.7	30.057	67.2	53	60.2	66.9	51.8	67.7		N	Fine and cloudless—light haze and wind. Evening, Cloudy.
	MEANS..	30.027	70.5	29.998	72.7	59.1	66.1	72.4	58.4	73.8	Sum. .320	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.916 / 3 P.M. 29.883

METEOROLOGICAL JOURNAL FOR SEPTEMBER AND OCTOBER, 1835.

1835.		9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit.		External Thermometer. Self-registering.		Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
		Barom.	Attach. Therm.	Barom.	Attach. Therm.		9 A.M.	3 P.M.	Lowest.	Highest.			
SEPTEMBER	T 1	30.160	65.6	30.174	68.0	57	62.8	67.8	53.4	70.2		NE	Fine—light clouds—haze and wind. Evening, Fine and clear.
	W 2	30.247	63.4	30.216	67.4	53	60.8	70.3	50.8	70.6		NE	A.M. Fine and cloudless—light brisk wind. P.M. Fine—light clouds and wind. Evening, Fine and clear.
	T 3	30.111	64.8	30.014	67.9	55	62.6	71.0	53.2	71.2		E	A.M. Fine—light clouds and wind. P.M. Fine—nearly cloudless. Evening, Overcast—rain and wind.
	F 4	29.837	67.0	29.832	71.5	61	68.4	75.2	59.6	76.3	.102	S	Overcast—light steady rain and wind.
	S 5	29.893	70.0	29.942	72.2	63	68.6	71.7	62.4	73.2	.133	SW	A.M. Fine—light clouds and wind. P.M. Cloudy—light wind.
	⊙ 6	30.148	68.7	30.103	72.2	60	63.9	71.6	58.4	73.6		ENE	A.M. Cloudy—light wind. P.M. Fine—light clouds and wind. Evening, Cloudy.
	○ M 7	30.087	67.4	30.027	70.7	58	63.8	69.7	60.3	71.3		NE	A.M. Overcast. P.M. Fine—light clouds and wind. Evening, Fine and clear.
	T 8	29.804	67.7	29.663	69.6	55	63.4	64.2	54.9	68.6		NW	A.M. Fine—light clouds—haze and wind. P.M. Overcast—light rain.
	W 9	29.714	64.6	29.696	65.8	48	56.7	60.7	50.0	63.4	.188	WSW	Fine and cloudless—light haze & wind. Evening, Fine & clear.
	T 10	29.438	62.2	29.348	66.0	57	60.8	63.6	50.2	65.7		S var.	A.M. Overcast—light rain and wind. P.M. Fine—light clouds —rain and wind. Evening, Fine and clear.
	F 11	29.461	60.4	29.570	62.2	50	54.7	57.2	51.2	59.0	.125	SW	A.M. Cloudy—light brisk wind. P.M. Overcast—rain and wind.
	S 12	29.336	59.6	29.307	62.7	54	55.3	63.4	49.9	63.6		SW var.	A.M. Overcast—heavy rain with high wind. P.M. Fine—light clouds and wind.
	⊙ 13	29.544	58.8	29.621	62.6	52	53.9	61.7	46.6	62.6	.230	SW	A.M. Cloudy—light wind. P.M. Fine—light clouds. Evening, Cloudy.
	M 14	29.861	59.6	29.885	64.0	52	57.2	66.3	48.7	67.6		SSW	A.M. Fine—light clds. & wind. P.M. Overcast—light brisk wind.
	T 15	29.899	63.9	29.847	65.6	58	64.2	63.7	57.0	65.7		S var.	Overcast—high wind. Evening, Overcast—heavy rain with high wind.
	W 16	29.814	61.2	29.774	65.2	53	53.8	62.7	50.3	63.6	.136	SW	A.M. Thick haze—light wind. P.M. Cloudy.
	T 17	29.752	61.6	29.746	63.8	54	56.7	61.4	50.5	62.6		S	A.M. Fine—lt. clds. P.M. Overct.—h. rain—lt. wind. Ev. Cldy.
	F 18	29.824	58.9	29.726	63.3	51	55.2	60.8	46.4	63.3	.130	S	A.M. Fine and cloudless—light haze. P.M. Overcast—light rain and wind. Evening, Heavy rain—high wind.
	S 19	29.554	62.5	29.629	65.4	58	61.7	65.4	54.3	67.2	.594	SW	A.M. Overcast—light brisk wind. P.M. Fine—light clouds and wind. Evening, Overcast—heavy rain.
	⊙ 20	29.635	62.4	29.675	65.4	57	60.2	65.2	58.3	65.6	.572	S	A.M. Overcast—lightly steady rain. P.M. Fine—light clouds—high wind. Evening, Fine and clear.
	M 21	29.897	62.2	29.816	62.6	56	59.5	56.3	54.2	64.6		S	Overcast—light rain and wind.
	● T 22	29.572	64.3	29.501	66.4	59	64.4	68.0	54.4	68.8	.340	E	A.M. Fine—nearly cloudless—light brisk wind. P.M. Cloudy —high wind. Evening, Overcast—rain and wind.
	W 23	29.667	64.8	29.891	67.2	58	62.0	65.3	56.9	66.7	.111	SW var.	Fine—light clouds & wind. Evening, Overcast—rain and wind.
	T 24	29.857	62.8	29.915	65.5	57	58.7	61.8	56.0	62.4	.250	SW	A.M. Cloudy—light steady rain and wind. P.M. Overcast.
	F 25	29.994	60.2	29.909	63.6	52	52.4	61.8	49.9	62.4		ENE	A.M. Thick haze—light wind. P.M. Fine—light clouds. Evening, Fine and clear.
	S 26	29.679	60.7	29.631	62.4	54	59.4	60.3	48.5	60.5		S var.	A.M. Lightly overcast—light wind. P.M. Overcast—light rain. Evening, Overcast—heavy rain.
	⊙ 27	29.594	58.7	29.530	61.6	52	52.8	59.4	49.2	59.5	.788	SW	Fine—light clouds and haze. Evening, Fine and clear.
	M 28	29.433	58.7	29.598	59.3	44	51.6	54.2	49.0	57.2	.055	SW	A.M. Cloudy—light haze and wind. P.M. Fine—light clouds and wind. Evening, Fine and clear.
	T 29	29.619	58.0	29.627	61.3	52	57.6	60.7	46.6	62.4		S	Cloudy—light wind.
	W 30	29.255	57.7	29.178	62.2	55	57.2	64.6	52.6	65.6		SE var.	A.M. Overcast—light rain and wind. P.M. Cloudy—light wind. Evening, Overcast—light rain.
	MEANS..	29.756	62.6	29.746	65.5	54.8	59.3	64.2	52.8	65.8	Sum. 3.754	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.....	9 A.M. 29.668 / 3 P.M. 29.650
OCTOBER	T 1	29.241	60.3	29.322	60.7	54	55.5	57.8	52.5	59.2	.044	SSE	A.M. Fine—light clouds and wind. P.M. Cloudy—light rain and wind. Evening, Overcast—heavy rain with high wind.
	F 2	29.208	60.4	29.235	62.2	55	58.6	59.5	54.0	62.6	.327	SE var.	Overcast—rain and wind. Evening, Fine and clear.
	S 3	29.362	55.5	29.311	58.8	49	51.3	55.7	48.0	57.6	.211	SSE	A.M. Cloudy—lt. fog & wind. P.M. Overcast—lt. rain & wind.
	⊙ 4	29.354	56.6	29.404	57.4	50	52.4	56.7	50.2	57.2	.208	WSW	A.M. Overcast—light wind. P.M. Cloudy—light wind. Evening, Fine and clear.
	M 5	29.677	56.3	29.740	58.4	50	50.4	58.2	45.3.	59.3		WSW	A.M. Fine—light haze and wind. P.M. Fine—nearly cloudless. Evening, Fine and clear.
	○ T 6	29.913	57.4	29.922	60.4	51	54.3	59.5	48.2	62.3		SW	A.M. Fine and cloudless—light haze and wind. P.M. Cloudy —light wind.
	W 7	30.033	57.3	30.053	59.2	50	51.3	57.4	48.7	57.7		N	A.M. Thick fog—light wind. P.M. Lightly cloudy.
	T 8	29.944	55.4	29.831	58.6	51	53.9	56.7	49.3	57.0		E	Fine—light clouds and wind. Evening, Cloudy.
	F 9	29.465	55.0	29.213	57.2	51	51.8	54.9	48.6	55.6	.061	SSW	A.M. Lightly overcast—light wind—rain early. P.M. Overcast—light rain & wind. Evening, continued rain & wind.
	S 10	29.388	54.7	29.053	56.3	48	48.7	51.6	46.9	52.8	.463	SSW	A.M. Cloudy—light wind. P.M. Fine—light clouds and wind.
	⊙ 11	29.344	52.7	29.452	53.0	41	45.7	50.6	40.0	50.3		S	Cloudy—light brisk wind. Evening, Fine and clear.
	M 12	29.734	48.0	29.740	50.2	42	48.7	51.3	48.4	54.7		SW	Cloudy—thick haze and wind. Evening, Overcast—light steady rain and wind.
	T 13	29.780	52.6	29.800	56.6	49	55.6	60.7	47.8	61.7	.166	SW	A.M. Overcast—light fog and wind. P.M. Cloudy—light wind. Evening, Overcast—very light rain and wind.
	W 14	30.081	56.7	30.144	57.4	52	53.6	57.3	53.7	56.8		ENE	Overcast—light wind. Evening, Cloudy.
	T 15	30.239	56.7	30.233	57.6	51	54.6	57.2	52.8	57.7		NE	Overcast—light fog and wind.
	F 16	30.247	54.9	30.208	55.8	49	53.0	55.2	51.6	55.2		E	Overcast—light wind.
	S 17	30.150	54.8	30.099	54.5	48	51.6	53.6	50.3	53.2		E	Overcast—fog and wind.
	⊙ 18	30.156	53.6	30.146	53.6	49	49.8	52.8	47.8	52.3		E	A.M. Overcast—light fog and wind. P.M. Fine—light clouds. Evening, Fine and clear.
	M 19	30.130	49.6	30.018	50.6	38	42.2	49.0	39.9	48.6		ESE	A.M. Thick fog. P.M. Fine and cloudless—light wind. Evening, Cloudy.
	T 20	29.758	49.5	29.681	49.7	37	39.2	48.9	35.4	47.9		ESE	Thick fog. Evening, Overcast—light rain. [—light clouds.
	● W 21	29.633	47.9	29.606	48.8	42	41.3	49.2	38.6	48.6	.116	SSW	A.M. Fine—lt. clds.—fog & wind. P.M. Fine—lt. haze. Ev. Fine
	T 22	29.425	47.5	29.352	48.8	42	46.7	51.9	40.2	51.7		E	A.M. Overcast—very light rain. P.M. Cloudy—light wind. Evening, Overcast—rain and wind.
	F 23	29.507	49.4	29.518	50.2	43	44.8	51.3	41.3	51.6	.285	SW	A.M. Fine & cloudless—lt. haze. P.M. Fine—lt. clds. & wind. Evening, Overcast—heavy rain & wind. [continued rain.
	S 24	29.507	51.0	29.520	52.0	45	49.3	53.4	43.3	53.7	.411	SW	A.M. Cldy.—lt. wind. P.M. Overcast—lt. rain & wind. Ev. Light
	⊙ 25	29.564	51.0	29.315	51.3	45	48.5	51.8	44.8	54.7	.043	S	A.M. Overcast—deposition—lt. wind. P.M. Overcast—heavy rain & wind. Evening, continued rain, with high wind.
	M 26	29.130	52.8	29.128	53.2	46	46.7	50.7	44.0	51.4	.677	S var.	A.M. Fine and cloudless—light haze. P.M. Overcast—light wind. Evening, Cloudy—light rain and wind.
	T 27	29.540	50.5	29.598	51.2	42	42.6	48.7	39.6	48.4	.038	SW	A.M. Fine and cloudless—light haze. P.M. Overcast—light wind. Evening, Cloudy.
	W 28	29.819	47.5	29.833	48.6	41	39.6	47.0	35.7	50.2		SW	Fine & cloudless—lt. haze and wind. Evening, Cloudy—lt. wind.
	T 29	29.699	48.4	29.766	50.3	46	51.2	54.4	39.4	55.2	.183	S	A.M. Overcast—light rain and wind. P.M. Fine and cloudless —light haze. Evening, Lightly cloudy.
	F 30	30.059	48.6	29.990	49.0	41	42.4	48.3	39.3	48.9		SE	A.M. Thick fog. P.M. Overcast—light fog. Evening, Overcast—light steady rain. [rain and wind.
	S 31	29.720	49.7	29.691	50.6	47	50.0	54.3	41.4	54.5	.588	E	Overcast—light rain & wind. Evening, Overcast—light continued
	MEANS..	29.703	53.0	29.675	54.3	46.6	49.2	53.7	45.4	54.5	Sum. 3.821	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.645 / 3 P.M. 29.612

METEOROLOGICAL JOURNAL FOR NOVEMBER AND DECEMBER, 1835.

1835.		9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit.		Self-registering.		Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
		Barom.	Attach. Therm.	Barom.	Attach. Therm.		9 A.M.	3 P.M.	Lowest.	Highest.			
NOVEMBER	⊙ 1	30.016	48.9	30.034	48.8	42	42.2	45.6	40.0	44.6	.038	NNE	A.M. Overcast—light fog and wind. P.M. Thick haze. Evening, Cloudy—light fog.
	M 2	30.094	46.2	30.014	46.7	36	37.7	47.6	34.9	46.7		S var.	A.M. Foggy—light wind. P.M. Fine—light clouds. Evening, Cloudy—rain during the night.
	T 3	29.909	47.6	29.879	48.4	42	46.4	47.6	36.7	47.4	.283	SE	A.M. Overcast—light rain and wind. P.M. Fine—light clouds —haze and wind. Evening, Cloudy.
	W 4	29.913	46.8	29.861	46.7	38	43.0	42.2	41.0	42.7	.041	E	Fine and cloudless—light haze and wind. Evening, Cloudy.
	○ T 5	29.800	43.8	29.760	44.3	35	40.2	42.3	38.2	41.3		E	Overcast—light brisk wind.
	F 6	29.726	43.2	29.728	43.4	35	38.5	41.2	35.2	41.4		E	Thick fog. Evening, Fine and clear.
	S 7	29.880	42.7	29.833	43.8	38	43.2	48.0	36.2	48.9		E	A.M. Lightly overcast—light wind. P.M. Fine light clouds and wind. Evening, Overcast—light rain and wind.
	⊙ 8	29.804	45.6	29.822	46.3	40	43.3	47.3	40.6	46.7	.191	NE	Fine—light clouds and wind. Evening, Fine and clear.
	M 9	29.942	45.3	29.998	44.7	36	40.6	41.0	39.4	40.5		NNE	A.M. Cloudy—light brisk wind. P.M. Overcast—high wind. Evening, Cloudy—very high wind.
	T 10	30.156	42.7	30.188	42.2	30	37.7	37.7	35.6	38.2		E	Cloudy—brisk wind.
	W 11	30.218	40.4	30.168	41.7	32	38.9	43.0	35.2	42.7		N	A.M. Light fog and wind. P.M. Overcast—light wind. Evening, Cloudy.
	T 12	30.039	41.7	30.035	42.9	35	39.8	43.3	36.9	44.2		N	Fine—light clouds, with light brisk wind. Evening, Cloudy.
	F 13	30.200	41.8	30.180	42.7	35	38.8	42.5	36.7	41.6		N var.	Overcast—light rain and wind.
	S 14	30.010	42.3	29.922	43.6	33	39.2	41.9	37.6	42.7	.038	SW	Overcast—light wind. Evening, Overcast—very light rain.
	⊙ 15	29.936	44.0	29.885	44.7	39	42.7	44.4	38.0	44.7		NE	A.M. Overcast—deposition—light wind. P.M. Cloudy—light wind. Rain during the night.
	M 16	29.837	43.7	29.810	47.4	35	40.4	45.2	38.3	44.2	.033	WSW	Overcast—light wind.
	T 17	29.756	45.2	29.712	46.3	40	43.9	48.0	39.3	46.6		WSW	Overcast—very light rain.
	W 18	29.659	46.6	29.538	48.0	42	47.7	52.4	42.8	52.4		SW	Overcast—light wind. Evening, Cloudy.
	T 19	29.841	46.9	29.855	47.6	39	41.6	47.7	39.0	46.4		SW	Fine and cloudless—light haze. Evening, Cloudy.
	● F 20	29.853	47.9	29.792	49.0	42	47.5	50.2	40.0	50.2		SW	A.M. Overcast—light wind. P.M. Fine—light clouds and wind. Evening, Cloudy.
	S 21	29.752	49.7	29.779	50.7	45	50.5	52.2	46.2	52.3		S var.	Overcast—light brisk wind.
	⊙ 22	29.689	51.0	29.615	52.0	47	51.8	53.0	48.6	53.2		S var.	A.M. Overcast—light brisk wind. P.M. Fine—light clouds. Evening, Overcast—light rain.
	M 23	29.685	52.2	29.691	53.4	47	51.5	52.3	50.0	53.7	.213	S var.	A.M. Overcast—light rain and high wind. P.M. Cloudy—light brisk wind. Evening, Fine and clear—high wind.
	T 24	29.705	52.4	29.687	53.0	46	50.7	53.0	46.3	52.7		NE	A.M. Cloudy—light brisk wind. P.M. Fine—light clouds. Evening, Fine and clear—high wind.
	W 25	29.675	52.8	29.617	53.6	46	51.2	54.0	49.3	53.4		ESE	Fine—nearly cloudless—lt. wind. Ev. Cloudy—lt. wind & rain.
	T 26	29.536	53.6	29.332	54.5	49	52.5	55.6	50.3	55.5	.111	ENE	A.M. Cloudy—light wind and rain. P.M. Fine—light clouds and wind. Evening, Overcast—rain and wind.
	F 27	29.251	56.0	29.178	56.0	51	53.8	52.3	52.0	53.8	.105	NE var.	A.M. Overcast—light steady rain and high wind. P.M. Fine and cloudless—light wind. Evening, Overcast—rain & wind.
	S 28	29.182	54.9	29.324	55.0	45	49.2	50.6	47.6	51.3	.408	SW	Overcast—light wind. Evening, Fine and clear.
	⊙ 29	29.382	52.5	29.182	52.6	45	48.7	49.3	44.8	50.6		S var.	Overcast—very light rain.
	M 30	28.976	52.3	28.982	53.6	46	51.7	54.8	47.8	55.3	.255	SE	Overcast—light rain and wind.
	MEANS..	29.807	47.4	29.747	48.1	40.4	44.8	47.5	41.5	47.5	Sum. 1.716	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.765 — 3 P.M. 29.703
DECEMBER	T 1	29.227	52.9	29.215	53.7	47	49.2	52.2	47.0	51.6		SSE	A.M. Fine and cloudless—light haze and wind. P.M. Lightly cloudy—light wind. Evening, Cloudy—high wind.
	W 2	29.232	53.0	29.408	52.9	43	46.6	48.8	44.8	48.8		S	Fine and cloudless—light haze. Evening, Fine and clear.
	T 3	29.540	51.0	29.513	52.0	46	49.4	50.8	43.0	50.6		SSE	Fine—light clouds and wind. Evening, Fine and clear.
	F 4	29.598	51.6	29.643	51.6	45	43.8	45.2	42.5	46.8		SW	A.M. Fine and cloudless—light haze. P.M. Overcast—very light rain. Evening, Fine and clear.
	○ S 5	30.041	48.3	30.065	48.6	41	41.8	46.5	37.0	45.3	.063	SW	Overcast—deposition—light wind. Evening, Fine and clear.
	⊙ 6	30.023	47.4	29.966	47.6	40	42.7	45.5	39.8	44.8		SSW	A.M. Overcast—deposition—light wind. P.M. Thick fog. Evening, Thick fog—deposition.
	M 7	29.986	45.3	29.962	45.4	36	40.4	42.3	36.0	41.8		ESE	Overcast—deposition. Evening, Cloudy.
	T 8	29.936	45.4	29.812	45.3	33	39.2	40.4	37.7	42.3		SSW	Overcast—light fog and rain.
	W 9	29.617	45.5	29.768	45.6	40	42.7	42.4	38.4	43.4	.202	SE var.	Fine—light clouds and wind. Evening, Cloudy.
	T 10	30.172	42.2	30.225	41.7	32	35.8	36.6	31.7	35.4		E	A.M. Overcast—light snow and wind. P.M. Fine and cloudless—light wind. Evening, Fine and clear.
	F 11	30.085	37.6	30.071	37.7	22	29.3	33.0	25.6	34.5		E	Fine and cloudless—light haze & wind. Evening, Fine & clear.
	S 12	30.130	35.3	30.119	36.3	25	31.0	37.6	25.2	36.6		SSW	A.M. Light fog and wind. P.M. Overcast—light wind. Evening, Cloudy.
	⊙ 13	30.158	37.8	30.126	38.2	29	35.2	39.6	29.6	38.6		SW	A.M. Fog and wind. P.M. Fine—light clouds and wind. Evening, Cloudy.
	M 14	30.188	38.8	30.170	39.6	32	38.6	42.4	33.5	40.8		SW	Fog—light wind. Evening, Cloudy—light wind.
	T 15	30.186	41.6	30.154	42.0	35	40.2	42.7	38.3	41.5		SW	Thick fog.
	W 16	30.200	42.9	30.200	43.4	38	41.6	43.2	39.2	42.4		SW	Fog—very light rain and wind.
	T 17	30.196	43.4	30.144	43.4	35	37.8	41.2	36.6	41.6		SW	A.M. Foggy—deposition. P.M. Overcast—light rain.
	F 18	29.928	44.3	29.845	44.7	35	42.3	43.2	37.2	45.3	.038	SW	A.M. Lt. fog & wind. P.M. Lt. clds. & wind. Evening, Cloudy.
	● S 19	29.821	43.5	29.772	42.5	32	35.7	36.3	34.3	35.8	.044	NE var.	A.M. Fine and cloudless—light haze and wind. P.M. Fine—light clouds and wind. Evening, Cloudy.
	⊙ 20	29.794	38.3	29.821	37.7	29	33.2	34.3	31.0	33.3		NE var.	A.M. Ovt.—it. snow, with lt. brk. wind. P.M. Ovt.—lt. sleet & wd.
	M 21	29.978	36.2	30.006	36.3	27	32.5	32.6	30.2	32.2		NE	A.M. Cloudy—light brisk wind. P.M. Fine and cloudless—light brisk wind. Evening, Fine and clear—high wind.
	T 22	30.198	34.6	30.263	35.2	25	28.8	36.3	26.3	34.6		NW	A.M. Fine and cloudless—light haze—light brisk wind. P.M. Fine—light clouds & wind. Evening, Fine and clear.
	W 23	30.400	32.8	30.280	32.7	19	25.7	31.2	24.0	30.2		N	Overcast—sharp frost—light wind. Evening, Cloudy—lt. wind.
	T 24	30.309	33.0	30.259	33.6	23	30.9	30.4	24.4	30.6		WSW	Thick fog throughout the day—sharp frost.
	F 25	30.220	31.7	30.170	31.4	18	22.2	26.7	21.0	24.6		WSW	Thick fog—sharp frost.—Evening, Cloudy—light wind.
	S 26	30.208	30.2	30.168	30.0	22	24.2	27.3	21.5	28.2		SW	Thick fog—sharp frost. Evening, Cloudy.
	⊙ 27	30.101	29.7	30.057	30.7	22	30.2	35.2	22.2	38.8		SSW	A.M. Thick fog. P.M. Fine—light clouds—thaw, and wind. Evening, Cloudy—deposition.
	M 28	29.956	32.8	29.893	35.2	32	40.9	46.3	38.7	45.4		S var.	A.M. Fine—light clouds. P.M. Cloudy—light wind.
	T 29	30.083	37.3	30.095	38.3	32	38.2	42.3	36.3	41.4		SW	Overcast—deposition—light wind.
	W 30	30.031	39.4	30.027	40.2	36	41.6	45.3	37.7	45.3		S	Overcast—very light rain and wind.
	T 31	30.162	39.4	30.142	38.7	32	33.2	34.3	31.3	33.6	.033	SW	Thick fog—frost during the night.
	MEANS..	29.990	40.8	29.979	41.0	30.2	36.9	37.9	33.6	39.6	Sum. .380	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.968 — 3 P.M. 29.956